Overview of Lattice QCD Results

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Rencontres de Moriond
QCD and High Energy Interactions
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Flavor Physics: a tale of multiple scales

- $m_{NP}$
- $m_t$
- $m_W, m_Z$

Effective Weak Hamiltonian

- Local RGE

Optical Th./OPE

- HQET

- Local RGE

SCET

- pQCD

- Non-local RGE

LCSR

- Lattice QCD

Isospin and Dalitz plot Analyses

Naive Factorization for multi (≥3) hadron final states
Lattice QCD

- Many flavor physics observables require matrix elements controlled by non-perturbative QCD: decay constants, bag parameters, form factors, ...
- Lattice QCD is the only approach that allows for the calculation of these quantities that is systematic and from first principles

\[
\mathcal{L} = \frac{1}{2g^2} \text{tr}[F_{\mu\nu}F^{\mu\nu}] + \sum_{q=u,d,s} \bar{q}(i\slashed{D} - m_q)q + \sum_{q=c,b} \bar{q}(i\slashed{D} - m_q)q + \frac{i\bar{\theta}}{32\pi^2} \text{tr}[F_{\mu\nu}\tilde{F}^{\mu\nu}]
\]

**Wilson gauge action**
- + improvements
  - Symanzik, Iwasaki, ...

**Wilson-clover**
- Twisted mass
- Staggered (Asqtad/HISQ)
- Overlap
- Domain wall

**HQET**
- NRQCD
- Columbia/Fermilab/Tsukuba tuning
- HISQ/Twisted mass (only charm: \(am_q \approx 1\))
- [but latest HISQ lattice spacing is \(a \approx 0.03\) fm implying \(am_q \approx 0.76\)]

**Input parameters:**
- \(g\) from \(r_1,f_\pi, m_\Omega, Y(1S-2S)\)
- \(m_q\) from \(m_\pi, m_K, m_{Ds}, m_Y\)

**Modern simulations**
- \(N_f=2\) (\(m_u = m_d = m_l\))
- \(N_f=2+1\) (\(m_l \neq m_s\))
- \(N_f=2+1+1\) (\(m_l \neq m_s \neq m_c\))
- \(N_f=1+1+1+1\) (\(m_u \neq m_d \neq m_s \neq m_c\))

**Main sources of uncertainties:**
- Statistics (MC simulation)
- Continuum Limit \((a \rightarrow 0)\)
- \(\infty\) Volume Limit \((m_\pi L \rightarrow \infty)\)
- Chiral Extrapolation \((m_{\pi}^{\text{lat}} > m_{\pi}^{\text{exp}})\)
- Operator matching (PT vs NPR)
Lattice QCD for flavor physics

- Quark masses: $m_u, m_d, m_s, m_c, m_b$
- Low Energy Constants ($\chi_{PT}$): $\Sigma^{1/3}, F_\pi/F, \ell_{3,4,6}$
- Strong coupling constant: $\alpha_s$
- Decay constants: $f_\pi, f_K, f_D, f_{Ds}, f_B, f_{Bs}$
- Matrix elements for meson mixing for SM and BSM operators: $B_K, B_K^{2,3,4,5}, B_d, B_s$
- Form Factors: $K \rightarrow \pi, D \rightarrow (\pi, K), B \rightarrow (\pi, K, K^*), B_s \rightarrow (K, \phi), B \rightarrow D(*)$, $\Lambda_b \rightarrow (p, \Lambda_c, \Lambda)$
- $K \rightarrow \pi\pi$ matrix elements for $\varepsilon_K$ and $\varepsilon_K'/\varepsilon_K$
- Exploratory studies of
  - Hadronic vacuum polarization and light-by-light contributions to $(g-2)_\mu$
  - Long distance contributions to $\Delta M_K$
  - Long distance contributions to charm effects in $K^+ \rightarrow \pi^+ \nu\bar{\nu}$
- Other applications include: QCD at finite temperature and density (heavy-ion collisions, early universe, neutron stars structure), nucleon structure, nucleon-nucleon interactions, Yang-Mills theories with additional fermions (for technicolor-like models), gravity, …
Flavor Lattice Averaging Group (FLAG)

- **Provide lattice averages of quantities of interest for flavor physics**
- All calculations are classified according to their treatment of systematic uncertainties
- **Meant to be easily accessible to non-experts and experts alike**
- Quantities considered are: quark masses, LEC, $\alpha_s$, decay constants, meson mixing matrix elements, form factors
- Updates at [http://itpwiki.unibe.ch/flag](http://itpwiki.unibe.ch/flag)
- Members (33) cover a sizable cross section of lattice collaborations and practitioners
  - **Advisory Board:** S. Aoki, Bernard, Golterman, Leutwyler, Sachrajda
  - **Editorial Board:** Colangelo, Jüttner, Hashimoto, Sharpe, Vladikas, Wenger
  - **Working Groups:**
    - Quark masses (Lellouch, Blum, Lubicz)
    - $V_{us}, V_{ud}$ (Simula, Boyle, Kaneko)
    - LEC (Dürr, Fukaya, Heller)
    - $B_K$ (Wittig, Dimopoulos, Mawhinney)
    - $f_{B(s)}, f_{D(s)}, B_B$ (Y. Aoki, Della Morte, Lin)
    - Form Factors (Becirevic, Gottlieb, Lunghi, Pena)
    - $\alpha_s$ (Sommer, Horsley, Onogi)
Rating Criteria for (1) continuum extrapolation, (2) chiral extrapolation, (3) finite volume and (4) renormalization are updated at each FLAG iteration.

- ★ = great (can enter the average)
- ○ = ok
- ■ = mmm

Final average: \( N_f = 2 + 1 : \hat{B}_K = 0.7625(97) \) 

\[ \text{Refs. [10, 43–45]} \]

1.3\% negligible impact on the \( \varepsilon_K \) error budget!!
## FLAG: Summary Tables

