Precision measurement of the form factors of semileptonic charged kaon decays from NA48/2

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on behalf of the NA48/2 collaboration

Rencontres de Moriond 2018, La Thuile

QCD and High Energy Interactions
Outline

- NA48/2 experiment
- $K_{l3}$ form factors precision measurement
- Conclusion
2003+2004 ~ 6 months, ~ $2 \cdot 10^{11}$ K± decays, Flux ratio: K+/K− ~ 1.8

Simultaneous K+ and K− beams: large charge symmetrization of experimental conditions

Beams coincide within ~ 1 mm all along the 114 m decay volume
**NA48/2 detectors**

**Main detector components:**

- **Magnetic spectrometer (4 DCHs):**
  4 views/DCH inside a He tank
  $\Delta p/p = 1.02\% \oplus 0.044\% \cdot p$
  [$p$ in GeV/c].

- **Hodoscope**
  fast trigger;
  precise time measurement (150ps).

- **Liquid Krypton EM calorimeter (LKr)**
  High granularity, quasi-homogenous
  $\sigma_E/E = 3.2\%/E^{1/2} \oplus 9\%/E \oplus 0.42\%$
  $\sigma_x=\sigma_y=0.42/E^{1/2} \oplus 0.06\text{cm}$
  [E in GeV]. (0.15cm@10GeV).

- **Hadron calorimeter, muon veto counters, photon vetoes.**
\[ K^\pm \to \pi^0 l^\pm \nu \ (K_{l3}^\pm) \text{ form factors} \]

\[ d^2\Gamma/(dE_l\, dE_{\pi}) \sim A \ f_+^2(t) + B \ f_+(t) \ f_-(t) + C \ f_-^2(t) \] (without radiative effects), where

\[
t = (P_K - P_\pi)^2 = M_K^2 + M_\pi^2 - 2 \ M_K \ E_\pi
\]

\[ f_-(t) = (f_+(t) - f_0(t))(m_K^2 - m_\pi^2)/t . \]

\[ f_0 \text{ is «scalar» and } f_+ \text{ is «vector» FF, } E_l, E_\pi, E_\nu - \text{ in the kaon rest frame.}\]

\[
A = M_K\left(2 \ E_l \ E_\nu - M_K(E_{\pi \text{max}} - E_\pi)\right) + M_l^2 \ ((E_{\pi \text{max}} - E_\pi)/4 - E_\nu)
\]

\[
B = M_l^2 \ (E_\nu - (E_{\pi \text{max}} - E_\pi)/2) \quad \text{negligible for Ke3}
\]

\[
C = M_l^2 \ (E_{\pi \text{max}} - E_\pi)/4 \quad \text{negligible for Ke3}
\]

\[
E_{\pi \text{max}} = (M_K^2 + M_\pi^2 - M_l^2)/(2 \ M_K)
\]

<table>
<thead>
<tr>
<th>FF Parameterization</th>
<th>( f_+(t, \text{parameters}) )</th>
<th>( f_0(t, \text{parameters}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quadratic</strong> (linear for ( f_0(t) ))</td>
<td>( 1 + \lambda_+ t/m_\pi^2 + \frac{1}{2} \lambda''<em>+(t/m</em>\pi^2)^2 )</td>
<td>( 1 + \lambda'<em>0 t/m</em>\pi^2 )</td>
</tr>
<tr>
<td><strong>Pole</strong></td>
<td>( M_{\nu}^2 / (M_{\nu}^2- t) )</td>
<td>( M_{s}^2 / (M_{s}^2- t) )</td>
</tr>
<tr>
<td><strong>Dispersive</strong>*</td>
<td>( \exp( (\Lambda_+ + H(t)) \ t/m_\pi^2 ) )</td>
<td>( \exp( (\ln[C]-G(t)) \ t/(m_K^2 - m_\pi^2) ) )</td>
</tr>
</tbody>
</table>


We use MC radiative decay generator of C.Gatti [Eur.Phys.J. C45 (2006) 417–420] provided by KLOE collaboration. It includes \( f_0 = f_+ = 1 + \lambda'_+ t/m_\pi^2 \).
**Data:** 16 special runs from the NA48/2 data taken in 2004 (3 days)

**Trigger:** 1 charged track and $E_{\text{LKr}} > 10$ GeV

Registered:
- **1 track** (> 0 candidates): $P_e \geq 5$ GeV, $P_\mu \geq 10$ GeV.
- **2 LKr clusters** (> 1 candidates): $E > 3$ GeV, to closest track > 15 cm.
- Beam geometry and average momentum $P_b$ are measured from $K^\pm \rightarrow \pi^+\pi^+\pi^-$.

