Employing Helicity Amplitudes for Resummation

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Based on arXiv:1508.02397 with Moult, Stewart, Tackmann
Motivation

• Powerful methods for calculating helicity amplitudes exist
  • Automation at NLO, progress at NNLO
  • Used in various codes [MCFM, BlackHat, Rocket, NJet, MadLoop, …]

• I will discuss how to combine this with resummation in Soft-Collinear Effective Theory using helicity operators

• Outline:
  • Resummation in SCET
  • Helicity operators
  • Applications
Example: jet veto resummation in slepton searches

Large jet veto logarithms $\rightarrow$ large uncertainty

$$\sigma_0(p_T^{\text{cut}}) = \sum_{m\leq 2n} c_{n,m} \alpha_s^n \ln m \frac{p_T^{\text{cut}}}{2m_{\tilde{\ell}}} + \ldots$$
Example: jet veto resummation in slepton searches

- Large jet veto logarithms $\rightarrow$ large uncertainty

$$\sigma_0(p_T^{\text{cut}}) = \sum_{m \leq 2n} c_{n,m} \alpha_s^n \ln m \frac{p_T^{\text{cut}}}{2m_{\tilde{\ell}}^2} + \ldots$$

- Want resummation beyond NLL $\sim$ parton shower
Soft-Collinear Effective Theory

- **Collinear** and **soft** radiation are degrees of freedom
- Resummation achieved by factorization

\[
d\sigma(N\text{ jet}) = f^2 \, \hat{C}^\dagger \, \hat{S}_N \, \hat{C} \, \mathcal{I}^2 \prod_{j=1}^{N} J_j
\]
Soft-Collinear Effective Theory

- Collinear and soft radiation are degrees of freedom
- Resummation achieved by factorization

\[ d\sigma(N\text{ jet}) = f^2 \sum_{i=1}^{N} C_i O_i \]

- Hard scattering encoded in \( \mathcal{L}_{\text{hard}} = \sum_{i} C_i O_i \)
Hard scattering operators

Building blocks: **collinear quark** $\chi_{n,\omega}$ **gluon** $B_{n\perp,\omega}$

$n = \text{direction}, \quad \omega = \text{energy (x2)}$

Textbook approach to spin unnecessarily complicated

$$O_1 = \bar{\chi}_{n_3,\omega_3} \eta_2 \chi_{n_4,\omega_4} B_{n_1\perp,\omega_1} \cdot B_{n_2\perp,\omega_2} H$$

$$O_2 = \bar{\chi}_{n_3,\omega_3} B_{n_1\perp,\omega_1} \chi_{n_4,\omega_4} n_4 \cdot B_{n_2\perp,\omega_2} H$$

$$O_3 = \bar{\chi}_{n_3,\omega_3} \eta_1 \eta_2 B_{n_1\perp,\omega_1} \chi_{n_4,\omega_4} n_4 \cdot B_{n_2\perp,\omega_2} H$$

... [Marcantonini, Stewart]
Organizing spin: helicity operators

- Using spinor representation of polarization vectors
  \[ \varepsilon_\pm^\mu (p, k) = \pm \frac{\langle p \pm |\gamma^\mu | k \pm \rangle}{\sqrt{2} \langle k \mp | p \pm \rangle} \]

- Define gluon field and quark current with definite helicity
  \[ B^a_{i \pm} = -\varepsilon_{\mp \mu} (n_i, \bar{n}_i) B^{a \mu}_{n_i \perp, \omega_i} \]
  \[ J_{i j \pm} = \pm \frac{\sqrt{2} \varepsilon^\mu_{\perp} (n_i, n_j)}{\sqrt{\omega_i \omega_j}} \frac{\bar{\chi}_{n_i, -\omega_i} \gamma_\mu \frac{1}{2} (1 \pm \gamma_5) \chi_{n_j, \omega_j}}{\langle n_i \mp | n_j \pm \rangle} \]

- Operator basis is trivial (6 spin structures)
  \[ O_{++(+)}^{ab \bar{\alpha} \beta} = \frac{1}{2} B^a_1 + B^b_2 + J_{34 +} \]
  \[ O_{+-(+)}^{ab \bar{\alpha} \beta} = B^a_1 - B^b_2 - J_{34 +} \]
  ...
Matching

\[ \sum_{\text{diagrams}} \begin{array}{c}
+ a \\
- \bar{\beta} \\
+ \alpha 
\end{array} = \sum_{i} C_i \times \begin{array}{c}
+ a \\
- \bar{\beta} \\
+ \alpha 
\end{array} \]

- Only \( O_{++-}(+) \) contributes and \( C_{++-}(+) = A^{(0)}(g^a + g^b - q^\alpha + \bar{q}^\bar{\beta} - H) \)
Matching

\[ \sum \text{diagrams} \]

- Only \( O^{ab \bar{\alpha} \beta}_{+- (+)} \) contributes and \( C^{(0)ab \alpha \bar{\beta}}_{+- (+)} = \mathcal{A}^{(0)} (g^a g^b q^\alpha \bar{q}^\beta H) \)

