Fluctuations of electromagnetic fields in heavy ion collisions

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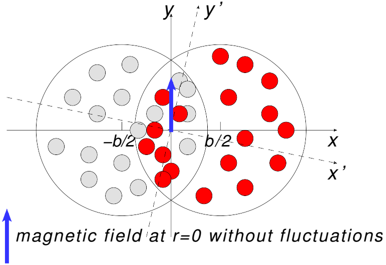
Magnetic field may lead to charge separation along **B**-direction due to the anomalous current $\propto$ **B** (the Chiral Magnetic Effect) [D.E. Kharzeev, L.D. McLerran, and H.J. Warringa, Nucl. Phys. A**803**, 227 (2008)].



Field fluctuations can lead to reduction of the **B**-induced observables. Fluctuations have been studied in Monte-Carlo simulation with the Woods-Saxon nuclear distribution [A. Bzdak and V. Skokov, Phys. Lett. B**710**, 171 (2012); W.-T. Deng and X.-G. Huang, Phys. Rev. C**85**, 044907 (2012); J. Bloczynski, X.-G. Huang, X. Zhang, and J. Liao, Phys. Lett. B**718**, 1529 (2013)].

But in these MC analyses the nuclear quantum collective dynamics is completely ignored. The classical treatment of the electromagnetic field may also be inadequate (similarly to calculations of the van der Waals forces [H.B.G. Casimir and D. Polder (1948)].

Mean fields and fluctuating fields

Right moving nucleus:$\mathbf{V}_R = (0, 0, V)$ $\mathbf{r}_R = (-\mathbf{b}/2, Vt)$.
Left moving nucleus: $\mathbf{V}_L = (0, 0, -V)$, $\mathbf{r}_L = (\mathbf{b}/2, -Vt)$.

$$F^{\mu\nu} = \langle F^{\mu\nu} \rangle + \delta F^{\mu\nu} \,.$$

For each nucleus $\langle \mathbf{E} \rangle$ and $\langle \mathbf{B} \rangle$ are given by the Lorentz transformation of its Coulomb field in the nucleus rest frame.
In lab-frame for a nucleus with the impact vector **b** the mean electric and magnetic fields at $x^\mu = (t, \boldsymbol{\rho}, z)$ read

$$\langle \mathbf{E}_T(t, \boldsymbol{\rho}, z) \rangle = \gamma \frac{E_A(r')(\boldsymbol{\rho} - \mathbf{b})}{r'} \,, \quad \langle E_z(t, \boldsymbol{\rho}, z) \rangle = \frac{E_A(r')z'}{r'} \,,$$

$$\langle \mathbf{B}(t, \boldsymbol{\rho}, z) \rangle = [\mathbf{V} \times \langle \mathbf{E}(t, \boldsymbol{\rho}, z) \rangle] \,, \quad E_A(r) = \frac{4\pi}{r^2} \int_0^r d\xi \xi^2 \rho_A(\xi)$$

$\gamma = 1/\sqrt{1 - V^2}$, $r'^2 = (\boldsymbol{\rho} - \mathbf{b})^2 + z'^2$, $z' = \gamma(z - Vt)$.
In lab-frame at $|z| \lesssim t$ we have $r' \gg R_A$ for $t \gtrsim 0.2$ fm.

For two colliding nuclei $\langle B_x(t, \mathbf{r}=0)\rangle = 0$. At $t \gg R_A/\gamma$, $\rho \ll t\gamma$

$$\langle B_y(t, \boldsymbol{\rho}, z=0)\rangle \approx Zeb/\gamma^2 t^3 \,.$$

For each nucleus field correlators in the lab-frame may be expressed via the correlators in the nucleus rest frame. At $\gamma \gg 1$ the transverse fluctuations dominate.