**Kaon momentum reconstruction**

Two solutions of the quadratic equation for $P_K$:

$$P_{1,2} = (\phi P_{Zb} \pm \sqrt{d}) / (E^2 - P_{Zb}^2),$$

where

$$\phi = 0.5 \left( M_K^2 + E^2 - P_t^2 - P_{Zb}^2 \right),$$

$$d = (\phi^2 P_{Zb}^2 - (E^2 - P_{Zb}^2)(M_K^2 E^2 - \phi^2))$$

When $d < 0$, we assume $d = 0$.

- **Best $P_K$** solution should be closest $P_{1,2}$ to the average beam momentum $P_b$.
- **A cut:** $-7.5$ GeV/c $< (P_K - P_b) < 7.5$ GeV/c
- **For each event,** separately for $K_{e3}$ and $K_{\mu3}$ selections, the combination with a minimum $\Delta P = |P_K - P_b|$ is the best candidate.
Decay vertex

CDA (previous analysis 2012):
- Systematic shift of the vertex closer to the beam
- High sensitivity to exact beam shape simulation

Neutral vertex (this analysis):
- $X_n, Y_n = \text{impact point of charged track at } Z=Z_n \text{ plane}$
- No transverse bias

Choice of the vertex affects the reconstructed $\gamma$ momenta directions

Neutral vertex radius $R$ restriction means (track - $\pi^0$) compatibility requirement
**Selection**

$\pi^0$ selection:
- A pair of clusters in-time (within 5 ns) without any in-time extra clusters
- Distance between the clusters in a pair $> 20$ cm
- $E(\pi^0) > 15$ GeV (for the trigger efficiency)
- Compatibility of neutral vertex $(X_n, Y_n, Z_n)$ with beam axis

Track selection:
- A good track in-time with the $\pi^0$ within 10 ns.
- No extra good track within 8 ns (against showers).
- If $2.0 > \frac{E_{LKr}}{P_{DCH}} > 0.9$, it is an electron of $K_{e3}$.
- If $\frac{E_{LKr}}{P_{DCH}} < 0.9$ (for true muons it cuts nothing) and there is a MUV muon associated, it is a $K_{\mu3}$ muon.
Final cuts

For $K_{e3}$
- $p_t^\nu$ (w.r.t. beam axis) > 0.03 GeV/c, against $K^\pm \rightarrow \pi^\pm \pi^0$ with $\pi^\pm$ misidentified as $e$ (when $E/P > 0.9$);

- $P_L(\nu)^2 = (E^\nu)^2/c^2 - (P_t^\nu)^2 > 0.0014 \text{ GeV}^2/c^2$, negative tail and zero region sensitive to beam shape

For $K_{\mu3}$
- against the background from $K^\pm \rightarrow \pi^\pm \pi^0$ with $\pi^\pm \rightarrow \mu^\pm \nu$
  - $m(\pi^\pm \pi^0) < 0.47 \text{ GeV/c}^2$
  - $m(\pi^\pm \pi^0) < (0.6 - P_t(\pi^0)) \text{ GeV/c}^2$
  - $m(\mu^\pm \nu) > 0.18 \text{ GeV/c}^2$ (to exclude $\pi^+$ mass region)

- a cut against $\pi^\pm \pi^0 \pi^0$: $(P_2 - P_1) < 60 \text{ GeV}$, a difference between two $P$ solution is large when one pion is missing
### Background

<table>
<thead>
<tr>
<th>Decay</th>
<th>Notation</th>
<th>Br, %</th>
<th>Ng, 10^6</th>
<th>$F_e$, 10^{-3}</th>
<th>$F_\mu$, 10^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pm \rightarrow \pi^\pm (\pi^0 \rightarrow 2\gamma)$</td>
<td>$2\pi$</td>
<td>20.66</td>
<td>393.2</td>
<td>0.270</td>
<td>0.264</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm 2(\pi^0 \rightarrow 2\gamma)$</td>
<td>$3\pi$</td>
<td>1.761</td>
<td>62.5</td>
<td>0.286</td>
<td>1.833</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm (\pi^0 \rightarrow e^+e^-\gamma)$</td>
<td>$2\pi D$</td>
<td>1.174</td>
<td>1.5</td>
<td>0.049</td>
<td>0.000</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \gamma (\pi^0 \rightarrow 2\gamma)$</td>
<td>$2\pi \gamma$</td>
<td>0.0275</td>
<td>35.3</td>
<td>0.004</td>
<td>0.044</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^0 \mu^\pm \nu (\mu \rightarrow e\nu)$</td>
<td>$K_{\mu 3}^e$</td>
<td>0.03353</td>
<td>174.3</td>
<td>0.004</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Br — branching ration  
Ng — number of MC generated events  
$F_e$ — estimated background contamination in $K_{e3}$ data  
$F_\mu$ — estimated background contamination in $K_{\mu 3}$ data

- BG contamination from $2\pi$ and $3\pi$: very small, $O(10^{-4} - 10^{-3})$  
- BG contamination from other channels: negligible
Experimental Dalitz plots and fits areas (5x5 MeV cells)

\[ 4.28 \times 10^6 \]
Data events
No Bg corr.

\[ 2.91 \times 10^6 \]
Data events
No Bg corr.

