Light-Front Holography in QCD and Hadronic Physics

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Rencontres de Moriond
QCD and High Energy Interactions
La Thuile, Valle d’Aosta, Italy
March 22-29, 2014

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**Introduction**

- SLAC and DESY 70's: elementary interactions of quarks and gluons at short distances remarkably well described by QCD
- No understanding of large distance strong dynamics of QCD: how quarks and gluons are confined and how hadrons emerge as asymptotic states
- Euclidean lattice important first-principles numerical simulation of nonperturbative QCD: excitation spectrum of hadrons requires enormous computational complexity beyond ground-state configurations
- Only known analytically tractable treatment in relativistic QFTh is perturbation theory
- Important theoretical goal: find initial analytic approximation to strongly coupled QCD, like Schrödinger or Dirac Eqs in atomic physics, corrected for quantum fluctuations
- Convenient frame-independent Hamiltonian framework for treating bound-states in relativistic theories (including well defined massless quark limit) is light-front (LF) quantization
- To a first semiclassical approximation one can reduce the strongly correlated multi-parton LF dynamics to an effective one-dim QFTh, which encodes the conformal symmetry of the classical QCD Lagrangian
• Effective LF theory for QCD endowed with $SO(2, 1)$ algebraic structure follows from one-dim semiclassical approximation to LF dynamics in physical space-time, higher dimensional gravity in AdS$_5$ space and one-dimensional conformal quantum mechanics

• Result is a relativistic LF wave equation with a confining structure which incorporates essential spectroscopic and dynamical features of hadron physics

• Remarkable connection follows from isomorphism of one-dim conformal group $Conf(R^1)$ with $SO(2, 1)$, which is also the isometry group of AdS$_2$

• One of the generators of $SO(2, 1)$ is compact and has therefore a discrete spectrum!

Conformal Quantum Mechanics: de Alfaro, Fubini and Furlan (dAFF)
Dirac Forms of Relativistic Dynamics
[Dirac (1949)]

- Poincaré generators $P^\mu$ and $M^{\mu\nu}$ separated into kinematical and dynamical
- Kinematical generators act along initial hypersurface and contain no interactions
- Dynamical generators are responsible for evolution of the system and depend on the interactions
- Each front has its Hamiltonian and evolve with a different time, but results computed in any front should be identical (different parameterizations of space-time)

- **Instant form:** initial surface defined by $x^0 = 0$: $P^0$, $K$ dynamical, $P$, $J$ kinematical
- **Front form:** initial surface tangent to the light cone $x^+ = x^0 + x^3 = 0$ ($P^\pm = P^0 \pm P^3$)
  
  $P^-, J^x, J^y$ dynamical $P_\perp, J^3, K$ kinematical

- **Point form:** initial surface is the hyperboloid $x^2 = \kappa^2 > 0$, $x^0 > 0$: $P^\mu$ dynamical, $M^{\mu\nu}$ kinematical
Light-Front Dynamics

- Hadron with 4-momentum $P = (P^+, P^-, P_\perp)$, $P^\pm = P^0 \pm P^3$, mass-shell relation $P_\mu P^\mu = M^2$ leads to LF Hamiltonian equation

$$P^- |\psi(P)\rangle = \frac{M^2 + P_\perp^2}{P^+} |\psi(P)\rangle$$

- Construct LF invariant Hamiltonian

$$P_\mu P^\mu = P^- P^+ - P_\perp^2 \quad (P^+ \text{ and } P_\perp \text{ kinematical})$$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

- Longitudinal momentum $P^+$ is kinematical: sum of single particle constituents $p^+_i$ of bound state, $P^+ = \sum_i p^+_i$, $p^+_i > 0$