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Sec.</th>
<th>$N_f = 2 + 1 + 1$</th>
<th>Refs.</th>
<th>$N_f = 2 + 1$</th>
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<td>$m_s$ [MeV]</td>
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<td>93.9(1.1)</td>
<td>[4, 5]</td>
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<td>[6–10]</td>
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<td>3.373(80)</td>
<td>[7–10, 13]</td>
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<td>$m_s / m_{ud}$</td>
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<td>[4, 14]</td>
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<tr>
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<td>[16]</td>
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<td>[16]</td>
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<tr>
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<tr>
<td>$m_c (3$ GeV) [GeV]</td>
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<td>0.996(25)</td>
<td>[4, 5]</td>
<td>0.987(6)</td>
<td>[9, 17]</td>
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<td>$m_c / m_s$</td>
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<td>[4, 5, 14]</td>
<td>11.82(16)</td>
<td>[17, 18]</td>
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<td>$\overline{m}_b (\overline{m}_b$ [GeV]</td>
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<td>4.190(21)</td>
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<td>4.164(23)</td>
<td>[9]</td>
<td>4.256(81)</td>
<td>[20, 21]</td>
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<tr>
<td>$f_{+}(0)$</td>
<td>4.3</td>
<td>0.9704(24)(22)</td>
<td>[22]</td>
<td>0.9677(27)</td>
<td>[23, 24]</td>
<td>0.9560(57)(62)</td>
<td>[25]</td>
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<tr>
<td>$f_{K^\pm}/f_\pi^\pm$</td>
<td>4.3</td>
<td>1.193(3)</td>
<td>[14, 26, 27]</td>
<td>1.192(5)</td>
<td>[28–31]</td>
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<td>[32]</td>
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<td>$f_\pi^\pm$ [MeV]</td>
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<td>155.6(4)</td>
<td>[14, 26, 27]</td>
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<td>$f_{K^\pm}$ [MeV]</td>
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<td>$\Sigma^{1/3}$ [MeV]</td>
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<td>274(3)</td>
<td>[10, 13, 34, 35]</td>
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<td>[33, 36–38]</td>
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<td>[10, 29, 34, 35, 40]</td>
<td>3.4(82)</td>
<td>[36, 37, 41]</td>
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<td>$\bar{\ell}_4$</td>
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<td>$\bar{\ell}_6$</td>
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<tr>
<td>$\bar{D}_K$</td>
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<td>0.7625(97)</td>
<td>[10, 43–45]</td>
<td>0.727(22)(12)</td>
<td>[46]</td>
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</tbody>
</table>
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<td>[17, 48, 49]</td>
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<td>(f_{+}^{DK}(0))</td>
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<td>(f_{B}[\text{MeV}])</td>
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<td>[20, 57, 58]</td>
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<td>[20, 57, 58]</td>
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<td>(f_{B_d}\sqrt{\hat{B}_{B_d}[\text{MeV}])}</td>
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<td></td>
<td></td>
<td>219(14)</td>
<td>[54, 59]</td>
<td>216(10)</td>
<td>[20]</td>
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<tr>
<td>(f_{B_s}\sqrt{\hat{B}_{B_s}[\text{MeV}])}</td>
<td>8.2</td>
<td></td>
<td></td>
<td>270(16)</td>
<td>[54, 59]</td>
<td>262(10)</td>
<td>[20]</td>
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<td>(\hat{B}_{B_d})</td>
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<td>1.26(9)</td>
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<td>1.32(5)</td>
<td>[54, 59]</td>
<td>1.32(5)</td>
<td>[20]</td>
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<td>(\hat{B}_{B_s})</td>
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<td>1.32(6)</td>
<td>[54, 59]</td>
<td>1.225(31)</td>
<td>[54, 60]</td>
<td>1.225(31)</td>
<td>[20]</td>
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<tr>
<td>(\xi)</td>
<td>8.2</td>
<td>1.239(46)</td>
<td>[54, 60]</td>
<td>1.039(63)</td>
<td>[54, 60]</td>
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<tr>
<td>(B_{B_s}/B_{B_d})</td>
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<th>Refs.</th>
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<tbody>
<tr>
<td>(\alpha_{\text{MS}}^{(5)}(M_Z))</td>
<td>9.9</td>
<td>0.1182(12)</td>
<td>[5, 9, 61–63]</td>
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<tr>
<td>(\Lambda_{\text{MS}}^{(3)}[\text{MeV}])</td>
<td>9.9</td>
<td>211(14)</td>
<td>[5, 9, 61–63]</td>
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</tbody>
</table>
Form Factors and z-Expansion

- The vector current $B \rightarrow P$ form factors are defined as:
  \[ \langle M(p)|V^\mu|B(p_B)\rangle = \left( p_B^\mu + p^\mu - \frac{m_B^2 - m_M^2}{q^2} q^\mu \right) f_+(q^2) + \frac{m_B^2 - m_M^2}{q^2} q^\mu f_0(q^2) \]

- They obey the exact constraint $f_+(0) = f_0(0)$

- A conformal transformation of the complex $q^2$ plane exploits the fact that the larger the gap between the kinematic endpoint $t_- = (m_B - m_M)^2$ and the pair creation threshold $t_+ = (m_B + m_M)^2$ the smaller the form factor variation over the physical $q^2$ window is:
  \[ z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \]
  \[ t_0 = (m_B + m_M)(\sqrt{m_B} - \sqrt{m_M})^2 \]

- An expansion in powers of $z$ is going to be controlled by the first few terms

\[ |z_{\text{max}}| = \begin{cases} 0.28 & B \rightarrow \pi \\ 0.14 & B \rightarrow K \\ 0.03 & B \rightarrow D \end{cases} \]
We also know that resonances that appear in the crossed channel, contribute poles to the form factors. For $f_+$ the resonance has $J^P=1^-$:

\[ f_{+}^{BCL}(q^2) = \frac{1}{B(q^2)} \sum_{n=0}^{N} a_n \left[ z^n - (-1)^{n-N-1} \frac{n}{N+1} z^{N+1} \right] \]

\[ f_{+}^{BGL}(q^2) = \frac{1}{B(q^2) \phi(q^2, t_0)} \sum_{n=0}^{N} a_n z^n \]

where $B(q^2)$ contains all sub-threshold poles* (Blaschke factor) and $\phi(q^2, t_0)$ is the so-called outer function.

* The threshold depends on the current alone.

E.g. the threshold for the $B \rightarrow \pi$ and $\Lambda_b \rightarrow p$ form factors is $(m_B-m_\pi)^2$.

The most common parametrizations of the form factor read:

\[ f_{+}^{BGL}(q^2) = \frac{1}{B(q^2) \phi(q^2, t_0)} \sum_{n=0}^{N} a_n z^n \]

enforces the proper behavior of the form factor at $q^2 \approx t_+$

[Boyd, Grinstein, Lebed hep-ph/9412324] [Bourrely, Caprini, Lellouch 0807.2722]
Different choices for the outer function impact the form of a certain constraint on the z-expansion coefficients. Popular choices are the BGL and BCL ($\phi(q^2, t_0) = 1$) parametrizations.

The constraint originates from using a dispersion integral to relate the correlator of two currents calculated with an OPE (assuming quark-hadron duality) and the form factor (using the crossing symmetry):

\begin{align*}
\text{BGL:} & \quad \sum a_n^2 < 1 \\
\text{BCL:} & \quad \sum B_{nm} a_n a_m < 1
\end{align*}

Different lattice collaborations use different parametrizations making it difficult to average: need to generate synthetic data and construct the correlated covariance matrix.

