Differential top-quark pair production at NNLO

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Top-quark pairs at the LHC

- Total cross section measured (~ 7 %)
  \[ \sigma_{tt}(13 \text{ TeV}) \approx (803 \pm 50) \text{pb} \]
  [Talk: P. Ferreira da Silva (last week)]

- Prediction (NNLO+NNLL)
  - ~ 3 % uncertainty

- Computational methods
  - Complete new implementation of event generator STRIPPER

- Partial results exist using independent approaches
  - [Abelof, Dekkers, GermannDeRidder, '11-'15]
  - [Bonciani, Catani, Grazzini, Sargsyan, Torre; '14. '15]
Differential top-quark pair production @ 8 TeV

[Czakon,DH, Mitov; published 2015]
Differential Distributions @ 8 TeV

- NNLO has important impact (Good perturbative convergence)
- Good agreement with data → [CMS 2015, ATLAS 2015]
- However: Results with fixed scales applicable only to limited kinematical range
Differential Distributions @ 8 TeV

- Indeed: Direct interpolation of fixed scales to higher values leads to worse convergence
- But: Boosted regime, 13 TeV
  - wider kinematic range required
- 2 main Improvements
  - Technical: Enhance statistics in the Monte Carlo
  - Conceptual: Choose the “right” dynamical scale to maintain perturbative convergence
What is the “right” scale?
Scale dependence – Theoretical uncertainty

- Hadronic cross section
  \[
  \sigma_{h_1 h_2}(P_1, P_2) = \sum_{ab} \int_0^1 dx_1 dx_2 \, f_{a/h_1}(x_1, \mu_F^2) \, f_{b/h_2}(x_2, \mu_F^2) \, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)
  \]

- Reduction of scale dependence by expanding partonic cross section in \( \alpha_s \)

  \[
  \hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)}
  \]

- Theoretical uncertainty ↔ dependence on scale (only ambiguity in fixed order pQCD)
- Monte Carlo allows to pick a dynamical scale
- How can we determine the “right” scale for top-quark pair production?
Scale dependence

Problem

- A number of dynamical scales have been used in the past (NLO, resummation)
- However: picking a dynamical scale would not solve the problem because:
  - Difference between different dynamical scales could be as large as difference between dynamical scale and fixed scale
  - Effect never been studied before
- Needed: Comparative study of perturbative convergence based on different scales

Plan for action

- Selection of the “correct” scale is based on the following criteria:
  - Perturbative convergence for both total and differential cross section
  - Limiting behavior: Low $p_T (m_{tt})$: $\sim m_{top}$ ↔ High $p_T (m_{tt})$: $\sim p_T$
  - Restriction to simple functional forms studied in the past ($H_T$, $m_{tt}$, ... )

[Czakon, DH, Mitov; in preparation]
Scale dependence – Total cross section

• Look for convergence
  • Scale value which minimizes difference
    • NLO → NNLO → (NNLO + NNLL)
  • Best convergence: \( \mu_0 < m_{\text{top}} \)
  • Little dependence on PDFset at NNLO

• Value of NNLO cross section at point of best convergence equals the NNLO+NNLL at the usual canonical scale \( \mu_0 = m_{\text{top}} \)

→ Therefore: Resummation has negligible impact on the total cross section at the point of fastest convergence
Scale dependence – Differential Distributions

[Czakon, DH, Mitov; in preparation]

- Main guidance is perturbative convergence to discriminate between scales

- Invariant mass distribution
  \[ \mu_0 = \frac{H_T}{4} \]
  \[ H_T = \sqrt{m_t^2 + p_T^2} + \sqrt{m_{t\bar{t}}^2 + p_{T\bar{t}}^2} \]

- Limiting behaviour
  \[ \mu_0(p_T \to 0) \to \frac{m_t}{2} \]
  \[ \mu_0(p_T \to \infty) \to \frac{(p_{T,t} + p_{T,\bar{t}})}{4} \]

- Scales based on the invariant mass itself
  \[ \mu \propto m_{t\bar{t}} \]
Scale dependence – Differential Distributions