- Loop corrections in SCET vanish and IR div. same as QCD

\[ \mathcal{A}^{(1)} = C^{(0)} \langle O \rangle^{(1)} + C^{(1)} \langle O \rangle^{(0)} \]

\[ \mathcal{A}^{(0)} \left( \frac{\#}{\epsilon_{\text{IR}}^2} + \frac{\#}{\epsilon_{\text{IR}}} \right) + \mathcal{A}^{(1)}_{\text{fin}} = C^{(0)} \left( \frac{\#}{\epsilon_{\text{IR}}^2} + \frac{\#}{\epsilon_{\text{IR}}} \right) + C^{(1)} \]

\[ C^{(1)} = \mathcal{A}^{(1)}_{\text{fin}} \]
Organizing color

- Pick a (color-conserving) basis

\[ C^{a_1 a_2 \cdots \alpha_{n-1} \bar{\alpha}_n}_{+ \cdots (+-)} = \bar{T}^{a_1 a_2 \cdots \alpha_{n-1} \bar{\alpha}_n} \, \tilde{C}^{+ \cdots (+-)} \]

E.g.

\[ \bar{T}^{a \alpha \bar{\beta}} = (T^a_{\alpha \bar{\beta}}) \quad \bar{T}^{abc} = (i f^{abc}, d^{abc}) \]

\[ \bar{T}^{ab \alpha \bar{\beta}} = ((T^a T^b)_{\alpha \bar{\beta}}, (T^b T^a)_{\alpha \bar{\beta}}, \text{tr}[T^a T^b] \, \delta_{\alpha \bar{\beta}}) \]
Organizing color

• Pick a (color-conserving) basis

\[
C^a_1a_2\cdots\alpha_{n-1}\bar{\alpha}_n = \bar{T}^a_1a_2\cdots\alpha_{n-1}\bar{\alpha}_n \tilde{C}^+\cdots(\cdots-) 
\]

E.g.

\[
\bar{T}^a_\alpha\bar{\beta} = (T^a_\alpha\bar{\beta}) \quad \bar{T}^{abc} = (if^{abc}, d^{abc}) 
\]

\[
\bar{T}^{ab}_\alpha\bar{\beta} = \left( (T^a T^b)_\alpha\bar{\beta}, (T^b T^a)_\alpha\bar{\beta}, \text{tr}[T^a T^b] \delta_{\alpha\bar{\beta}} \right) 
\]

• Color basis not orthonormal \(\rightarrow\) careful calculating conjugate!

\[
\tilde{C}^\dagger = (C^a_1\cdots\alpha_n)^* \bar{T}^a_1\cdots\alpha_n = \tilde{C}^*T^{\dagger}\hat{T} 
\]

• Color cannot be summed over, soft gluons exchange color

\[
d\sigma(N \text{ jet}) = f^2 \tilde{C}^\dagger \hat{S}_N \tilde{C} \mathcal{L}^2 \prod_{j=1}^{N} J_j 
\]
Features of helicity operators

- Basis is crossing symmetric

- Discrete symmetries are simple.

E.g. \( C \mathcal{B}_{i}^{a} T_{\alpha \beta}^{a} C = - \mathcal{B}_{i}^{a} T_{\beta \alpha}^{a} \)

\( C J_{ij}^{\alpha \beta} C = - J_{ji}^{\beta \alpha} \)

- No evanescent operators, as soft gluons don’t exchange spin

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<th>HV</th>
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<td>external</td>
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Applications to matching

• Matching to fixed-order calculations

\[ C = A_{\text{fin}} \]

• We provide matching coefficients for \( pp \rightarrow H+0,1,2 \) jets,
  \( pp \rightarrow W/Z+0,1,2 \) jets, \( pp \rightarrow 2,3 \) jets at (N)LO

• Basis for power corrections to matching
  [Kolodrubetz, Moult, Stewart]

• E.g. for \( pp \rightarrow H+0 \) jets

\[ O^\text{sub}_{+(+)} = B_{n} + J_{n} \bar{n} + H \]
\[ O^\text{sub}_{-(+)} = B_{n} - J_{n} \bar{n} + H \]
\[ \ldots \]

• Not all allowed: \( O^\text{sub}_{+(+)} \) cannot conserve angular momentum
Application to jets with kinematic hierarchies

[Bauer, Tackmann, Walsh, Zuberi; Pietrulewicz, Tackmann, WW]

- Large kinematic logarithms of ratios of dijet invariant masses
Application to jets with kinematic hierarchies

[Bauer, Tackmann, Walsh, Zuberi; Pietrulewicz, Tackmann, WW]

• Large kinematic logarithms of ratios of dijet invariant masses

\[ s_{12} \sim s_{13} \sim s_{23} \]

SCET

\[ s_{12} \ll s_{13} \sim s_{23} \]

SCET+

• E.g. two nearby jets are resolved in a second matching step

\[
\bar{x}_{t+} = \sum_{\lambda_g} \int d\omega_1 \ d\omega_2 \ C_{\lambda_g}^{\alpha \beta \gamma} \ (X_{n_1} \ B_1 \lambda_g \ X_{n_1}^{\dagger})^\alpha (\bar{X}_2 + X_{n_2}^{\dagger})^{\beta} \ V_{n_t}^{\gamma \bar{\alpha}}
\]

where Wilson lines \( X \) and \( V \) describes collinear-soft radiation

• SCET+ used for multidifferential measurements [Procura, WW, Zeune]

nonglobal logs [Larkoski et al, Becher et al], jet radius logs [Chien et al], …
Conclusions

• Helicity operators are convenient
  • Easy to write down basis and perform matching
  • Crossing symmetry and discrete symmetries manifest

• Applications:
  • Combining fixed-order calculations with SCET resummation
  • Jets with kinematic hierarchies

• Towards automated resummation in SCET
  [GENEVA: Alioli, Bauer, Berggren, Tackmann, Walsh; LO+NLL: Farhi, Feige, Freytsis, Schwartz; 0-jet resummation: Becher, Frederix, Neubert, Rothen]