At present FLAG uses consistently BCL parametrizations for all form factors.
Two new independent calculations

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>$N_f$</th>
<th>publication st.</th>
<th>continuum extrapolation</th>
<th>chiral extrapolation</th>
<th>finite volume</th>
<th>renormalization</th>
<th>heavy-quark treatment</th>
<th>$\xi$-parameterization</th>
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<tbody>
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<td>FNAL/MILC 15</td>
<td>[503]</td>
<td>2+1</td>
<td>A</td>
<td>★</td>
<td>★</td>
<td>○</td>
<td>✓</td>
<td>✓</td>
<td>BCL</td>
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</table>

Generate synthetic data for FNAL/MILC and combine with RBC/UKQCD:

$B \to \pi (N_f = 2 + 1)$

<table>
<thead>
<tr>
<th>Central Values</th>
<th>Correlation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0^+$</td>
<td>0.404 (13)</td>
</tr>
<tr>
<td>$a_1^+$</td>
<td>-0.68 (13)</td>
</tr>
<tr>
<td>$a_2^+$</td>
<td>-0.86 (61)</td>
</tr>
<tr>
<td>$a_0^0$</td>
<td>0.490 (21)</td>
</tr>
<tr>
<td>$a_1^0$</td>
<td>-1.61 (16)</td>
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Need to fit $f_+$ and $f_0$ simultaneously and impose the $f_+(0) = f_0(0)$ constraint.

sea: staggered
b quark: Fermilab
Matching: mostly NPR

sea: DWF
b quark: Columbia
Matching: mostly NPR
**$B \to \pi \ell \nu$ ($N_f = 2 + 1$)**

<table>
<thead>
<tr>
<th>Central Values</th>
<th>$V_{ub} \times 10^3$</th>
<th>$a_0^+$</th>
<th>$a_1^+$</th>
<th>$a_2^+$</th>
<th>$a_0^0$</th>
<th>$a_1^0$</th>
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<tr>
<td></td>
<td>3.73 (14)</td>
<td>0.414 (12)</td>
<td>-0.494 (44)</td>
<td>-0.31 (16)</td>
<td>0.499 (19)</td>
<td>-1.426 (46)</td>
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<table>
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<th>0.211</th>
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<td>$a_0^+$</td>
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<td>-0.456</td>
<td>0.259</td>
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<tr>
<td>$a_1^+$</td>
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<td>0.154</td>
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<td>-0.0995</td>
<td>0.223</td>
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<tr>
<td>$a_2^+$</td>
<td>-0.374</td>
<td>-0.456</td>
<td>-0.797</td>
<td>1</td>
<td>0.0160</td>
<td>-0.0994</td>
</tr>
<tr>
<td>$a_0^0$</td>
<td>0.211</td>
<td>0.259</td>
<td>-0.0995</td>
<td>0.0160</td>
<td>1</td>
<td>-0.467</td>
</tr>
<tr>
<td>$a_1^0$</td>
<td>0.247</td>
<td>0.144</td>
<td>0.223</td>
<td>-0.0994</td>
<td>-0.467</td>
<td>1</td>
</tr>
</tbody>
</table>
For the first time two form factors calculations at non-zero recoil!

We assume 100% correlation between the statistical uncertainties because both calculations are based on the same MILC Asqdat ensembles.

We do not include poles in the Blaschke factors because of the poor knowledge of $I^-$ and $0^+$ excited $B_c$ states.
$B \to D_{\ell \nu}$ ($N_f = 2 + 1$)

| $|V_{cb}| \times 10^3$ | Central Values | Correlation Matrix |
|-----------------------|-----------------|---------------------|
|                       | 40.1 (1.0)      | 1 -0.525 -0.431 -0.185 -0.526 -0.497 |
| $a_0^+$               | 0.8944 (95)     | -0.525 1 0.282 -0.162 0.953 0.446 |
| $a_1^+$               | -8.08 (22)      | -0.431 0.282 1 0.613 0.350 0.934 |
| $a_2^+$               | 49.0 (4.6)      | -0.185 -0.162 0.613 1 -0.0931 0.603 |
| $a_0^0$               | 0.7802 (75)     | -0.526 0.953 0.350 -0.0931 1 0.446 |
| $a_1^0$               | -3.42 (22)      | -0.497 0.446 0.934 0.603 0.446 1 |

There is some sensitivity of $R(D)$ to the choice of $f_+$ parametrization that disappears once experimental $B \to D_{\ell \nu}$ data are included:

$R(D)_{\text{lat+exp}} = 0.299(3)$

$R(D)_{\text{exp}} = 0.397(40)(28)$

$\Rightarrow 2 \text{ sigma discrepancy}$
$V_{ub}$ and $V_{cb}$: exclusive vs inclusive

- **Exclusive:**
  \[
  |V_{ub}|_{B\to\pi\ell\nu} = 3.73(14) \times 10^{-3} \quad \text{[FLAG]}
  \]
  \[
  |V_{ub}|_{B\to\tau\nu} = 4.33(72) \times 10^{-3} \quad \text{[PDG+FLAG]}
  \]
  \[
  |V_{cb}|_{B\toD\ell\nu} = 40.1(1.0) \times 10^{-3} \quad \text{[FLAG]}
  \]
  \[
  |V_{cb}|_{B\toD^*\ell\nu} = 39.27(56)(49) \times 10^{-3} \quad \text{[FLAG]}
  \]
  \[
  |V_{ub}/V_{cb}|_{\Lambda_b\to(p,\Lambda_c)\ell\nu} = 0.083(6) \quad \text{[PDG]}
  \]
  \[\text{[Detmold, Lehner, Meinel]}\]

- **Inclusive [PDG]:**
  \[
  |V_{ub}|_{B\toX_u\ell\nu} = 4.49(16)(+16)\times 10^{-3}
  \]
  \[
  |V_{cb}|_{B\toX_c\ell\nu} = 42.2(0.7) \times 10^{-3}
  \]
  \[
  \text{The overall tension between all these determinations is } 3.2 \sigma
  \]

- **Future progress**
  - $B\toD^*$ form factor: $q^2$ dependence and use of BCL/BGL parametrization [Berlochner et al. 1703.05330]
  - $B_s\toK\ell\nu$ [Bigi, Gambino, Schacht 1703.06124]
  - [Grinstein, Kobach 1703.08170]

- Uses CLN parametrization [Caprini, Lellouch, Neubert 9712417]
$V_{ub}$ and $V_{cb}$: exclusive vs inclusive

- **Exclusive:**
  
  $|V_{ub}|_{B \rightarrow \pi \ell \nu} = 3.73(14) \times 10^{-3}$
  
  $|V_{ub}|_{B \rightarrow \tau \nu} = 4.33(72) \times 10^{-3}$
  
  $|V_{cb}|_{B \rightarrow D \ell \nu} = 40.1(1.0) \times 10^{-3}$
  
  $|V_{cb}|_{B \rightarrow D^* \ell \nu} = 39.27(56)(49) \times 10^{-3}$
  
  $|V_{ub}/V_{cb}|_{\Lambda_b \rightarrow (p, \Lambda_c) \ell \nu} = 0.083(6)$

- **Inclusive [PDG]:**
  
  $|V_{ub}|_{B \rightarrow X_u \ell \nu} = 4.49(16)(+16\, -18) \times 10^{-3}$
  
  $|V_{cb}|_{B \rightarrow X_c \ell \nu} = 42.2(0.7) \times 10^{-3}$

- The overall tension between all these determinations is 3.2 $\sigma$

- **Future progress**
  
  - $B \rightarrow D^*$ form factor: $q^2$ dependence and use of BCL/BGL parametrization
    
