Measuring the leading hadronic contribution to the muon g-2 anomaly via $\mu e$ scattering

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Reference papers

A new approach to evaluate the leading hadronic corrections to the muon $g-2$ ★

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Muons $g-2$ anomaly

\[ \mu = g_\mu \frac{e\hbar}{2m_\mu c} s \]

\[ g_\mu = 2 \left( 1 + a_\mu \right) \]

muon–γ vertex

\[ \gamma(q) \rightarrow \mu(p_2) \rightarrow \mu(p_1) \]

\[ = (-ie) \bar{u}(p_2) \left[ \gamma^\mu F_E(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_M(q^2) \right] u(p_1) \]

Static (classical) limit

\[ F_E(0) = 1, \quad F_M(0) = a_\mu \]

Charge renormalization condition

QED/SM

Largest contribution

J. Schwinger (1948)

\[ \alpha/2\pi \]
**Summary of the present status**

- E821 experiment at BNL:
  \[ a_μ^{E821} = (11659208.9 \pm 6.3) \times 10^{-10} \] [0.54 ppm]

- The SM prediction:
  \[ a_μ^{SM} = (11659180.2 \pm 4.9) \times 10^{-10} \] [0.42 ppm] (DHMZ)

- 3.5σ discrepancy:
  \[ a_μ^{E821} - a_μ^{SM} = (28 \pm 8) \times 10^{-10} \]

- Significance is limited by:
  - Experimental uncertainty:
    New experiments planned at FNAL E989 and J-PARC, aiming to improve the precision x4.
  - Theoretical uncertainty:
    Theoretical precision is limited by low energy hadronic effects.

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\[ a_μ^{SM} = a_μ^{QED} + a_μ^{Weak} + a_μ^{HAD} \]

**Hadronic Vacuum Polarization**

\[ a_μ^{HLO} = (692.3 \pm 4.2) \times 10^{-10} \]

\[ \frac{δa_μ^{HLO}}{a_μ^{HLO}} \sim 0.6\% \]

We aim to
\[ \frac{δa_μ^{HLO}}{a_μ^{HLO}} \sim 0.3\% \]

by means of the new approach
**$a_\mu^{HLO}$ calculation with time-like data**

- Optical theorem and analyticity:
  \[
  \sigma(s)_{(e^+e^-\rightarrow had)} = \frac{4\pi}{s} \text{Im} \Pi_{\text{hadron}}(s)
  \]
  \[
  a_\mu^{HLO} = \frac{1}{4\pi^3} \int_{4m_e^2}^\infty ds \, K(s) \cdot \sigma(s)_{(e^+e^-\rightarrow had)}
  \]

- The main contribution to the integral is in the low energy region: highly fluctuating.

\[ K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \sim \frac{1}{s} \]

- Current precision at 0.6% needs to be reduced by a factor ~2 to be competitive with the planned g-2 experiments
The elastic scattering $\mu + e \rightarrow \mu + e$

$\alpha(t)$ through:

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha(0)} \right|^2$$

$t = q^2 < 0$

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta \alpha(t)}$$

$\Delta \alpha(t) = \Delta \alpha_{lep}(t) + \Delta \alpha_{had}(t)$

$$t = -m_\mu^2 \frac{x^2}{1-x} \left(10^{-3} \text{GeV}^2\right)$$
\( a_{\mu}^{HLO} \) space-like approach

- It requires just the single process \( \mu + e \rightarrow \mu + e \) elastic
  High intensity CERN muon beam of \( E_\mu \sim 150 \text{ GeV} \) colliding on atomic electrons at rest.
- **Highly boosted final state:**
  \( 0 < -t < 0.161 \text{ GeV}^2 \)
  \( 0 < x < 0.93 \) (peak is at \( x = 0.914 \))
  The range covers 87% of the integral.
- Beyond the kinematics limit the integral (13%) can be determined using pQCD & time-like data, and/or lattice QCD results.

\[
a_{\mu}^{HLO} = \frac{\alpha}{\pi} \int_0^1 dx \ (1-x) \cdot \Delta \alpha_{\text{had}} \left( -\frac{x^2 m_{\mu}^2}{1-x} \right)
\]

The expected shape of integral function

- \( x_{\text{peak}} \approx 0.914 \)
- \( t_{\text{peak}} \approx -0.108 \text{ GeV}^2 \)
Elastic scattering in the \((\theta_e, \theta_\mu)\) plane

\[ \theta_e^f = \arccos \left( \frac{1}{r} \sqrt{\frac{E_e^f - m_e}{E_e^f + m_e}} \right) \]

**Signal**

- \(x = 0.9, E_e = 88.5\) GeV
- \(x = 0.8, E_e = 35.0\) GeV
- \(x = 0.7, E_e = 17.8\) GeV
- \(x = 0.6, E_e = 9.8\) GeV
- \(x = 0.5, E_e = 5.5\) GeV
- \(x = 0.4, E_e = 2.9\) GeV
- \(x = 0.3, E_e = 1.4\) GeV
- \(x = 0.2, E_e = 0.5\) GeV
- \(x = 0.1, E_e = 0.1\) GeV

**Normalization region**

Muon beam momentum = 150 GeV
Detection technique

Modular apparatus covering the full angular acceptance with high uniformity. 20 layers of low Z material (Be or C) paired to Si strip planes.