Kinematic limit without radiative \( \gamma \)

At least 20 experimental events per bin are required
Events-weighting fit procedure

- Experimental Dalitz plot is corrected for the simulated background.

- For each fit iteration, the model Dalitz plot is filled in with an MC simulated reconstructed center-of-mass pion and lepton energies. Each event is weighted by

$$w(\vec{\lambda}) = \frac{\rho_0(\vec{\lambda}; E_{\pi}^{\text{true}}, E_{l}^{\text{true}})}{\rho_0(\vec{\lambda}_{\text{gen}}; E_{\pi}^{\text{true}}, E_{l}^{\text{true}})}$$

where $\rho_0$ is the non-radiative Dalitz density formula.

- MINUIT package is searching for the $\vec{\lambda}$ parameters minimizing the standard $\chi^2$ value:

$$\chi^2 = \sum_{i,j} \frac{(D_{i,j} - MC_{i,j})^2}{(\delta D_{i,j})^2 + (\delta MC_{i,j})^2}$$

where $i, j$ means the Dalits plot cell indices, $D_{i,j}$ is the background-corrected experimental data content of the cell, $MC_{i,j}$ is the weighted MC bin content, and $\delta D_{i,j}, \delta MC_{i,j}$ are the corresponding statistical errors.

At least 20 data events per cell are required in the fit area, so $\chi^2$ works well.
Dalitz plot projections

- Data-Bg
- MC fit result (quadr.)
- (Data-Bg)/MC

Marginally significant slope within the radiative correction precision. Radiative effect uncertainty is taken into account as a contribution to systematics.

Small deviation in the Bg area. Bg-related uncertainty is included into syst. error.
## Results for the $K_{e3}$ analysis

<table>
<thead>
<tr>
<th></th>
<th>Quadratic parameterization (in units of $10^{-3}$)</th>
<th>Pole parameterization (in MeV)</th>
<th>Dispersive parameterization (in units of $10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda'_+$</td>
<td>$\lambda''_+$</td>
<td>$M_v$</td>
</tr>
<tr>
<td><strong>Central value</strong></td>
<td>23.52</td>
<td>1.60</td>
<td>896.8</td>
</tr>
<tr>
<td><strong>Stat. error</strong></td>
<td>0.78</td>
<td>0.30</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>Syst. error</strong></td>
<td>1.29</td>
<td>0.39</td>
<td>7.6</td>
</tr>
<tr>
<td><strong>Total error</strong></td>
<td>1.51</td>
<td>0.49</td>
<td>8.3</td>
</tr>
<tr>
<td>$\chi^2$/ndf</td>
<td>609.4/687</td>
<td>609.3/688</td>
<td>609.1/688</td>
</tr>
<tr>
<td><strong>Correlation coefficients</strong></td>
<td>-0.927</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
## Results for the $K_{\mu 3}$ analysis

<table>
<thead>
<tr>
<th>Quadratic parameterization (in units of $10^{-3}$)</th>
<th>Pole parameterization (in MeV)</th>
<th>Dispersive parameterization (in units of $10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda'<em>+ \quad \lambda''</em>+ \quad \lambda'_0$</td>
<td>$M_v \quad M_s$</td>
<td>$\Lambda_+ \quad \ln[C]$</td>
</tr>
<tr>
<td>Central value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$23.32 \quad 2.14 \quad 14.33$</td>
<td>$879.1 \quad 1196.4$</td>
<td>$23.55 \quad 186.68$</td>
</tr>
<tr>
<td>Stat. error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.08 \quad 1.06 \quad 1.11$</td>
<td>$8.1 \quad 18.1$</td>
<td>$0.50 \quad 5.12$</td>
</tr>
<tr>
<td>Syst. error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.50 \quad 0.96 \quad 1.25$</td>
<td>$13.5 \quad 28.8$</td>
<td>$0.97 \quad 9.23$</td>
</tr>
<tr>
<td>Total error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4.67 \quad 1.43 \quad 1.67$</td>
<td>$15.7 \quad 34.0$</td>
<td>$1.10 \quad 10.55$</td>
</tr>
<tr>
<td>$\chi^2$/ndf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$391.2/384$</td>
<td>$388.0/385$</td>
<td>$385.8/385$</td>
</tr>
<tr>
<td>Correlation coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.969 (\lambda'<em>+/\lambda''</em>+)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.851(\lambda'_+/\lambda'_0)$</td>
<td>$0.320$</td>
<td>$0.408$</td>
</tr>
<tr>
<td>$-0.810 (\lambda''_+/\lambda'_0)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Results for the joint $K_{l3}$ analysis