$$\langle \delta E_i \delta E_k\rangle = \gamma^2 \left[\langle \delta E_i \delta E_k\rangle + V^2 e_{3il} e_{3kj} \langle \delta B_l \delta B_j\rangle\right]_{rf} ,$$

$$\langle \delta B_i \delta B_k\rangle = \gamma^2 \left[\langle \delta B_i \delta B_k\rangle + V^2 e_{3il} e_{3kj} \langle \delta E_l \delta E_j\rangle\right]_{rf} ,$$

where i, k are $\perp$ indices and the subscript *rf* indicates that the correlators are calculated in the nucleus rest frame.
We calculate of the rest frame correlators $\langle \delta E_l \delta E_j\rangle$, $\langle \delta B_i \delta B_k\rangle$ with the help of the FDT formalism of E.M. Lifshits and L.P. Pitaevski [*Statistical Physics, Part 2 (Landau Course of Theoretical Physics Vol. 9)*]

Fluctuation-dissipation theorem for fluctuating fields

The Lifshits-Pitaevski FDT formalism is formulated in the gauge $\delta A^0 = 0$. It allows to relate the time Fourier component of the vector potential correlator

$$\langle \delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega = \frac{1}{2}\int dt e^{i\omega t}\langle \delta A_i(t,\mathbf{r}_1)\delta A_k(0,\mathbf{r}_2)+\delta A_k(0,\mathbf{r}_2)\delta A_i(t,\mathbf{r}_1)\rangle$$

and that of the retarded Green's function

$$D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = -i\int dt e^{i\omega t}\theta(t)\langle \delta A_i(t,\mathbf{r}_1)\delta A_k(0,\mathbf{r}_2)-\delta A_k(0,\mathbf{r}_2)A_i(t,\mathbf{r}_1)\rangle\,.$$

In the zero temperature limit the FDT gives

$$\langle \delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega = -\mathrm{sign}(\omega)\mathrm{Im} D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2).$$

Formulas for field correlator

$$\langle\delta E_i(\mathbf{r}_1)\delta E_k(\mathbf{r}_2)\rangle_\omega=\omega^2\langle\delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega\,, \tag{1}$$

$$\langle\delta B_i(\mathbf{r}_1)\delta B_k(\mathbf{r}_2)\rangle_\omega=\mathrm{rot}^{(1)}_{il}\mathrm{rot}^{(2)}_{kj}\langle\delta A_l(\mathbf{r}_1)\delta A_j(\mathbf{r}_2)\rangle_\omega. \tag{2}$$

The same point field correlators that we need read

$$\langle\delta E_i(t,\mathbf{r})\delta E_k(t,\mathbf{r})\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\langle\delta E_i(\mathbf{r})\delta E_k(\mathbf{r})\rangle_\omega\,,$$

$$\langle\delta B_i(t,\mathbf{r})\delta B_k(t,\mathbf{r})\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\langle\delta B_i(\mathbf{r})\delta B_k(\mathbf{r})\rangle_\omega\,.$$

At $t\gtrsim 0.2$ fm (in the lab-frame) for each nucleus the distance between the observation point **r** and the center of the nucleus (in its rest frame) $r\gg R_A$. $\Rightarrow$ Each nucleus acts as a point like dipole described by dipole polarizability $\alpha_{ik}(\omega)$.

The retarded Green's function in the presence of nuclear dipole

The equation determining the retarded Green's function reads

$$\left[\frac{\partial^2}{\partial x_i \partial_l} - \delta_{il}\triangle - \delta_{il}\omega^2 - 4\pi\omega^2\alpha_{il}(\omega)\delta(\mathbf{r}-\mathbf{r}_A)\right] D_{lk}(\omega,\mathbf{r},\mathbf{r}') = -4\pi\delta_{ik}\delta(\mathbf{r}-\mathbf{r}')\,.$$

The correction to D_{ik} due to α_{ik} reads

$$\Delta D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = -\omega^2 D^v_{il}(\omega,\mathbf{r}_1,\mathbf{r}_A)\alpha_{lm}(\omega)D^v_{mk}(\omega,\mathbf{r}_A,\mathbf{r}_2)\,.$$