- Bound-state is arbitrarily off the LF energy shell, $P^- = -\sum_i p^-_i < 0$

- Vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$

- Simple structure of vacuum allows definition of partonic content of hadron in terms of wavefunctions: quantum-mechanical probabilistic interpretation of hadronic states
Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Reduce effectively LF multiparticle dynamics to a 1-dim QFT with no particle creation and absorption: LF quantum mechanics!
- Central problem is derivation of effective interaction which acts only on the valence sector: express higher Fock states as functionals of the lower ones
- Compute $M^2$ from hadronic matrix element
  \[ \langle \psi'(P') | P_\mu P^\mu | \psi(P) \rangle = M^2 \langle \psi'(P') | \psi(P) \rangle \]
- Semiclassical approximation
  \[
  \psi_n(k_1, k_2, \ldots, k_n) \rightarrow \phi_n \left( \frac{(k_1 + k_2 + \cdots + k_n)^2}{x} \right), \quad m_q \rightarrow 0
  \]
  \[
  M_n^2 = \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i} \quad \text{(Invariant mass)}
  \]
  \[ M^2 - M_n^2 \] is the measure of the off-energy shell: key variable which controls the bound state
- For a two-parton bound-state in the $m_q \rightarrow 0$ case
  \[
  M_{qq}^2 = \frac{k_{\perp}^2}{x(1-x)}
  \]
• Conjugate invariant variable in transverse impact space is

\[ \zeta^2 = x(1 - x)b_{\perp} \]

• To first approximation LF dynamics depend only on the invariant variable \( \zeta \), and dynamical properties are encoded in the hadronic mode \( \phi(\zeta) \)

\[
\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}},
\]

where we factor out the longitudinal \( X(x) \) and orbital kinematical dependence from LFWF \( \psi \)

• Ultra relativistic limit \( m_q \to 0 \) longitudinal modes \( X(x) \) decouple \( (L = L^z) \)

\[
\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta),
\]

where effective potential \( U \) includes all interaction terms upon integration of the higher Fock states
• LF eigenvalue equation \( P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle \) is a LF wave equation for \( \phi \)

\[
\begin{pmatrix}
-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \\
\text{kinetic energy of partons}
\end{pmatrix}
+ U(\zeta)
\text{confinement}
\phi(\zeta) = M^2 \phi(\zeta)
\]

• Critical value \( L = 0 \) corresponds to lowest possible stable solution, the ground state of the LF Hamiltonian

• Relativistic and frame-independent LF Schrödinger equation: \( U \) is instantaneous in LF time

• A linear potential \( V_{eff} \) in the \textit{instant form} implies a quadratic potential \( U_{eff} \) in the \textit{front form} at large \( q\bar{q} \) separation (thus linear Regge trajectories for small quark masses!)

\[
U_{eff} = V_{eff}^2 + 2\sqrt{p^2 + m_q^2} V_{eff} + 2 V_{eff} \sqrt{p^2 + m_q^2}
\]

• Result follows from comparison of invariant mass in the \textit{instant form} in the CMS, \( P = 0 \), with invariant mass in \textit{front form} in the constituent rest frame (CRF): \( p_q + p_{\bar{q}} = 0 \)

Conformal Quantum Mechanics and Light Front Dynamics

[S. J. Brodsky, GdT and H.G. Dosch, PLB 729, 3 (2014)]

- Incorporate in a 1-dim QFT – as an effective theory, the fundamental conformal symmetry of the 4-dim classical QCD Lagrangian in the limit of massless quarks

- Invariance properties of 1-dim field theory under the full conformal group from dAFF action
  [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A 34, 569 (1976)]

\[ S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{g}{Q^2} \right), \quad g \text{ dimensionless} \]

- Absence of dimensional constants implies action is invariant under a large group of transformations, the general conformal group

\[ t' = \frac{\alpha t + \beta}{\gamma t + \delta}, \quad Q'(t') = \frac{Q(t)}{\gamma t + \delta}, \quad \alpha \delta - \beta \gamma = 1 \]

I. Translations in t: \( H = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) \),

II. Dilatations: \( D = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) t - \frac{1}{4} \left( \dot{Q} Q + Q \dot{Q} \right) \),