<table>
<thead>
<tr>
<th>Quadratic parameterization (in units of $10^{-3}$)</th>
<th>Pole parameterization (in MeV)</th>
<th>Dispersive parameterization (in units of $10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda'_+$</td>
<td>$\lambda''_+$</td>
<td>$\lambda'_0$</td>
</tr>
<tr>
<td>Central value</td>
<td>23.35</td>
<td>1.73</td>
</tr>
<tr>
<td>Stat. error</td>
<td>0.75</td>
<td>0.29</td>
</tr>
<tr>
<td>Syst. error</td>
<td>1.23</td>
<td>0.41</td>
</tr>
<tr>
<td>Total error</td>
<td>1.44</td>
<td>0.50</td>
</tr>
<tr>
<td>$\chi^2$/ndf</td>
<td>1004.6/1073</td>
<td>1001.1/1074</td>
</tr>
<tr>
<td>Correlation coefficients</td>
<td>$-0.954$ ($\lambda'<em>+ / \lambda''</em>+$)</td>
<td>$0.076$ ($\lambda'_+ / \lambda'_0$)</td>
</tr>
</tbody>
</table>

**Analysis has been performed:**

- for $K_{e3}$ and $K_{\mu3}$ separately
- for the combined $K_{l3}$ sample (joint fit)
**K_{e3} and K_{\mu3} results**

- comparison for quadratic fit: $\lambda^{'}, \lambda^{''}$, $\lambda^{'}_{0}$
- confidence contours of $1 \sigma$ (68% CL)
- black ellipse: NA48/2
- comparison to other experiments
Joint $K_{l3}$ results

- comparison for quadratic fit: $\lambda^\prime_+, \lambda^\prime_0, \lambda^\prime_0$
- confidence contours of 1 $\sigma$ (68% CL)
- black ellipse: NA48/2
- comparison to other experiments
Conclusion

- $K_{l3}$ form factors measurement is performed by NA48/2 on the basis of 2004 run selected $4.28 \cdot 10^6 (K_{e3})$ and $2.91 \cdot 10^6 (K_{\mu3})$ events.
- Result is competitive with the other ones in $K_{\mu3}$ mode, and a smallest error in $K_{e3}$ has been reached, that gives us also the most precise combined $K_{l3}$ result.
- For the first time both $K^+$ and $K^-$ $K_{e3}$ decays were studied together.
Spares
Bell distributions in a wide area

~ 3σ range is relatively well simulated as well as the very far tail.

But the discrepancy near ~5-10 is not described by the known background.

Sensitivity to the background variation at the very far tail (>20) is used to measure the Bg-related systematic uncertainty.

It looks like a small wide component of the beam, that becomes negligible for Bell > 11. For wider cuts final results are stable.
Focused scattering simulated in MC: 3% of beam kaons are additionally scattered into a series of rings with a different radius at focus > 2.2 cm.

This MC simplified modification is not used for the FF central values extraction (only for systematics estimate). So we need a wide radius cut to avoid the acceptance distortion, and also we need a vertex reconstruction, that is not too sensitive to the transverse general shift of the decay — it is a Neutral vertex rather than CDA.
Selection

Min bias trigger: 1 track and $E_{\text{LKr}} > 10 \text{ GeV}$ ((sevt->trigWord $>$ $11$) & $1$)

N of good clusters $> 1$:

- LKr standard nonlinearity correction for Data clusters (user_lkrcalcor_SC)
- LKr small final nonlinearity correction for MC clusters, extracted from $\pi^+\pi^0\pi^0$
  (see April 2007 talk of Di Lella and Madigozhin)
- LKr scale corrections from $K_{e3}$ E/P (different for Data and MC, sub-permill precision)
- Cluster status $<=$ $4$
- Cluster energy $<=$ $3 \text{ GeV}$
- Distance to dead cell $<=$ $2 \text{ cm}$
- Radius at LKr $<=$ $15 \text{ cm}$
- In standard LKr acceptance
- Distance to any in-time (within 10 ns) track impact point at LKr $<=$ $15 \text{ cm}$
- Distance to any another in-time (within 5 ns) cluster $>$ $10 \text{ cm}$

N of good tracks $> 0$:

- $P_e > 5 \text{ GeV}$, $P_\mu > 10 \text{ GeV}$ (muon case cut applied after identification)
- Track momenta $\alpha, \beta$ corrections both for data and MC
- If there is the associated LKr cluster, its cluster status $<=$ $4$
- Track quality $<=$ $0.6$
- Distance to dead cell $<=$ $2 \text{ cm}$
- Radius at every DCH(1,2,3,4) $>$ $15 \text{ cm}$
- Reject DCH tracks with $0 \text{ cm} < X(DCH4) < 6 \text{ cm} \& \& Y(DCH4) > 0$ (inefficient band)
- $K_{\mu3}$ DCH track: for all 3 MUV planes $R_{\text{MUV}} > 30 \text{ cm}$, $|X_{\text{MUV}},Y_{\text{MUV}}|<115 \text{ cm}$.
- LKr impact point is in LKr acceptance

In Monte Carlo everything is in-time
\( \pi^0 \) selection

- Check all the pairs of good in-time (within 5 ns) clusters

- Calculate \( \pi^0 \) time \( t_\pi \) (average of two \( \gamma \) ones) and reject the combination, if there is a good extra cluster in 5 nanoseconds around \( t_\pi \) (to suppress \( \pi^+\pi^0\pi^0 \) and showers).