Here D^v_{ik} is the vacuum Green's function that is given by

$$D^v_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = \delta_{ik}D_1(\omega,r) + \frac{x_i x_k}{r^2}D_2(\omega,r)\,,\ \text{with } \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2\,,$$

$$D_1(\omega,r) = -\frac{e^{i\omega r}}{r}\left(1+\frac{i}{\omega r}-\frac{1}{\omega^2 r^2}\right),\, D_2(\omega,r) = \frac{e^{i\omega r}}{r}\left(1+\frac{3i}{\omega r}-\frac{3}{\omega^2 r^2}\right)\,.$$

For spherical nuclei $\alpha_{ik}(\omega)=\delta_{ik}\alpha(\omega)$.

Final formulas for the rest frame field correlators

$\alpha(\omega)$ is an analytical function of ω in the upper half-plane, and $\alpha^*(-\omega^*) = \alpha(\omega)$ $\Rightarrow$ on the upper imaginary axis $\alpha(\omega)$ is real.

$$\langle \delta E_i(t,\mathbf{r})\delta E_k(t,\mathbf{r})\rangle = \delta_{ik} J_1(r) + \frac{x_i x_k}{r^2} J_2(r)\,,$$

$$\langle \delta B_i(t,\mathbf{r})\delta B_k(t,\mathbf{r})\rangle = \left(\delta_{ik} - \frac{x_i x_k}{r^2}\right) J_3(r)\,,$$

$$J_1 = \frac{1}{2\pi r^7}\left[I_0 + I_1 + \frac{3}{4} I_2 + \frac{1}{4} I_3 + \frac{1}{16} I_4\right]\,,$$

$$J_2 = \frac{1}{2\pi r^7}\left[3 I_0 + 3 I_1 + \frac{1}{4} I_2 - \frac{1}{4} I_3 - \frac{1}{16} I_4\right]\,,$$

$$J_3 = -\frac{1}{8\pi r^7}\left[I_2 + I_3 + \frac{1}{4} I_4\right]\,,$$

$$I_n = \int_0^\infty d\xi \xi^n e^{-\xi} \alpha\left(\frac{i\xi}{2r}\right)\,.$$

Parametrization of the dipole polarizability

$$\alpha(\omega) = \frac{1}{3}\sum_s \left[\frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0}-\omega-i\delta} + \frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0}+\omega+i\delta}\right],$$

d is the dipole operator

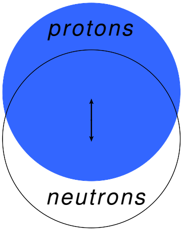
$$\mathbf{d} = e\frac{N}{A}\sum_p \mathbf{r}_p - e\frac{Z}{A}\sum_n \mathbf{r}_n \,.$$

At $\omega > 0$ the dipole polarizability tensor coincides with the photon scattering tensor

$$\sigma_{abs}(\omega) = 4\pi\omega \mathrm{Im}\alpha(\omega)\,.$$

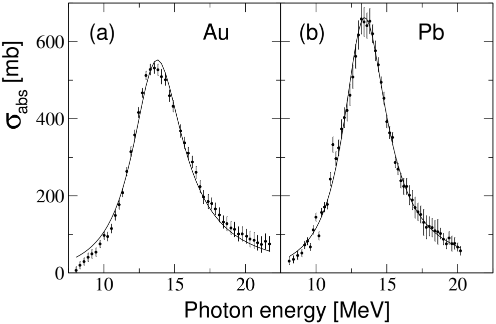
Giant dipole resonance

For heavy nuclei the dipole strength is dominated by the GDR. It appears as a broad peak in the σ_{abs} with a mean energy $\sim$ 14 MeV. We write $\alpha(\omega)$ in the form



$$\alpha(\omega) = c\left[\frac{1}{\omega_{10} - \omega - i\Gamma/2} + \frac{1}{\omega_{10} + \omega + i\Gamma/2}\right] .$$