III. Special conformal transformations: \( K = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) t^2 - \frac{1}{2} \left( \dot{Q} Q + Q \dot{Q} \right) t + \frac{1}{2} Q^2, \)
• Using canonical commutation relations \([Q(t), \dot{Q}(t)] = i\) find

\[
[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK,
\]

the algebra of the generators of the conformal group \(Conf(R^1)\)

• Introduce the linear combinations

\[
J^{12} = \frac{1}{2} \left( \frac{1}{a} K + aH \right), \quad J^{01} = \frac{1}{2} \left( \frac{1}{a} K - aH \right), \quad J^{02} = D,
\]

where \(a\) has dimension \(t\) since \(H\) and \(K\) have different dimensions

• Generators \(J\) have commutation relations

\[
[J^{12}, J^{01}] = iJ^{02}, \quad [J^{12}, J^{02}] = -iJ^{01}, \quad [J^{01}, J^{02}] = -iJ^{12},
\]

the algebra of \(SO(2, 1)\)

• \(J^{0i}, \ i = 1, 2\), boost in space direction \(i\) and \(J^{12}\) rotation in the (1,2) plane

• \(J^{12}\) is compact and has thus discrete spectrum with normalizable eigenfunctions

• The relation between the generators of conformal group and generators of \(SO(2, 1)\) suggests that the scale \(a\) may play a fundamental role
dAFF construct a generator as a superposition of the 3 constants of motion

\[ G = uH + vD + wK \]

and introduce new time variable \( \tau \)

\[ d\tau = \frac{dt}{u + vt + wt^2} \]

Find usual quantum mechanical evolution for time \( \tau \)

\[ G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle \]

with the new Hamiltonian \( G' \)

\[ G' = \frac{1}{2}u \left( \dot{Q}^2 + \frac{g}{Q^2} \right) - \frac{1}{4}v \left( Q\ddot{Q} + \dot{Q}Q \right) + \frac{1}{2}wQ^2 \]

Scale appears in the Hamiltonian without affecting the conformal invariance of the action!

Operator \( G' \) is compact for

\[ \frac{4uw - v^2}{4} > 0 \]

Bound states!
Connection to Light-Front Dynamics

• The Schrödinger picture follows from the representation of $Q$ and $P = \dot{Q}$

\[ Q \to x, \quad \dot{Q} \to -i \frac{d}{dx} \]

\[ G = \frac{1}{2} u \left( -\frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4} v \left( x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2} wx^2. \]

• Compare dAFF Hamiltonian $G$ with LF Hamiltonian $P^2$

\[ P^2 = -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \]

and identify dAFF variable $x$ with LF invariant variable $\zeta$

• $u = 2, \ v = 0, \ g = L^2 - \frac{1}{4}$ from kinematical constraints

• $w = 2\lambda^2 \sim \frac{1}{a^2}$ fixes the LF potential to quadratic $\lambda^2 \zeta^2$ dependence

\[ U \sim \lambda^2 \zeta^2 \]
Gravity in AdS and Light Front Holographic Mapping

\[ R_{N K L M} = - \frac{1}{R^2} (g_{N L} g_{K M} - g_{N M} g_{K L}) \]

- Description of strongly coupled gauge theory using a dual gravity description in a higher dimensional space (holographic)

- Why is AdS space important? AdS\(_5\) is a 5-dim space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- Isomorphism of \( SO(4, 2) \) group of conformal transformations with generators \( P^\mu, M^{\mu\nu}, K^\mu, D \) with the group of isometries of AdS\(_5\)

- AdS\(_5\) metric \( x^M = (x^\mu, z) \):
  \[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \]

- Since the AdS metric is invariant under a dilatation of all coordinates \( x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z \), the variable \( z \) acts like a scaling variable in Minkowski space

- Short distances \( x_\mu x^\mu \rightarrow 0 \) map to UV conformal AdS\(_5\) boundary \( z \rightarrow 0 \)