- Make the projectivity correction for the experimental data and MC.

- Reject the pair, if the distance between the clusters is < 20 cm

- \( E_{\pi^0} > 15 \) GeV (for trigger efficiency: trigger \( E_{LKr} > 10 \) GeV).

- Calculate \( Z_n \) from two \( \gamma \), assuming \( \pi^0 \) mass

  - \(-1600 \text{ cm} < Z < 9000 \text{ cm}\)

- DCH flunge gamma cut for the both \( \gamma \)
Track selection and identification

For each found good $\pi^0$ check all the good tracks:

- In-time with $\pi^0$ (within 10 ns)
- There is no extra good track within 8 ns around the track time (against showers).
- If $2.0 > E/P > 0.9$, it is an electron ($K_{e3}$)
- If $E/P < 0.9$ and there is a muon associated, it is a muon ($K_{\mu3}$)

First iteration decay vertex position:

- $Z_{\text{decay}} = Z (\pi^0)$
- $X_{\text{decay}}, Y_{\text{decay}} = \text{impact point of reconstructed charged track on the transversal plane, defined by } Z_{\text{decay}}$
Blue field correction:

With the «first iteration vertex», we implement the Blue field correction, obtain corrected track slopes and recalculate vertex X,Y again.

Beam position correction:

We know the position of beam axis in space (it is always displaced slightly from the nominal Z axis). For the CMC tuning, these positions were measured for each run from $3\pi^\pm$ data many years ago.

We use these data to calculate all the relevant values with respect to the current run beam axis rather than with respect to nominal Z arrow. First of all, we calculate the vertex (x,y) with respect to the beam center $X_b, Y_b$ at this $Z_n$.  

**Vertex position cut (very wide):**

$$\sqrt{\left(\frac{(X-a_x(Z))}{\sigma_x(Z)}\right)^2 + \left(\frac{(Y-a_y(Z))}{\sigma_y(Z)}\right)^2} < 11.0$$

Here $a_x, a_y, \sigma_x$ and $\sigma_y$ are the functions of Z and represent the average position and width of the beam with respect to standard ($3\pi^\pm$) beam position.

They are obtained by Gaussian fit ($\pm 1.2$ cm around maximum) for Z slices, separately for MC and Data, for X and Y and for positive and negative beams. Then these points are parametrised as functions of Z by polinomes of 5-th degree of Z.
Final stage of the selection

- $P_L(v)^2 > 0.0014 \text{ GeV}^2$ for $K_{e3}$ only
- Quadratic equation for $P_K$ is solved, if no solutions, the combination is taken with zero discriminant. With the above $P_L(v)^2$ requirement, such a cases are rare for $K_{e3}$.
- Average beam momentum $P_b$ measured from $3\pi^\pm$ decays for each run is used to choose the best $P_K$ solution (closest to $P_b$ from two ones).
- $-7.5 \text{ GeV/c} < (P_K - P_b) < 7.5 \text{ GeV/c}$

- For $K_{\mu3}$, the cut against $K^\pm \rightarrow \pi^\pm\pi^0$ with $\pi^\pm \rightarrow \mu^\pm\nu$: $m(\pi^+\pi^0) < 0.47 \text{ GeV}$ and $m(\pi^+\pi^0) < (0.6 - P_t(\pi^0)) \text{ GeV}$;
- For $K_{\mu3}$, one more cut against $K^\pm \rightarrow \pi^\pm\pi^0$ with $\pi^\pm \rightarrow \mu^\pm\nu$: $m(\mu^\pm\nu) > 0.18 \text{ GeV}$;
- For $K_{\mu3}$ only: a cut against $\pi^+\pi^0\pi^0$: $(P_2 - P_1) < 60 \text{ GeV}$
  $\Leftrightarrow$ in terms of $P_K$ equation discriminant squared $d = ((P_2 - P_1)/2)^2$: $d < 900 \text{ GeV}^2$;
- For $K_{e3}$, the $v$ transversal momentum with respect to beam axis must be $P_t \geq 0.03 \text{ GeV}$: a cut against $K^\pm \rightarrow \pi^\pm\pi^0$ with $\pi^\pm$ misidentified as $e$ (when $E/P > 0.9$).