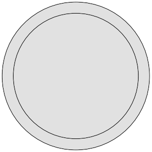
We fit parameters using σ_{abs} from A. Veyssiere *et al.*, Nucl. Phys. A**159**, 561 (1970) (for ^{197}Au) and from A. Tamii *et al.*, Phys. Rev. Lett. **107**, 062502 (2011) for ^{208}Pb.
We obtained: $\omega_{10} \approx 13.6(13.3)$, $\Gamma \approx 4.38(3.72)$ MeV, $c \approx 18.2(18.93)$ GeV^{-2} for Au(Pb).



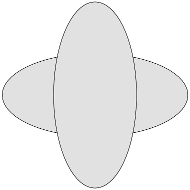
$$\langle 0|\mathbf{d}^2|0\rangle = \frac{3}{\pi}\int_0^\infty d\omega \mathrm{Im}\alpha(\omega) = \frac{6c}{\pi}\mathrm{arctg}\left(2\omega_{10}/\Gamma\right)\ .$$

$\langle 0|\mathbf{d}^2|0\rangle_{Au} \approx 1.91$ fm^2, $\langle 0|\mathbf{d}^2|0\rangle_{Pb} \approx 2.02$ fm^2
The classical MC with the WS nuclear density gives $\langle \mathbf{d}^2\rangle_{Au} \approx 9.89$ fm^2 and $\langle \mathbf{d}^2\rangle_{Pb} \approx 10.39$ fm^2. $\Rightarrow$ The classical model overestimates the dipole moment squared by a factor of ~ 5. This may be important for the event-by-event hydro.

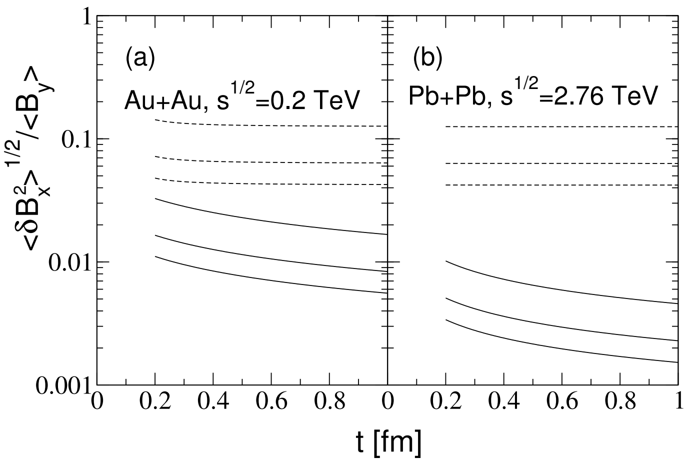
It would be interesting to clarify the situation with MC for the giant monopole and quadrupole modes. These modes also may be important for the initial entropy deposition and for fluctuations of the experimental participant plane.



monopole mode



quadrupole mode



The t-dependence of $\langle \delta B_x^2 \rangle^{1/2} / \langle B_y \rangle$ at $\mathbf{r} = 0$ for Au+Au collisions at $\sqrt{s} = 0.2$ TeV (left) and for Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV for impact parameters $b = 3$, 6 and 9 fm (from top to bottom). Solid lines are for quantum calculations, dashed lines for classical MC calculations with the WS nuclear density.

Summary

- Within the FDT formalism we have performed a quantum analysis of fluctuations of the electromagnetic field in *AA* collisions at RHIC and LHC energies. The fluctuations are expressed via the nuclear dipole polarizability.
- Our quantum calculations show that the field fluctuations are very small, and they practically do not affect the direction of the magnetic field as compared to the mean field classical predictions.
- We have demonstrated that the classical picture overestimates strongly the field fluctuations.
- Our results for the nuclear dipole fluctuations question the applicability of the MC with the Woods-Saxon density for the event-by-event hydro simulation of *AA* collisions.