PROBING THE TRANSVERSE DYNAMICS AND POLARIZATION OF
GLUONS INSIDE THE PROTON AT THE LHC

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Transverse momentum dependent gluon distributions encode fundamental information on the
structure of the proton. Here we show how they can be accessed in heavy quarkonium pro-
duction in proton-proton collisions at the LHC. In particular, their first determination could
come from the study of an isolated $J/\psi$ or $\Upsilon$ particle, produced back to back with a photon.

1 Formalism

Transverse momentum dependent (TMD) gluon distributions inside an unpolarized proton are
defined by the hadron matrix element of a correlator of the gluon field strengths
$F^{\mu\rho}(0)$ and $F^{\nu\sigma}(\xi)$. Expanding the gluon four-momentum as
$p = xP + p_\perp + p^\parallel n$, with $n$ being a lightlike
vector conjugate to the momentum of the proton $P$, such correlator can be written as

$$
\Phi^{\mu\nu}(x,P_\perp) = \frac{n_\rho n_\sigma}{P_\cdot n} \int \frac{d^2 \xi}{(2\pi)^2} \frac{d^2 \xi_T}{2\pi} e^{ip_\cdot \xi} \langle P | \text{Tr} \left[ F^{\mu\rho}(0) F^{\nu\sigma}(\xi) \right] | P \rangle|_{\xi_T=0} = -\frac{1}{2x} \left\{ g^{\mu\nu} f^g_1(x,P_\perp^2) - \left( \frac{p_\perp^2 p_\perp^\mu}{M_p^2} + g_\perp^\mu \frac{P_\perp^2}{2M_p^2} \right) h_1^{\perp g}(x,P_\perp^2) \right\},
$$

(1)

where gauge links have been omitted. The transverse projector $g^{\mu\nu}$ is defined as $g^{\mu\nu} = g^{\mu\nu} - P^\mu n^\nu/P \cdot n - n^\mu P^\nu/P \cdot n$. Moreover, $p_\perp^2 = -p_\perp^2$ and $M_p$ is the proton mass. The gluon correlator of
an unpolarized proton can therefore be expressed in terms of two independent TMD distribution
functions: $f^g_1(x,P_\perp^2)$ is the unpolarized one, while $h_1^{\perp g}(x,P_\perp^2)$ denotes the $T$-even, helicity-flip
distribution of linearly polarized gluons, which satisfies the model-independent positivity bound

$$
\frac{P_\perp^2}{2M_p^2} |h_1^{\perp g}(x,P_\perp^2)| \leq f^g_1(x,P_\perp^2).
$$

Like any TMD distribution, $h_1^{\perp g}$ might receive contributions from initial and final state interactions
that can render it nonuniversal and even hamper its extraction in processes for which TMD
factorization does not apply.

2 Phenomenology

Several processes have been suggested to measure the experimentally unknown distributions $f^g_1$
and $h_1^{\perp g}$. Although it has been discussed how to isolate the contribution from $h_1^{\perp g}$ by means of
an azimuthal angular dependent weighting of the cross section for dijet production in hadronic
collisions, TMD factorization is expected to be broken in this case due to the presence of both
initial and final state interactions. A theoretically cleaner and safer way would be to study dijet or heavy quark pair production in electron-proton collisions, for instance at a future Electron-Ion Collider. Another process where the problem of factorization breaking is absent is \( pp \to \gamma \gamma X \), which however suffers from a huge background from \( \pi^0 \) decays and contaminations from quark-induced channels.

In the following we show how TMD gluon distributions can be probed in heavy quarkonium production at the LHC. TMD factorization should hold in this case, provided that the two quarks that form the bound state are produced in a colorless state already at short distances.

### 2.1 Transverse momentum distributions of \( C = + \) quarkonia

We consider first the process \( p(P_A) + p(P_B) \to Q(q) + X \), where \( Q \) is a heavy quark-antiquark bound state with \( C = + \), and the four-momenta of the particles are given between brackets. Assuming TMD factorization, the corresponding cross section can be written as

\[
\frac{d\sigma}{d\eta(Q)} = \frac{1}{2s} \frac{d^3q}{(2\pi)^3 2q^0} \int dx_a \, dx_b \, d^2p_{aT} d^2p_{bT} \frac{1}{(2\pi)^4} \delta^4(p_a + p_b - q) \times \Phi_{gT}(x_a, p_{aT}) \Phi_{gT}(x_b, p_{bT}) \sum_{\text{colors}} A_{\mu\rho} A_{\nu\sigma}^* (p_a, p_b; q),
\]