    [Berlochner et al. 1703.05330]
    [Bigi, Gambino, Schacht 1703.06124]
    [Grinstein, Kobach 1703.08170]
  
  - $B_s \rightarrow K \ell \nu$

- $B \rightarrow D^* \ell \nu$ using the new Belle result 1702.01521
B→K form factor

- Simultaneous correlated calculation of the three pseudo scalar form factors:

\[
\langle K(p)|\bar{s}\gamma^{\mu}b|B(p_B)\rangle = \left(\frac{p_B^{\mu} + p_K^{\mu} - \frac{m_B^2 - m_K^2}{q^2} q^{\mu}}{m_B + m_K}\right) f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^{\mu} f_0(q^2)
\]

\[
\langle K(p)|i\bar{s}\sigma^{\mu\nu}b|B(p_B)\rangle = \frac{2 f_T(q^2)}{m_B + m_K} (p_B^\mu p_K^\nu - p_B^\nu p_K^\mu)
\]

\[
\langle K(p)|\bar{s}b|B(p_B)\rangle = \frac{m_B^2 - m_K^2}{m_b - m_s} f_0(q^2)
\]

- Relevant sub-threshold poles are \( L^- \) for \( f_+ \) and \( f_T \) (\( B_s^* \) from PDG) and \( 0^+ \) for \( f_0 \) (\( B_{s0}^* \) from a lattice calculation [Lang et al, 1501.01646])

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>( N_f )</th>
<th>( b ) quark</th>
<th>( c ) quark</th>
<th>Matching</th>
<th>Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPQCD 13E</td>
<td>[514]</td>
<td>2+1</td>
<td>A</td>
<td></td>
<td>PT@1loop</td>
<td>BCL</td>
</tr>
<tr>
<td>FNAL/MILC 15D</td>
<td>[515]</td>
<td>2+1</td>
<td>A</td>
<td>★</td>
<td>PT@1loop</td>
<td>BCL</td>
</tr>
</tbody>
</table>

sea: staggered
b quark: NRQCD
c quark: HISQ
Matching: PT@1loop

sea: staggered
b,c quarks: Fermilab
Matching: PT@1loop
$B \rightarrow K$ form factor

- The results are dominated by the more recent FNAL/MILC calculation

<table>
<thead>
<tr>
<th>$B \rightarrow K$ ($N_f = 2 + 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Central Values</strong></td>
</tr>
<tr>
<td>$a_0^+$</td>
</tr>
<tr>
<td>$a_1^+$</td>
</tr>
<tr>
<td>$a_2^+$</td>
</tr>
<tr>
<td>$a_0^0$</td>
</tr>
<tr>
<td>$a_1^0$</td>
</tr>
<tr>
<td>$a_2^0$</td>
</tr>
<tr>
<td>$a_1^T$</td>
</tr>
<tr>
<td>$a_2^T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Correlation Matrix</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
</tr>
<tr>
<td>0.467</td>
</tr>
<tr>
<td>0.058</td>
</tr>
<tr>
<td>0.755</td>
</tr>
<tr>
<td>0.553</td>
</tr>
<tr>
<td>0.609</td>
</tr>
<tr>
<td>0.253</td>
</tr>
<tr>
<td>0.102</td>
</tr>
</tbody>
</table>
The effective Hamiltonian responsible for $b \rightarrow q$ ($q = d, s$) transitions in the SM is:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \sum_{i=3}^{6} C_i Q_i Q_i + C_b Q_b + C_{\nu \nu} Q_{\nu \nu} \right]$$

- $\lambda_q$ contributions are relevant only for $b \rightarrow d$ transitions and yield large CP asymmetries ($\lambda_s = -0.0074 + 0.020 i$, $\lambda_d = -0.036 - 0.43 i$)

- Phenomenologically important operators are:

\[
\begin{align*}
Q_7 &= \frac{e}{16\pi^2} (\bar{q}_L \sigma^{\mu \nu} b_R) F_{\mu \nu} \\
B &\rightarrow (K^*, K_1, \rho, X_s, X_d, \ldots) \gamma
\end{align*}
\]

\[
\begin{align*}
Q_{\nu \nu} &= (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \nu_\ell) \\
B &\rightarrow (K^*, X_s, \ldots) \nu \bar{\nu}
\end{align*}
\]

\[
\begin{align*}
Q_9 &= \frac{\alpha_{\text{em}}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \ell) \\
Q_{10} &= \frac{\alpha_{\text{em}}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \gamma_5 \ell) \\
B &\rightarrow (K^{(*)}, \pi, X_s, X_d, \ldots) \ell \ell
\end{align*}
\]

charmonium resonances:

\[
\begin{align*}
Q_2 &= (\bar{q}_L \gamma_\mu c_L) (\bar{c}_L \gamma_\mu b_L) \\
B &\rightarrow (K^{(*)}, \pi, X_s, X_d, \ldots) (\psi_{cc} \rightarrow \ell \ell)
\end{align*}
\]
$B \to K\ell\ell$: power corrections

- The very schematic expression for the amplitude is ($B\to K\ell\ell$ at low-$q^2$):

$$A(B \to K\ell\ell) \sim C_7 f_T + C_9 f_+ + C_{10} f_+ + \sum_{i \neq 7,9,10} C_i \langle K| T J_{em} Q_i |B \rangle$$

$$\sim C_7 f_T + C_9 f_+ + C_{10} f_+ + \sum_{i \neq 7,9,10} C_i \left[ A_i^T f_T + A_i^+ f_+ \right] + \phi_B \otimes H_i \otimes \phi_K$$

- Every term in the amplitude not proportional to $C_{7,9,10}$ receives a $O(10\%)$ power correction [Fermilab/MILC, EL 1507.01618 and 1510.02349]

- If you factorize the form factors themselves, power corrections are much larger (this is required to construct “optimized observables” like $P_{5'}$ in $B \to K^*\ell\ell$)

- One can parametrize and fit power corrections to data:

  - [1006.4945; Khodjamirian, Mannel, Pivovarov, Wang] $\to$ Estimate using LCSR and dispers. relations
  - [1212.2263; Jäger, Camalich] $\rightarrow$ $q^2$ dependent parametrization of power corrections.
  - [1512.07157; Ciuchini et al.] $\to$ Ascribe $b \to K^*\ell\ell$ tensions to $q^2$ dependent power corrections.