- Large confinement dimensions \( x_\mu x^\mu \sim 1/\Lambda^2_{\text{QCD}} \) map to IR region of AdS\(_5\), \( z \sim 1/\Lambda_{\text{QCD}} \), thus AdS geometry has to be modified at large \( z \) to include the scale of strong interactions
Higher Integer-Spin Wave Equations in AdS Space

[GdT, H.G. Dosch and S. J. Brodsky, PRD 87, 075004 (2013)]

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Integer spin-$J$ fields in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{\text{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \left( g^{M M'} \, D_M \Phi^*_{N_1 \cdots N_J} \, D_{M'} \Phi_{N_1' \cdots N_J'} - \mu_{\text{eff}}^2(z) \, \Phi^*_{N_1 \cdots N_J} \, \Phi_{N_1' \cdots N_J'} \right)$$

$D_M$ is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ breaks conformality

- Effective mass $\mu_{\text{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement
• Physical hadron has plane-wave and polarization indices along 3+1 physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable $z$

$$\Phi_{\mu_1...\mu_J}(x,z) = e^{iP \cdot x} \Phi_{\mu_1...\mu_J}(z), \quad \Phi_{\mu_2...\mu_J} = \cdots = \Phi_{\mu_1\mu_2...z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu = M^2$

• Further simplification by using a local Lorentz frame with tangent indices

• Variation of the action gives AdS wave equation for spin-$J$ field $\Phi(z)_{\nu_1...\nu_J} = \Phi_J(z) \epsilon_{\nu_1...\nu_J}$

$$\left[ -z^{d-1-2J} \frac{\partial}{\partial z} \left( \frac{e^\varphi(z)}{z^{d-1-2J}} \partial_z \right) + \left( \frac{mR}{z} \right)^2 \right] \Phi_J = M^2 \Phi_J$$

with

$$(mR)^2 = (\mu_{\text{eff}}(z) R)^2 - Jz \varphi'(z) + J(d - J + 1)$$

and the kinematical constraints

$$\eta^{\mu\nu} P_\mu \epsilon_{\nu_2...\nu_J} = 0, \quad \eta^{\mu\nu} \epsilon_{\mu\nu_3...\nu_J} = 0.$$

• Kinematical constrains in the LF imply that $m$ must be a constant

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Light-Front Mapping

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Upon substitution \( \Phi_J(z) \sim z^{(d-1)/2-J}e^{-\varphi(z)/2} \phi_J(z) \) and \( z \to \zeta \) in AdS WE

\[
\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{mR}{z} \right)^2 \right] \Phi_J(z) = M^2 \Phi_J(z)
\]

we find LFWE \((d = 4)\)

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)
\]

with

\[
U(\zeta) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{4} \varphi'(\zeta)^2 + \frac{2J - 3}{2z} \varphi'(\zeta)
\]

and \( (mR)^2 = -(2 - J)^2 + L^2 \)

- Unmodified AdS equations correspond to the kinetic energy terms of the partons inside a hadron
- Interaction terms in the QCD Lagrangian build the effective confining potential \( U(\zeta) \) and correspond to the truncation of AdS space in an effective dual gravity approximation
- AdS Breitenlohner-Freedman bound \((mR)^2 \geq -4\) equivalent to LF QM stability condition \( L^2 \geq 0 \)
Meson Spectrum

- Dilatonic profile in the dual gravity model determined from one-dim QFT (dAFF)
  \[ \phi(z) = \lambda z^2, \quad \lambda^2 = w/2 \]

- Effective potential:
  \[ U = \lambda^2 \zeta^2 + 2\lambda(J - 1) \]

- LFWE
  \[ \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \lambda^2 \zeta^2 + 2\lambda(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta) \]

- Normalized eigenfunctions
  \[ \langle \phi | \phi \rangle = \int d\zeta \phi^2(z) = 1 \]

  \[ \phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2) \]