- Main guidance is perturbative convergence to discriminate between scales
  - Choose individual scales for top and antitop $p_T$
  - Transverse mass scale

$$\mu_0 = \frac{1}{2} m_T(t/\bar{t}) = \frac{1}{2} \sqrt{m_t^2 + p_{T,t/\bar{t}}^2}$$

- Average distributions afterwards

Different scale choices for different observables

[Diagram showing scale dependence with different colors for NNLO, NLO, and LO]

[Caption: Preliminary]

[Czakon, DH, Mitov; in preparation]
Scale dependence – Differential Distributions

- Comparison between different scale choices
- Difference within uncertainty
- Main impact on scale dependence at high values and the K-factor

[Czakon, DH, Mitov; in preparation]

[Preliminary]
Scale dependence – Conclusion

Based on our study the following dynamical scales are chosen for top-pairs:

\[ \mu = \frac{H_T}{4} \quad \text{for} \quad m_{t\bar{t}}, y_t, y_{\bar{t}}, y_{\text{avet}}, y_{t\bar{t}}, \ldots \]

\[ \mu = \frac{1}{2} m_T(t/\bar{t}) \quad \text{for} \quad \rho_T, t, \rho_{T,\bar{t}}, \rho_{T,\text{avet}} \]

Additional feature: The numerical value of the total cross section is (~) equal to the best prediction at NNLO+NNLL at \( \mu_0 = m_{\text{top}} \) (rel. Difference ~ 0.5 %)
New: Differential top-quark pair production @ 13 TeV

- Dynamical scales
- Covers multi-TeV range

[Czakon, DH, Mitov; in preparation]
Differential Distributions @ 13 TeV

PT of the top

Rapidity of the top

Invariant mass

Dynamical scales → extended kinematical regime

• Preliminary comparison with data → Good agreement

[Talk: P. Ferreira da Silva (last week)]
Summary and Outlook

• Differential distributions for top-quark pairs at 8 TeV and 13 TeV at NNLO
  • Dynamical scales, multi-TeV range → available

• Comparative study of different dynamical scales
  • Conclusion:

\[
\mu = \frac{H_T}{4} \quad \text{for} \quad m_t\bar{t}, \; y_t, \; y_{\bar{t}}, \; y_{avt}, \; y_{t\bar{t}}, \ldots
\]

\[
\mu = \frac{1}{2} m_T(t/\bar{t}) \quad \text{for} \quad \not{p}_T, t, \; \not{p}_T, \bar{t}, \; \not{p}_T, avt
\]

• Outlook
  • Combination with EW-NLO
  • Inclusion of top-quark decay in narrow width approximation at NNLO

[Brucherseifer, Caola, Melnikov; '13]  [Gao, Li, Zhu; '12]
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[Brucherseifer, Caola, Melnikov; ‘13]
[Gao, Li, Zhu; ‘12]

Thank you for your attention!
Differential Distributions @ 8 TeV

- Comparison with data
Scale dependence – Total cross section

- PDF dependence

\[ \frac{\sigma(\mu)}{\sigma(\mu_0)} \]

\[ \text{PP} \to t\bar{t}+X \ (8 \text{ TeV}) \]
\[ m_t = 173.3 \text{ GeV} \]
\[ \mu_0 = m_t \]

NNPDF3.0

\[ \text{LO} \]
\[ \text{NLO} \]
\[ \text{NNLO} \]
\[ \text{NNLO+NNLL} \]
Scale dependence – Total cross section

- Total cross section integrated with dynamical scales

\[ H_T = \sqrt{m_t^2 + p_{Tt}^2} + \sqrt{m_{\bar{t}}^2 + p_{T\bar{t}}^2} \]

\[ H'_T = H_T + \sum_i p_{T_j_i} \]

\[ E_T = \sqrt{m_t^2 + p_{Tt}^2} \sqrt{m_{\bar{t}}^2 + p_{T\bar{t}}^2} \]

\[ m_T(t/\bar{t}) = \sqrt{m_t^2 + p_{T,t/\bar{t}}^2} \]

[Preliminary]
Scale dependence – Differential distributions

• Scales that depend on the invariant mass