  The fit points to PC’s of order (20-50)$\%$ of the whole amplitude

- Non-factorizable corrections

- Exact power corrections

- Power corrections

see talks by Siavash and Javier

Enrico Lunghi

Moriond QCD 2017
$B \rightarrow K\ell\ell$: lattice results confronting experiment

- Using the most recent Fermilab/MILC form factors:

\[\Delta B(B^+ \rightarrow K^+\mu^+\mu^-)^{\text{SM}} \times 10^9 = \begin{cases} 174.7(9.5)(29.1)(3.2)(2.2) , & 1.1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2, \\ 106.8(5.8)(5.2)(1.7)(3.1) , & 15 \text{ GeV}^2 \leq q^2 \leq 22 \text{ GeV}^2, \end{cases}\]

\[\Delta B(B^0 \rightarrow K^0\mu^+\mu^-)^{\text{SM}} \times 10^9 = \begin{cases} 160.8(8.8)(26.6)(3.0)(1.9) , & 1.1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2, \\ 98.5(5.4)(4.8)(1.6)(2.8) , & 15 \text{ GeV}^2 \leq q^2 \leq 22 \text{ GeV}^2, \end{cases}\]

- Errors are (CKM elements)(Form Factors)(matching scale)(everything else)
- Power correction error is about 1%

- Experimental LHCb results are:

\[\Delta B(B^+ \rightarrow K^+\mu^+\mu^-)^{\text{exp}} \times 10^9 = \begin{cases} 118.6(3.4)(5.9) , & 1.1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2, \\ 84.7(2.8)(4.2) , & 15 \text{ GeV}^2 \leq q^2 \leq 22 \text{ GeV}^2, \end{cases}\]

\[\Delta B(B^0 \rightarrow K^0\mu^+\mu^-)^{\text{exp}} \times 10^9 = \begin{cases} 91.6 \left(\frac{+17.2}{-15.7}\right)(4.4) , & 1.1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2, \\ 66.5 \left(\frac{+11.2}{-10.5}\right)(3.5) , & 15 \text{ GeV}^2 \leq q^2 \leq 22 \text{ GeV}^2, \end{cases}\]

- We observe a 2σ tension
$B \to (\pi, K, K^*)_{ll}$: Wilson coefficients fits

- $B \to K^*_{ll}$ fit results taken from [1503.06199; Altmannshofer, Straub]
$K \rightarrow \pi\pi$ amplitudes and $\varepsilon'_K / \varepsilon_K$

- Need to calculate $\Delta I=1/2$ and $\Delta I=3/2$ $K\rightarrow\pi\pi$ matrix elements for the 10 operators that appear in the $\Delta S=1$ effective Hamiltonian [RBC/UKQCD 1502.00263 and 1505.07863]

- The long standing mysterious enhancement of the $\Delta I=1/2$ channel (the so-called $\Delta I=1/2$ rule) is explained as an accidental cancellation between contributions to $\text{Re}(A_2)$ which does not occur for $\text{Re}(A_0)$

- Real and imaginary parts of the matrix elements are required to calculate:

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left( \frac{\omega e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2} \varepsilon} \left[ \frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] \right)$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\langle Q_i \rangle_0$</th>
<th>$\langle Q_i \rangle_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.151(44)</td>
<td>0.00965(59)</td>
</tr>
<tr>
<td>2</td>
<td>0.169(56)</td>
<td>0.00965(59)</td>
</tr>
<tr>
<td>3</td>
<td>-0.0492(661)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.271(111)</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-0.191(64)</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-0.379(128)</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.219(61)</td>
<td>0.286(11)</td>
</tr>
<tr>
<td>8</td>
<td>1.72(39)</td>
<td>1.314(76)</td>
</tr>
<tr>
<td>9</td>
<td>-0.202(70)</td>
<td>0.01447(89)</td>
</tr>
<tr>
<td>10</td>
<td>0.118(50)</td>
<td>0.01447(89)</td>
</tr>
</tbody>
</table>

$\text{Re}(A_0) = 4.66(1.00)(1.26) \times 10^{-7}$ GeV

$\text{Im}(A_0) = -1.90(1.23)(1.08) \times 10^{-11}$ GeV

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \approx \left\{ \begin{array}{ll} (16.6 \pm 2.3) \times 10^{-4} & \text{exp} \\ (1.36 \pm 5.15 \pm 4.59) \times 10^{-4} & \text{th} \end{array} \right.$$
$K \rightarrow \pi \pi$ amplitudes and $\varepsilon'_K / \varepsilon_K$

- Important interplay between $\varepsilon_K$, $\varepsilon'_K / \varepsilon_K$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [Lehner, EL, Soni, 1508.01801]

- Theory uncertainties will be dominated by lattice improvements (exclusive $V_{cb}$, Im($A_0$) and long distance charm contributions to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$)
backup...

“Jim was told that he could back up his data by making an image of his computer.”
Theoretical Tools

- **OPE:**
  \[
  T \left( O_i(x)O_j(y) \xrightarrow{\frac{y \rightarrow x}{y_{\mu}}} \sum_i C_i(x - y) \right) Q_i(x)
  \]

  We usually have \((x - y)^2 \sim 0\) instead of \(x_{\mu}^\mu - y_{\mu}^\mu \sim 0\): quark-hadron duality

\[
B \rightarrow X_s \ell \ell
\]

\[
\Gamma[B \rightarrow X_s \ell \ell] = \Gamma[b \rightarrow X_s \ell \ell] + O\left(\frac{\Lambda^2}{m_{b,c}^2}\right)
\]

\[
(x - y)^2 \sim p^{-2}_{Xs} \sim (m_b - \sqrt{q^2})^{-2}
\]

The OPE breaks down at large \(q^2\). Charmonium resonances can be included using \(ee \rightarrow \text{hadrons}\)

[hep-ph/9603237; Krüger, Sehgal]

[0902.4446; Beneke, Buchalla, Neubert, Sachrajda]

\[
B \rightarrow K^{(*)} \ell \ell
\]

\[
(x - y)^2 \sim q^{-2}
\]

The OPE breaks down at small \(q^2\). Charmonium resonances correspond to large \(x_{\mu}^\mu - y_{\mu}^\mu\) and must be dealt with invoking quark-hadron duality

[1101.5118; Beylich, Buchalla, Feldmann]
Non-perturbative inputs

- $f_B$ and $f_{B_s}$ from lattice [ALPHA$_2$, ETM$_{2+1}$, FNAL/MILC$_{2+1}$, HPQCD$_{2+1}$, RBC/UKQCD$_{2+1}$]
- Form Factors. LQCD: $B \rightarrow (\pi, K)$ [FNAL/MILC$_{2+1}$, HPQCD$_{2+1}$, RBC/UKQCD$_{2+1}$]

- modified z-expansion
- non-optimal treatment of the unstable $K^*$

$B_s \rightarrow K$ [HPQCD$_{2+1}$, RBC/UKQCD$_{2+1}$]

$B \rightarrow K^*$ and $B_s \rightarrow \phi$ [Horgan, Liu, Meinel, Wingate]$_{2+1}$

$B \rightarrow D$ [HPQCD$_{2+1}$, FNAL/MILC$_{2+1}$, (Atoui et al.)$_2$]

$B \rightarrow D^*$ [FNAL/MILC$_{2+1}$]