- Eigenvalues for \( \lambda > 0 \)
  \[ M^2_{n,J,L} = 4\lambda \left( n + \frac{J + L}{2} \right) \]

- Results are easily extended to light quarks

- \( \lambda < 0 \) incompatible with LF constituent interpretation
Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)
Higher Half-Integer Spin Wave Equations in AdS Space

[J. Polchinski and M. J. Strassler, JHEP 0305, 012 (2003)]
[GdT and S. J. Brodsky, PRL 94, 201601 (2005)]
[GdT, H.G. Dosch and S. J. Brodsky, PRD 87, 075004 (2013)]

- The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods: analytical exploration of systematics of light-baryon resonances
- Extension of holographic ideas to spin-\(\frac{1}{2}\) (and higher half-integral \(J\)) hadrons by considering wave equations for Rarita-Schwinger spinor fields in AdS space and their mapping to light-front physics
- LF clustering decomposition of invariant variable \(\zeta\): same multiplicity of states for mesons and baryons (spectators vs active quark)
- But in contrast with mesons there is important degeneracy of states along a given Regge trajectory for a given \(L\): no spin-orbit coupling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
• Half-integer spin $J = T + \frac{1}{2}$ conveniently represented by RS spinor $[\Psi_{N_1...N_T}]_{\alpha}$ with effective AdS action

$$S_{\text{eff}} = \frac{1}{2} \int d^dx \, dz \, \sqrt{|g|} \, g^{N_1 N'_1} \ldots g^{N_T N'_T}$$

$$\left[ \Psi_{N_1...N_T} \left( i \Gamma^A e^M_A \, D_M - \mu - U(z) \right) \Psi_{N'_1...N'_T} + \text{h.c.} \right]$$

where the covariant derivative $D_M$ includes the affine connection and the spin connection

• $e^A_M$ is the vielbein and $\Gamma^A$ tangent space Dirac matrices $\{ \Gamma^A, \Gamma^B \} = \eta^{AB}$

• LF mapping $z \rightarrow \zeta$ find coupled LFWE

$$- \frac{d}{d\zeta} \psi_- - \frac{\nu + \frac{1}{2}}{\zeta} \psi_- - V(\zeta) \psi_- = M \psi_+$$

$$\frac{d}{d\zeta} \psi_+ - \frac{\nu + \frac{1}{2}}{\zeta} \psi_+ - V(\zeta) \psi_+ = M \psi_-$$

provided that $|\mu R| = \nu + \frac{1}{2}$ and

$$V(\zeta) = \frac{R}{\zeta} U(\zeta)$$

a $J$-independent potential – No spin-orbit coupling along a given trajectory!
**Baryon Spectrum**

- Choose linear potential \( V = \lambda \zeta, \quad \lambda > 0 \) to satisfy dAFF
- Eigenfunctions
  \[
  \psi_+ (\zeta) \sim \zeta^{3/2+\nu} e^{-\lambda \zeta^2/2} L_n^\nu (\lambda \zeta^2), \quad \psi_- (\zeta) \sim \zeta^{3/2+\nu} e^{-\lambda \zeta^2/2} L_{n+1}^\nu (\lambda \zeta^2)
  \]
- Eigenvalues: \( M^2 = 4\lambda(n+\nu+1) \)
- Lowest possible state \( n = 0 \) and \( \nu = 0 \): orbital excitations \( \nu = 0, 1, 2 \cdots = L \)
- \( L \) is the relative LF angular momentum between the active quark and spectator cluster
- \( \nu \) depends on internal spin and parity
  
  The assignment
  
  \[
  \begin{array}{l|ll}
  S & 1/2 & 3/2 \\
  \hline
  \nu = L & \nu = L + \frac{1}{2}
  \\
  \nu = L + \frac{1}{2} & \nu = L + 1
  \end{array}
  \]

  describes the full light baryon orbital and radial excitation spectrum
Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)
Thanks!