In every event, separately for $K_{e3}$ and $K_{\mu3}$, the combination with the minimum $\Delta P = |P_K - P_b|$ is choosen as the best candidate.
A complex nature of $(P_L^\nu)^2$ - dependent $K_{e3}$ systematic effect

1) Mismeasurement of decay transversal coordinates happens (in the neutral vertex case it also involves the LKr clusters mismeasurement).
2) As a consequence, a small mismeasurement of transversal $(P_t^\nu)^2$
3) As a consequence, a small mismeasurement of $(P_L^\nu)^2 = (E^\nu)^2 - (P_t^\nu)^2$
4) As a consequence, a small mismeasurement of $D = ((P^K_1 - P^K_2)/2)^2$
5) When $D$ itself is small or negative, even small $D$ mismeasurement is relatively not small.
6) Distorted $D$ changes in a different way the probability of the «best» $P^K$ choise (we take the closest to average true $<P^K>$) for different vertex definitions and for MC and Data, depending on true $P^K$ spectrum. The wrong choise may also depend on the correlations between true $P^K$ and the transversal decay coordinates.
7) Mistake in $P^K$ choise from two options may be not small, it is of the order of spectrum width (few GeV), and it leads to relatively big mismeasurement of Dalits plot variables, especially for $E_{\pi^*}$.
   - Correct simulation of this effect seems to be difficult, we have only a simple beam correction for the scattered component.
   - But we know, where the problem is concentrated (small $(P_L^\nu)^2$), so we just cut the problematic region.
For $K_{\mu 3}$ only: a cut against $\pi^\pm \pi^0\pi^0: (P_2-P_1)<60$ GeV $\leftrightarrow D = ((P_2-P_1)/2)^2 < 900$ GeV$^2$

Equally normalized distributions of signal and background events are shown in order to check that the cut is doing its work in both cases.

But the absolute $K_{e 3}$ background level is much smaller than for $K_{\mu 3}$.
So we don't use this cut for $K_{e 3}$ and save some experimental statistics.
For $K_{e3}$, the $\nu$ transversal momentum with respect to beam axis must be $P_t \geq 0.03$ GeV.

It is a cut against $K^{\pm} \rightarrow \pi^{\pm} \pi^0$ with $\pi^{\pm}$ misidentified as $e$ (when $E/P > 0.9$).
Cuts for $K_{\mu 3}$ against the background from $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ with $\pi^{\pm} \rightarrow \mu^{\pm} \bar{\nu}$

$\sqrt{m(\pi^{+} \pi^{0})} < 0.47 \text{ GeV/c}^2$ and $\sqrt{m(\pi^{+} \pi^{0})} < (0.6 - P_t(\pi^{0})) \text{ GeV/c}^2$ (to exclude $\pi^+$ mass region)

$m(\mu^{\pm} \bar{\nu}) > 0.18 \text{ GeV/c}^2$
**Ke3** requirement: \( P_L(\nu)^2 > 0.0014 \text{ GeV}^2 \)

\[ P_L(\nu)^2 = (E\nu)^2 - (P_t\nu)^2 \]

The negative tail is difficult to simulate precisely, as it depends on the beam transverse shape (scattering) via \( P_t\nu \).

For **Ke3** only the region of small and negative \( P_L(\nu)^2 \) induces a systematic FF uncertainty (\( P_L(\nu)^2 \) dependence), that is avoided by this cut.

Peak sharpness residual mismatch is used to check \( P_L(\nu)^2 \) resolution systematics related to this cut.
Neutral Z normalized distributions comparison

Residual discrepancy (~1%) is taken into account as a contribution to systematic uncertainty = variation of final result due to the change of geometrical acceptance by the factor of 1.002, that corrects the $K_{e3}$ differences.
# Experimental systematics

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Approach to the uncertainty calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam scattering</td>
<td>Effect of the additional beam fraction imitating the beam scattering</td>
</tr>
<tr>
<td>LKr nonlinearity</td>
<td>Effect of the final nonlinearity correction</td>
</tr>
<tr>
<td>LKr scale</td>
<td>Effect of the LKr scale shift allowed by Data/MC electron E/P peak</td>
</tr>
<tr>
<td>Background</td>
<td>Effect of the background contribution change within $B_{ell}$ distribution tails</td>
</tr>
<tr>
<td></td>
<td>Data/MC agreement. It absorbs the PDG branching fraction errors</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>Effect of the measured quadratically smoothed trigger efficiency (~100%)</td>
</tr>
<tr>
<td>Accidentals</td>
<td>Effect of the time windows doubling for clusters and tracks acceptance</td>
</tr>
<tr>
<td>Acceptance</td>
<td>Effect of small transversal detector cuts increasing for MC, that (over) corrects Z distributions</td>
</tr>
<tr>
<td>$P_K$ average</td>
<td>Effect of beam $&lt;P_K&gt;$ possible mismeasurement</td>
</tr>
<tr>
<td>$P_K$ spectra</td>
<td>Effect of the MC true $P_K$ spectra variation within the agreement of measured MC/Data $P_K$ spectra</td>
</tr>
<tr>
<td>Neutrino P cut</td>
<td>Effect of the artificial $(P_L^\nu)^2$ resolution variation within $(P_L^\nu)^2$ peak sharpness MC/Data agreement</td>
</tr>
<tr>
<td>Binning</td>
<td>Effect of the bins doubling for the both Dalitz plot dimensions</td>
</tr>
<tr>
<td>Resolution</td>
<td>Difference between the main events weighting approach and the acceptance correction technique that is more sensitive to resolution</td>
</tr>
</tbody>
</table>
## External contributions to systematic uncertainty.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Approach to the uncertainty calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiative correction precision</td>
<td>Effect of the theoretical uncertainty in the radiative Dalitz plot corrections in terms of one-dimensional slopes.</td>
</tr>
<tr>
<td>Parameterization for Dispersive fits</td>
<td>100 fits with the independently sampled 5 external parameters known with a given uncertainty.</td>
</tr>
</tbody>
</table>