$\Lambda_b \rightarrow p$ and $\Lambda_b \rightarrow \Lambda_c$ [(Detmold, Lehner, Meinel)]$_{2+1}$

$\Lambda_b \rightarrow \Lambda$ [(Detmold, Meinel)]$_{2+1}$

LCSR: $B \rightarrow (\pi, K, \eta, Q, \omega, K^*)$ [Ball, Zwicky; Bharucha, Straub, Zwicky]

$B_s \rightarrow (K^*, \phi)$ [Ball, Zwicky; Bharucha, Straub, Zwicky]

- LCDA. $\phi_\pi$ and $\phi_K$: Lattice QCD gives the first few moments [Arthur et al.]
  $\phi_B$: Guesstimates! But see [1404.1343; Feldmann, Lange, Wang]

- Power Corrections
  - Inclusive: calculable in terms of local matrix elements.
  - Exclusive: within SCET they can be expressed in terms of new non-local matrix element but there are no current estimates beyond naive scaling
Recent Results (last year)

**Multi-hadron (n>2) exclusive decays**
- $B^\pm \rightarrow K^+ p\bar{p}$ [1505.07439; Di Salvo, Fontanelli]: naive factorization, some modeling, CP asymmetry
- $B \rightarrow D^- \pi^+ \pi^+ \pi^-$ [1506.03996; Talebtash, Mehraban]: naive factorization, HHχPT
- $B^\pm \rightarrow K^+ K^- K^\pm$ [1509.06979; Lesniak, Zenczykowski]: QCD factorization, KK rescattering in S,P and D wave, large strong phases, large CP asymmetry

**Two-hadron exclusive decays**
- $B_s \rightarrow \pi^+ \pi^- \ell \ell$ [1502.05104; Wang, Zhou]: non-resonant ($\rho \rightarrow \pi \pi$) study
- $B_{(s)} \rightarrow D_{(s)} D_{(s)}$ [1505.01361; Bel, De Bruyn, Fleischer, Mulder, Tuning]
- $B \rightarrow \pi \pi \ell \nu$ [1511.02509; Hambrock, Khodjamirian]: LCSR, di-pion form factor
- **Asymmetries in** $B \rightarrow K \pi$ [1510.05910; Liu, Li, Xiao]: pQCD and Glauber modes
- $\Lambda_b \rightarrow \Lambda(\phi, \eta, \eta')$ [1603.06682; Geng, Hsiao, Lin, Yu]: QCD factorization
- $B \rightarrow J/\psi K_1 \rightarrow J/\psi K \pi \pi$ [1604.07708; Kou, Le Yaouanc, Tayduganov]: Dalitz plot analysis to extract details of the $K_1$ decay. This can be used to extract the photon polarization in $B \rightarrow K\gamma$. 
Recent Results (last year)

- Single hadron exclusive decays
  - $B \to (\pi, D, D^*)\ell \nu$ and $B_s \to K\ell\nu$
    - A comment on the $R_D$ and $R_{D^*}$ anomaly: the measured $B \to (D, D^*)\tau \nu_{\tau}$ rates are (with reasonable estimates of higher resonances contributions) in disagreement with $B \to X_c\tau \nu_{\tau}$ predictions [1506.08896; Freytsis, Ligeti, Ruderman].
  - $\Lambda_b \to p\ell\nu$ [1503.01421; Detmold, Lehner, Meinel]: lattice QCD
    - $\Lambda_b \to (p\ell\nu, \Lambda\ell\ell)$ [1511.03540; Kozachuk, Melnikov, Nikitin]: Bethe-Salpeter equation approach
    - $\Lambda_b \to \Lambda\ell\ell$ [1602.01399; Detmold, Meinel. 1603.02974; Meinel, van Dyk]: lattice QCD
  - $B \to D_s^*\gamma$ and $B_s \to J/\psi\gamma$ [1511.03540; Kozachuk, Melnikov, Nikitin]: annihilation topologies
    - $B \to K_1\gamma$ [1604.07708; Kou, Le Yaouanc, Tayduganov]: photon polarization
  - $B \to (\pi, K, K^*)\ell\ell$
    - see upcoming slides

- Inclusive decays:
  - $B \to X_{s,d}\gamma$ and $B \to X_s\ell\ell$

- Leptonic decays
  - $B_q \to \ell\ell$: for reviews see [1405.4907, Bobeth. 1407.0916, Fleischer. 1407.2771, Knegjens]
  - $B \to \gamma\ell\nu$ [1604.08300; Yang, Yang]: Factorization at 1-loop, attempt to discuss soft photons
$B \rightarrow (\pi, K, K^*) \ell \ell$ : general considerations

**Typical spectrum:**

- **Low-$q^2$**
  - non-local power corrections
  - need more inputs (LCDA)
  - LQCD form factors need to be extrapolated from high-$q^2$

- **High-$q^2$**
  - need to integrate over several broad charmonium resonances

**SCET ($1/E_K$):**
- LCDA ($\pi, K, K^*$)
- Form Factors

**OPE ($1/q^2$):**
- Form Factors

**LCSR:** low-$q^2$ + z-parametrization

**LQCD:** high-$q^2$

---

- **$K^*$ vs $K$**
  - Disadvantages:
    - larger power corrections
    - $K^* \rightarrow K\pi$ decay (S vs P wave)
  - status of LQCD form factors
  - Advantages:
    - $K^* \rightarrow K\pi$ decay (angular observables)
$B \rightarrow (\pi, K, K^*)_{ll}$ : references

- Some references (last year):
  - 1502.05509; Descotes-Genon, Virto
  - 1502.00920; Hofer, Matias
  - 1503.03328; Descotes-Genon, Hofer, Matias, Virto
  - 1503.05534; Barucha, Straub, Zwicky
  - 1503.06199; Altmannshofer, Straub
  - 1503.09024; Becirevic, Fajfer, Kosnik
  - 1506.02661; Cabibbi, Crivellin, Ota
  - 1506.04535; Mandal, Sinha
  - 1506.06699; Das, Hiller, Jung
  - 1507.01618; Fermilab/MILC, EL
  - 1510.02349; Fermilab/MILC, EL
  - 1510.04239; Fermilab/MILC, EL
  - 1511.04015; Crivellin
  - 1511.04887; Dubnicka et al.
  - 1512.01560; Barbieri, Isidori, Pattori, Senia
  - 1512.07157; Ciuchini et al.
  - 1602.01372; Colangelo, de Fazio, Santorelli
  - 1603.00865; Hurth, Mahmoudi, Neshatpour
  - 1603.04355; Karan, Nayak, Sinha, Browder
  - 1605.02934; Crivellin
  - 1605.03156; Capdevila, Descotes-Genon, Matias, Virto

- find title 750 and GeV and date after 2014: 195 records found
$B \rightarrow (\pi, K, K^*) \ell \ell$ : differential rate

- $B \rightarrow K^* \ell \ell \rightarrow K \pi \ell \ell$ events can be described in terms of three angles: $(\theta_\ell, \phi, \theta_{K^*})$