with \( s = (P_A + P_B)^2 \) being the total energy squared in the hadronic center-of-mass frame and \( A \) denoting the hard scattering amplitude of the dominant subprocess \( g(p_a) + g(p_b) \to Q(q) \). The amplitude \( A \) is evaluated at order \( \alpha_s^2 \) within the framework of the color-singlet model. Color octet contributions should be negligible, according to nonrelativistic QCD arguments. For small transverse momentum, \( q_T^2 \ll M_Q^2 \), with \( M_Q \) being the quarkonium mass, the resulting transverse momentum distributions for \( \eta_Q \) and \( \chi_{Q0,2} \) (\( Q = c, b \)) are given by

\[
\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dy \, dq_T^2} = \frac{C[f_{1g}^g f_{1g}^g]}{\int dq_T^2 \, C[f_{1g}^g f_{1g}^g]} \left[ 1 - R(q_T^2) \right],
\]

\[
\frac{1}{\sigma(\chi_Q)} \frac{d\sigma(\chi_Q)}{dy \, dq_T^2} = \frac{C[f_{1g}^g f_{1g}^g]}{\int dq_T^2 \, C[f_{1g}^g f_{1g}^g]} \left[ 1 + R(q_T^2) \right],
\]

\[
\frac{1}{\sigma(\chi_Q)} \frac{d\sigma(\chi_Q)}{dy \, dq_T^2} = \frac{C[f_{1g}^g f_{1g}^g]}{\int dq_T^2 \, C[f_{1g}^g f_{1g}^g]},
\]

where \( \sigma = \int dq_T^2 \, d\sigma \) and \( y \) is the rapidity of the quarkonium along the direction of the incoming protons. Furthermore, \( x_{a,b} = M_Q/\sqrt{s} e^{\pm y} \),

\[
R(q_T^2) = \frac{C[w \, h_{1g}^1 g \, h_{1g}^1 g]}{C[f_{1g}^g f_{1g}^g]}, \quad w = \frac{1}{2M^4} \left( p_{aT} \cdot p_{bT} \right)^2 - \frac{1}{2} p_{aT}^2 p_{bT}^2.
\]

and we have used the following definition of convolution of two TMD distributions \( f \) and \( g \),

\[
C[w \, f \, g] = \int d^2p_{aT} \int d^2p_{bT} \, \delta^2(p_{aT} + p_{bT} - q_T) \, w(p_{aT}, p_{bT}) f(x_a, p_{aT}^2) g(x_b, p_{bT}^2).
\]

Our numerical estimates are shown in Fig.1, where we have assumed that the gluon distributions have a simple Gaussian dependence on transverse momentum. Namely,

\[
f_{1g}^g(x, p_T^2) = f_{1g}^g(x) \frac{2(1 - r)}{r} \exp \left( -\frac{p_T^2}{\langle p_T^2 \rangle} \right),
\]

where \( f_{1g}^g(x) \) is the collinear gluon distribution and the width \( \langle p_T^2 \rangle \) is taken to be independent of \( x \) and the energy scale, set by \( M_Q \). The bound in Eq. (2) is satisfied, although not everywhere saturated, by the form

\[
h_{1g}^g(x, p_T^2) = \frac{2M^2 f_{1g}^g(x) 2(1 - r)}{1/r \, \langle p_T^2 \rangle} \exp \left( -\frac{1 - 1/r \, p_T^2}{\langle p_T^2 \rangle} \right),
\]
with $0 < r < 1$. The distributions for $\eta_{c,b}$ and $\chi_{c,b0}$ are similar to the ones for a pseudoscalar and a scalar Higgs boson, and can be used to extract $h_1^{+g}$, while $f_1^{qg}$ can be accessed by looking at $\chi_{c,b2}$. A comparison among the different spectra could help to cancel out uncertainties. This experiment requires forward detectors like the LHCb, which hopefully will be able to provide such data in the near future.

2.2 $C = -$ quarkonium production in association with a photon

Along the lines of the previous section, we study the process $p(P_A) + p(P_B) \to Q(P_Q) + \gamma(P_\gamma) + X$, where now $Q$ is a $C = -$ quarkonium ($J/\psi$ or $\Upsilon$) produced almost back to back with the photon. Hence the imbalance $q_T = P_{Q_T} + P_{\gamma_T}$ will be small, but not the individual transverse momenta of the two particles. No forward detector is therefore needed in this case. The cross section has the following structure,

$$\frac{d\sigma}{dQdYd^2q_Td\Omega} \propto F_1 C \left[ f_1^{qg} f_1^{qg} \right] + F_3 C \left[ w_3 f_1^{qg} h_1^{+g} + x_a \leftrightarrow x_b \right] \cos 2\phi + F_4 C \left[ w_4 h_1^{+g} h_1^{+g} \right] \cos 4\phi,$$