The full analysis is performed and form factor parameters are extracted:

- For $K_{e3}$

- For $K_{\mu3}$

- For the combined $K_{l3}$ result: A joint fits are done by minimizing of the sum $\chi^2(K_{e3}) + \chi^2(K_{\mu3})$ with a common set of fit parameters. This is repeated also for each of the systematic uncertainty studies.
**LKr Nonlinearity**

Use 2004 $\pi^0\pi^0\pi^-$ data (done for cusp analysis):

- $22 < E(\pi^0_1) < 26 \text{ GeV}$
- $E(\pi^0_2) < E(\pi^0_1)$
- $E(\gamma)^{\text{max}} < 0.55 E(\pi^0)$ for both $\pi^0$

**Final correction for MC:**

- $P_0 = 1.0170$
- $P_1 = -0.48025\times10^{-2}$
- $P_2 = 0.45538\times10^{-3}$
- $P_3 = -0.14474\times10^{-4}$

- $E$: cluster energy in GeV

- $f = P_0 + P_1 E + P_2 E^2 + P_3 E^3$
  
- if($f > 1$) $E = E/f$

100% of the final correction effect is taken as the nonlinearity-related uncertainty.

---

**Graphs:**

1. **Graph 1:**
   - $M(\pi^0_2)/M(\pi^0_1)$
   - Black: Data
   - Red: MC
   - Blue: Data
   - Mainly resolution-caused drop. But the MC/Data discrepancy may be due to difference in nonlinearity (reasonable resolution variation did not help).

2. **Graph 2:**
   - MC/Data
   - $E(\pi^0)/2 \sim E(\gamma)$
Blue field correction:

With the «first iteration vertex», we implement the Blue field correction, obtain corrected track slopes and recalculate vertex X,Y again.

Beam position correction:

We know the position of beam axis in space (it is always displaced slightly from the nominal Z axis). For the CMC tuning, these positions were measured for each run from $3\pi^\pm$ data many years ago.

We use these data to calculate all the relevant values with respect to the current run beam axis rather than with respect to nominal Z arrow. First of all, we calculate the vertex (x,y) with respect to the beam center $X_b, Y_b$ at this $Z_n$.

Vertex position cut (very wide):

$$\sqrt{\left(\frac{(X-a_X(Z))/\sigma_X(Z)}{2} + \left(\frac{(Y-a_Y(Z))/\sigma_Y(Z)}{2}\right)^2\right)} < 11.0$$

Here $a_X$, $a_Y$, $\sigma_X$ and $\sigma_Y$ are the functions of Z and represent the average position and width of the beam with respect to standard $(3\pi^\pm)$ beam position.

They are obtained by Gaussian fit ($\pm 1.2$ cm around maximum) for Z slices, separately for MC and Data, for X and Y and for positive and negative beams. Then these points are parametrised as functions of Z by polynomials of 5-th degree of Z.

17 - 24 March, 2018  Moriond 2018, La Thuile, Italy
### Results for $K_{e3}$ and $K_{\mu3}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\chi^2$/NDF($K_{e3}$): 609.4/687</th>
<th>$\chi^2$/NDF($K_{\mu3}$): 391.2/384</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central values</td>
<td>$\lambda_+''(K_{e3})$</td>
<td>$\lambda_+''(K_{\mu3})$</td>
</tr>
<tr>
<td>Stat. error</td>
<td>23.52</td>
<td>1.60</td>
</tr>
<tr>
<td>Beam scattering</td>
<td>0.78</td>
<td>0.30</td>
</tr>
<tr>
<td>LKr nonlinearity</td>
<td>0.90</td>
<td>0.32</td>
</tr>
<tr>
<td>LKr scale</td>
<td>0.28</td>
<td>0.01</td>
</tr>
<tr>
<td>Background</td>
<td>0.68</td>
<td>0.12</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>Accidentals</td>
<td>0.24</td>
<td>0.08</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.28</td>
<td>0.08</td>
</tr>
<tr>
<td>Pk average</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Pk spectra</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td>Neutrino P cut</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Binning</td>
<td>0.05</td>
<td>0.00</td>
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<tr>
<td>Resolution</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Radiative</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>Syst. error</td>
<td>1.29</td>
<td>0.39</td>
</tr>
<tr>
<td>Total error</td>
<td>1.51</td>
<td>0.49</td>
</tr>
</tbody>
</table>