Transverse dimension $\sim 10 \times 10$ cm$^2$

Measuring angles with high angular resolution $\sim 0.02$ mrad
Luminosity and statistical error

- With the CERN 150 GeV muon beam, which has an average intensity of \(~1.3 \times 10^7\) \(\mu/s\), incident on 20 Be layers, each 3 cm thick, and 2 years of data taking with a running time of \(2 \times 10^7\) s/yr, one can reach an integrated luminosity of

\[ \mathcal{L}_{\text{int}} \sim 1.5 \times 10^7\ \text{nb}^{-1} \]

- \(\mathcal{L}_{\text{int}}\) implies a statistical sensitivity of \(~0.3\%\) on \(a_\mu^{\text{HLO}}\) (\(\delta a_\mu^{\text{HLO}} \sim 2 \times 10^{-10}\))

- \(\sigma_{\text{LO}}(E_e > 1\ \text{GeV}) = 245\ \mu\text{b}\)
The role of Multiple Scattering (MSC)

**GEANT4, 1 GeV electrons, Be target**

Vertices of the $\mu + e \rightarrow \mu + e$ collisions will be uniformly distributed inside the target along the direction of the beam axis. The observable angles (electron and muon angles) depend therefore on the particles’ path length inside the material and on their energies. **We need a MSC model** to relate the observed angles to the scattering ones.
Modeling the MSC

![Graph showing the comparison between GEANT4 events and the Highland formula.]

- **Gaussian core, \( \sigma = 3.6 \text{ mrad} \)**
- **GEANT4 events**
- **Highland formula (PDG) = 3.593 mrad**

### Equation

\[
\sigma = \frac{13.6}{\beta pc} \sqrt{\frac{d}{X_0}} \left[ 1 + 0.038 \ln \left( \frac{d}{X_0} \right) \right]
\]

- For 1 GeV, \( d = 3 \text{ cm} \)
Resolution models

LO differential cross section
- Observable angular distribution
- Observable angular distribution, $E_e > 1$ GeV
- Observable angular distribution, $E_e > 2$ GeV

$E_e > 500$ MeV

electron angle [mrad]
Events in the \((\theta_e, \theta_\mu)\) plane

- Low energy electron scattered away from the original direction
- \(E_e > 1\) GeV
- Background events are mainly due to pair production: \(\mu\) and \(e^+\) looking as 2 body final state: \(\sim 10^{-8}\)
Convolu2on coefficients

Values of the coefficients ratio = observed / truth turn out to be 1. within few parts per mille

10^8 Geant-4 events
Detector optimization

• Modeling MSC effects.
  – Geant4 is the the tool: likely to be tuned on data.
• Target material and geometry.
  – Best low Z material and geometry.
  – Active target to determine vertex positions.
• Define the calorimeter layout: to measure electron energy (energy cut) and for particle ID
• Test beam at CERN M2 beam line by September 2017
  • Identify elastic scattering events: \(~10^4\) events expected.
  • Measure MSC effects, variable beam energy and targets’ thickness
Test Beam

Check Geant4 MSC prediction and populate the 2D \((\theta_e, \theta_\mu)\) scattering plane

- 27 Sep-3 October 2017 allocated at CERN in "H8 Beam Line"
- 5 Si strips planes: 2 before (upstream) and 3 after the target
- Max rate 10 kHz
- Beam energy in the range 90 - 190 GeV
Theory

1. Resummation of dominant corrections up to all orders, matched with NLO corrections.
   − Non-trivial issue: mass effects in this case are important.
2. NNLO corrections: some classes of NNLO re-usable from existing Bhabha calculations, some new due to different mass scales ($m_m$ and $m_e$). In any case, NNLO must be matched with 1. and 2. Reference: Eur. Phys. J. C 66 (2010) 585 and references therein.
3. Development of dedicated MC tools including all the above ingredients.
4. Detailed study of all the mentioned corrections, comparison among independent calculations, estimate of further-missing higher-order corrections.
Conclusions

• The proposed experiment can reach the required statistical precision of 0.3%

• We need to estimate systematics errors related to MSC.
  – In collaboration with Geant4 developers.
  – Planned test beam at CERN this year.

• From the theoretical side: NNLO MC event generation and fit for HLO.

• A proto-experiment at CERN M2 will require one module station only.

• The plan for the next year is of starting a Collaboration to write a TDR.