where $Q$ and $Y$ are the invariant mass and the rapidity of the pair, to be measured, like $q_T$, in the hadronic center-of-mass frame. On the other hand, the solid angle $\Omega = (\theta, \phi)$ is measured in the Collins-Soper frame, where the final pair is at rest and the $\hat{x}\hat{z}$-plane is spanned by $P_A$ and $P_B$, with the $\hat{x}$-axis set by their bisector. The transverse weights are given by

$$w_3 = \frac{Q^2 P_{\gamma T}^2 - 2(q_T \cdot P_{\gamma T})^2}{2M_p^2 q_T^2}, \quad w_4 = 2 \left[ \frac{P_{\alpha T} \cdot P_{\gamma T}}{2M_p^2 q_T^2} - \frac{(P_{\alpha T} \cdot q_T)(P_{\gamma T} \cdot q_T)}{M_p^2 q_T^2} \right]^2 - \frac{P_{\alpha T}^2 P_{\gamma T}^2}{4M_p^4},$$

and the light-cone momentum fractions are $x_{a,b} = \exp[\pm Y] Q/\sqrt{s}$. Explicit expressions for $F_{1,3,4}$ can be found elsewhere. We propose the measurement of the following three observables,

$$S_{q_T}^{(n)} = \frac{\int d\phi \cos(n \phi) \frac{d\sigma}{dQdYd^2q_Td\Omega}}{\int dq_T^2 \int d\phi \frac{d\sigma}{dQdYd^2q_Td\Omega}},$$

Figure 1 – Transverse momentum distributions for $C = +$ quarkonia evaluated at $y = 0$, obtained using the input distributions in Eqs. (7)-(8), with $\langle p_T^2 \rangle = 1$ GeV$^2$ and two different values of $r$: $r = 2/3$ (left) and $r = 1/3$ (right).
with $n = 0, 2, 4,$ and where the $q^2$ integration in the denominator is up to $(Q/2)^2$. In this way we are able to single out the three terms in Eq. 9:

$$S_{qT}^{(0)} = \frac{C[f_1^g f_1^g]}{\int dq_T^2 C[f_1^g f_1^g]}, \quad S_{qT}^{(2)} = \frac{F_3 C[w_3 f_1^g h_1^g + x_a \leftrightarrow x_b]}{2F_1 \int dq_T^2 C[f_1^g f_1^g]}, \quad S_{qT}^{(4)} = \frac{F_4 C[w_4 f_1^g h_1^g + h_1^g]}{2F_1 \int dq_T^2 C[f_1^g f_1^g]}. \quad (12)$$

Our model predictions are presented in Fig. 2 for $\Upsilon + \gamma$ production, in a kinematic region where color octet contributions are suppressed. The size of $S_{qT}^{(0)}$ should be sufficient to allow for a determination of the shape of $f_1^g$ as a function of $q_T$. Since $S_{qT}^{(2)}$ and $S_{qT}^{(4)}$ are considerably smaller, one would need to integrate them over $q_T^2$ [up to $(Q/2)^2$] to get at least an experimental evidence of a nonzero $h_1^g$.

![Figure 2](image)

Figure 2 – Model predictions for $S_{qT}^{(0)}$, $S_{qT}^{(2)}$, and $S_{qT}^{(4)}$ for the process $p(P_A) + p(P_B) \rightarrow Q(P_Q) + \gamma(P_\gamma) + X$ at $\sqrt{s} = 14$ TeV in the kinematic region defined by $Q = 20$ GeV, $Y = 0$, $\theta = \pi/2$, and $x_\alpha = x_\beta \simeq 1.4 \times 10^{-3}$.

3 Conclusions

The distribution of linearly polarized gluons inside an unpolarized proton $h_1^g$ leads to a modulation of the transverse momentum distribution of scalar ($\chi_{c0}, \chi_{b0}$) and pseudoscalar ($\eta_c, \eta_b$) quarkonia that depends on their parity. It does not contribute to the transverse spectra of $\chi_{c2}$ and $\chi_{b2}$, which can be used to probe the unpolarized gluon distribution $f_1^g$. No angular analysis is needed for such measurements and experimental opportunities are offered by LHCb and the proposed fixed-target experiment AFTER at LHC. Furthermore, a first determination of $h_1^g$ and $f_1^g$ could come from $J/\psi(\Upsilon) + \gamma$ production at the running experiments at the LHC, where yields are large enough to perform these analyses with existing data at $\sqrt{s} = 7$ and 8 TeV. We have shown that, together with similar studies in the Higgs sector, quarkonium production can be used to extract gluon TMDs and investigate their process and energy scale dependences.

Acknowledgments

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References