- $B \rightarrow K^* \ell \ell$ fully differential rate:

$$
\frac{1}{d(\Gamma + \overline{\Gamma})/dq^2} \frac{d^3(\Gamma + \overline{\Gamma})}{d\Omega} = W_P + W_S
$$

$$
W_P = \frac{32}{\pi} \left[ J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_\ell \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + J_5 \sin 2\theta_K \sin \theta_\ell \cos \phi \\
+ (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_\ell + J_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\
+ J_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]
$$

$$
W_S = \frac{1}{4\pi} \left[ \tilde{J}_{1a}^c + \tilde{J}_{1b}^c \cos \theta_K + (\tilde{J}_{2a}^c + \tilde{J}_{2b}^c \cos \theta_K) \cos 2\theta_\ell + \tilde{J}_4 \sin \theta_K \sin 2\theta_\ell \cos \phi \\
+ \tilde{J}_5 \sin \theta_K \sin \theta_\ell \cos \phi + \tilde{J}_7 \sin \theta_K \sin \theta_\ell \sin \phi + \tilde{J}_8 \sin \theta_K \sin 2\theta_\ell \sin \phi \right]
$$

Enrico Lunghi
An attempt to use naive factorization with no relative strong phases to describe the resonant structure at high-$q^2$ fails:
[1406.0566; Zwicky, Lyon]

This should be interpreted as a failure of QCD factorization to describe the hadronic $B \to \psi_{cc} K$ process (e.g. color octet contributions might be important) and, most of all, its interference with the non-resonant rate
[1101.5118; Beylich, Buchalla, Feldmann]
\( B \rightarrow (\pi, K, K^*)_{ll} \): on power corrections

- Factorizable power corrections are a self inflicted wound:

\[
F_i(B \rightarrow K^*) = C_i \perp \xi \perp + C_i \parallel \xi \parallel + \sum_{a=\pm} \phi_B^a \otimes H_i^a \otimes \phi_{K^*} + \text{factorizable PC's}
\]

- Use the full form factors: LQCD (high-\( q^2 \)) and LCSR (low-\( q^2 \))

- Using z-expansions (e.g. 3 params) for the 7 form factors one can perform a LQCD+LCSR fit that gives the 19 z-fit parameters with a 19x19 correlation matrix

- Personally I prefer potential systematic issues in the LCSR approach to unknowable factorizable PC’s (that are many and enter everywhere)

- Clean observables (e.g. \( P_5' \)) are defined in such a way that form factors drop out up to factorizable and non-factorizable power corrections

  - If QCD factorization at leading power is a good description at low-\( q^2 \), uncertainties on these observables will be small once correlations between form factors uncertainties are taken into account
\[ B \rightarrow (\pi, K, K^*)_{\ell\ell} : \text{on power corrections} \]

- **LQCD vs LCSR** \( B \rightarrow K^* \) form factors:

  \[ [1310.3722; \text{Horgan, Liu, Meinel, Wingate}] \]
  \[ [\text{hep-ph/0412079; Ball, Zwicky}] \]
  \[ [1503.05534v2; \text{Barucha, Straub, Zwicky}] \]
\[ B \rightarrow (\pi, K, K^*) e e : \text{anomalies} \]

- \( P_5' \) is an “optimized observable”. At leading power is independent of FF’s:
  \[ P_5' = \frac{C_{10}(C_{9\perp} + C_{9\parallel})}{\sqrt{(C_{9\parallel}^2 + C_{10}^2)(C_{9\perp}^2 + C_{10}^2)}} + O(\alpha_s, \Lambda/m_b) \]

- Non-factorizable and non-factorizable power corrections

- \( R_K = 0.745 \pm 0.090 \pm 0.074 \pm 0.036 \)

- \( R_K = \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu \mu)}{\mathcal{B}(B^+ \rightarrow K^+ ee)} = 1 + O(10^{-4}) \)
The effects on $C_9$ and $C_9'$ are large enough to be easily checked at Belle II with inclusive decays [1410.4545; Hurth, Mahmoudi, Neshatpour]
The central problem is the calculation of hadronic matrix elements of the type 
\( \langle M | T O_i(x) J_{em}(y) | B \rangle \), \( \langle M_1 M_2 | O_i(x) | B \rangle \) or \( \langle B | T O_i(x) O_j(y) | B \rangle \)

- **HQET/SCET\(_{II}\)**
  
  Modes are systematically isolated in an effective theory framework
  
  Power expansion in \( 1/m_b \)
  
  Factorization proofs at all orders in perturbation theory and at leading power
  
  Matrix elements are expressed in terms of mesons Light Cone Distribution Amplitudes and Form Factors:
  
  \[
  A(B \rightarrow M_1 M_2) \sim F_{B \rightarrow M_1} \otimes H_4 \otimes \phi_{M_2} \\
  + \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2}
  \]

- **pQCD**
  
  Endpoint singularities are smeared by integrating over parton transverse momenta resulting in a Sudakov double log that can be resummed (**S**):
  
  \[
  A(B \rightarrow M_1 M_2) \sim \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes S
  \]
Theoretical Tools

- **Lattice QCD**: direct calculation of matrix elements, decay constants, form factors and some LCDA moments from first principles. Note that form factors are calculable at large \( q^2 \).

- **LCSR**. The calculation of form factors starts from the following correlator:

\[
\langle M|TO_i J_B|\Omega \rangle \sim \left\{ \sum_{\text{twist } n} H_i^{(n)} \otimes \phi_M^{(n)} \equiv \Pi^{\text{LCE}} \text{ at small } q^2 \right\} \\
\sum_X \left\{ \langle M|O_i|X \rangle \langle X|J_B|\Omega \rangle \sim F_{B \rightarrow M} \frac{f_B}{m_B^2-p_B^2} + \text{non pole} \right\}
\]

Introduce a Borel transformation, use a dispersion relation to describe the non-pole terms and assume quark-hadron duality:

\[
\hat{B} \Pi^{\text{LCE}} \sim F_{B \rightarrow M} f_B e^{-m_B^2/M^2} + \int_{s_0}^{\infty} \text{Im} \left[ \Pi^{\text{LCE}} \right] e^{-t^2/M^2}
\]

Ingredients: light-cone expansion at small \( q^2 \), Borel parameter \( M \), continuum threshold \( s_0 \), quark-hadron duality, decay constants and LCDA’s.
Theoretical Tools

- **OPE:** the T-product of operators evaluated at \( x^\mu \sim y^\mu \) is given in terms of a sum over local operators

\[
T \, O_i(x) O_j(y) \xrightarrow{y^\mu \to x^\mu} \sum_i C_i(x - y) \, Q_i(x)
\]

In inclusive \((B \to X_s \ell \ell)\) and exclusive \((B \to K(\ast) \ell \ell\) at high-\(q^2\)) decays we have \((x - y)^2 \sim 0\) instead of \(x^\mu - y^\mu \sim 0\): quark-hadron duality.

\[
\Gamma \left[ \bar{B} \to X_s \ell^+ \ell^- \right] = \Gamma \left[ \bar{b} \to X_s \ell^+ \ell^- \right] + O \left( \frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots \right)
\]
Theoretical Tools

- **OPE in inclusive vs exclusive decays:**

  **Inclusive**

  \[(x - y)^2 \sim \frac{1}{p_{X_s}^2} \sim \frac{1}{(m_b - \sqrt{q^2})^2}\]

  The OPE breaks down at large \(q^2\).