#### Quadratic parameterization
(in units of $10^{-3}$)

- $\chi^2/NDF(K_{e3})$: 609.4/687
- $\chi^2/NDF(K_{\mu3})$: 391.2/384

#### Correlation

- $-0.927$

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### Pole parameterization (in units of $10^{-3}$)

<table>
<thead>
<tr>
<th>$\chi^2$/NDF($K_{e3}$):</th>
<th>$\chi^2$/NDF($K_{\mu3}$):</th>
</tr>
</thead>
<tbody>
<tr>
<td>609.3/688</td>
<td>388.0/385</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>$m_V(K_{e3})$</th>
<th>$m_V(K_{\mu3})$</th>
<th>$m_S(K_{\mu3})$</th>
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<tbody>
<tr>
<td>Central values</td>
<td>896.8</td>
<td>879.1</td>
<td>1196.4</td>
</tr>
<tr>
<td>Stat. error</td>
<td>3.4</td>
<td>8.1</td>
<td>18.1</td>
</tr>
<tr>
<td>Beam scattering</td>
<td>1.4</td>
<td>7.6</td>
<td>22.6</td>
</tr>
<tr>
<td>LKr nonlinearity</td>
<td>3.5</td>
<td>9.6</td>
<td>6.2</td>
</tr>
<tr>
<td>LKr scale</td>
<td>5.3</td>
<td>4.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Background</td>
<td>0.4</td>
<td>1.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.8</td>
<td>0.1</td>
<td>12.7</td>
</tr>
<tr>
<td>Accidentalss</td>
<td>0.5</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Acceptance</td>
<td>1.3</td>
<td>2.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Pk average</td>
<td>0.3</td>
<td>0.2</td>
<td>9.0</td>
</tr>
<tr>
<td>Pk spectra</td>
<td>0.1</td>
<td>0.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Neutrino P cut</td>
<td>1.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Binning</td>
<td>0.7</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.6</td>
<td>2.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Radiative</td>
<td>3.2</td>
<td>0.8</td>
<td>1.6</td>
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<tr>
<td>Syst. error</td>
<td>7.6</td>
<td>13.5</td>
<td>28.8</td>
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<tr>
<td>Total error</td>
<td>8.3</td>
<td>15.7</td>
<td>34.0</td>
</tr>
</tbody>
</table>

### Dispersion parameterization (in units of $10^{-3}$)

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda_+(K_{e3})$</th>
<th>$\Lambda_+(K_{\mu3})$</th>
<th>ln<a href="K_%7B%5Cmu3%7D">C</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>Central values</td>
<td>22.54</td>
<td>23.55</td>
<td>186.68</td>
</tr>
<tr>
<td>Stat. error</td>
<td>0.20</td>
<td>0.50</td>
<td>5.12</td>
</tr>
<tr>
<td>Beam scattering</td>
<td>0.09</td>
<td>0.48</td>
<td>7.05</td>
</tr>
<tr>
<td>LKr nonlinearity</td>
<td>0.20</td>
<td>0.60</td>
<td>2.08</td>
</tr>
<tr>
<td>LKr scale</td>
<td>0.31</td>
<td>0.26</td>
<td>0.50</td>
</tr>
<tr>
<td>Background</td>
<td>0.02</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.04</td>
<td>0.01</td>
<td>3.62</td>
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<tr>
<td>Accidentals</td>
<td>0.03</td>
<td>0.00</td>
<td>0.09</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.08</td>
<td>0.16</td>
<td>0.35</td>
</tr>
<tr>
<td>Pk average</td>
<td>0.02</td>
<td>0.01</td>
<td>2.62</td>
</tr>
<tr>
<td>Pk spectra</td>
<td>0.00</td>
<td>0.00</td>
<td>0.46</td>
</tr>
<tr>
<td>Neutrino P cut</td>
<td>0.07</td>
<td>0.00</td>
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</tr>
<tr>
<td>Binning</td>
<td>0.04</td>
<td>0.03</td>
<td>1.24</td>
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<tr>
<td>Resolution</td>
<td>0.03</td>
<td>0.10</td>
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<tr>
<td>Radiative</td>
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<tr>
<td>Parameterization</td>
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<td>Syst. error</td>
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<tr>
<td>Total error</td>
<td>0.65</td>
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<td>10.55</td>
</tr>
</tbody>
</table>

Correlation 0.320

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