  Charmonium resonances can be included using \(e^+ e^- \rightarrow \text{hadrons}\)

  [hep-ph/9603237; Krüger, Sehgal]

  **Exclusive**

  \[(x - y)^2 \sim \frac{1}{q^2}\]

  Charmonium resonances correspond to large \(x^\mu - y^\mu\) and must be dealt with invoking quark-hadron duality

  [1101.5118; Beylich, Buchalla, Feldmann]
$B \rightarrow (\pi, K, K^*)\ell\ell$ : differential rate

- B→Kll rate at low-q$^2$:

\[
\frac{d\Gamma}{dq^2} \sim |f_+(q^2) C_{10}|^2 + \left| f_+(q^2) C_{9}^{\text{eff}} (q^2) + \frac{2m_b}{m_B + m_K} f_T(q^2) C_{7}^{\text{eff}} (q^2) \right|^2
\]

\[
+ \frac{2m_b \pi^2 f_B f_K}{m_B N_c m_B} \sum_{\pm} \int \frac{d\omega}{\omega} \Phi_{B,\pm} (\omega) \int_0^1 du \Phi_{K} (u) \left[ T_{P,\pm}^{(0)} + \tilde{\alpha}_s C_F T_{P,\pm}^{(nf)} \right]^2
\]

absent at high-q$^2$

- The form factor $f_T$ can be expressed in terms of $f_+$:

\[
\frac{m_B}{m_B + m_K} f_T = f_+ \left[ 1 + \tilde{\alpha}_s C_F \left( \log \frac{m_b^2}{\mu^2} + 2L \right) \right]
\]

\[
- \frac{\pi}{N_c} \frac{f_B f_K}{E} \alpha_s C_F \left\{ \int \frac{d\omega}{\omega} \Phi_{B,\pm} (\omega) \right\} \int_0^1 \frac{du}{\bar{u}} \Phi_{K} (u)
\]
\[ B \rightarrow (\pi, K, K^*)_{\ell\ell} : S/P \text{ wave pollution} \]

- In \( B \rightarrow K^*_{\ell\ell} \rightarrow K\pi_{\ell\ell} \), the \( K\pi \) system is produced in P wave (\( J^P(K^*)=1^- \))

- The \( K_0^*(800) \) resonance (\( J^P=0^+ \)) generates a \( K\pi \) pairs in S wave. This background can be removed by studying the \((\theta_\ell, \phi, \theta_{K^*})\) dependence of the differential width.

- Non-resonant \( K\pi \) decays are more problematic because their P-wave channel is an irreducible background to \( B \rightarrow K^*_{\ell\ell} \rightarrow K\pi_{\ell\ell} \)
  - At high-\( q^2 \) this background can be estimated using HH\( \chi \)PT but it is simpler to remove it using sideband subtraction.
  - At low-\( q^2 \) the situation is similar. See [1307.0947; Doering, Meissner, Wang] for a discussion based on pQCD.
Analytic structure of the $q^2$ plane:

Diagrammatically:

\[
\langle K^* | TJ^\mu(x)O_{1,2}(y)|B\rangle \sim h(q^2) f_+(q^2)
\]

highly non-local

Need to integrate over a large enough $q^2$ range
**B → Kll: power corrections**

- Factorizable power corrections are a self inflicted wound:
  \[ F_i(B \rightarrow K^*) = C_{i\perp} \xi_{\perp} + C_{i\parallel} \xi_{\parallel} + \sum_{a=\pm} \phi_B^a \otimes H_i^a \otimes \phi_{K^*} + \text{factorizable PC’s} \]

  - Use the full form factors: LQCD (high-\(q^2\)) and LCSR (low-\(q^2\))

  - Using z-expansions (e.g. 3 params) for the 7 form factors one can perform a LQCD+LCSR fit that gives the 19 z-fit parameters with a 19x19 correlation matrix

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  - If QCD factorization at leading power is a good description at low-\(q^2\), uncertainties on these observables will be small once correlations between form factors uncertainties are taken into account
$B \rightarrow (\pi, K, K^*) \ell \ell$ : on the fate of optimized observables

- $P_5' = \sqrt{2} \frac{\text{Re}(A_0^L A_{\perp}^{L*} - A_0^R A_{\perp}^{R*})}{\sqrt{|A_0|^2 (|A_{\perp}|^2 + |A_\parallel|^2)}}$

- **SCET/QCD factorization at low-$q^2$**:
  
  \[
  A_{\perp}^{L,R} = N_\perp \left[ C_{9_{\mp 10}}^+ V(q^2) + C_7^+ T_1(q^2) \right] + O(\alpha_s, \Lambda/m_b, \cdots) \\
  A_{\parallel}^{L,R} = N_\parallel \left[ C_{9_{\mp 10}}^- A_1(q^2) + C_7^- T_2(q^2) \right] + O(\alpha_s, \Lambda/m_b, \cdots) \\
  A_0^{L,R} = N_0 \left[ C_{9_{\mp 10}}^- A_{12}(q^2) + C_7^- T_{23}(q^2) \right] + O(\alpha_s, \Lambda/m_b, \cdots)
  \]

- **Factorization of the form factors (up to order $\alpha_s$ and $\Lambda/m_b$)**
  \[
  \frac{m_B}{m_B + m_{K^*}} V(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) = T_1(q^2) = \frac{m_B}{2E} T_2(q^2) = \xi_\perp(E), \\
  \frac{m_{K^*}}{E} A_0(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2) = \frac{m_B}{2E} T_2(q^2) - T_3(q^2) = \xi_\parallel(E)
  \]

- $P_5' = \frac{C_{10} (C_{9_{\perp}} + C_{9_{\parallel}})}{\sqrt{(C_{9_{\parallel}}^2 + C_{10}^2) (C_{9_{\perp}}^2 + C_{9_{10}}^2)}} + O(\alpha_s, \Lambda/m_b)$
$B \rightarrow (\pi, K, K^*) \ell \ell$ : anomalies

- $P_5^\prime$ is an “optimized observable”. At leading power is independent of FF’s:
  
  $$P_5^\prime = \frac{C_{10}(C_{9\perp} + C_{9\parallel})}{\sqrt{(C_{9\parallel}^2 + C_{10}^2)(C_{9\perp}^2 + C_{10}^2)}} + O(\alpha_s, \Lambda/m_b)$$

  factorizable and non-factorizable power corrections

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