New Probes of New Physics with Leptonic Rare $B$ Decays

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Setting the Stage
General Features of $B_{s,d}^0 \rightarrow \ell^+\ell^-$ Decays

- Situation in the Standard Model (SM): → only loop contributions:

- Moreover: helicity suppression → branching ratio $\propto m_{\ell}^2$

⇒ strongly suppressed decays

- Hadronic sector: → very simple, only the $B_q$ decay constant $F_{B_q}$ enters:

$$\langle 0 | \bar{b} \gamma_5 \gamma_\mu q | B_q^0(p) \rangle = iF_{B_q} p_\mu$$

⇒ $B_{s,d}^0 \rightarrow \ell^+\ell^-$ belong to the cleanest rare $B$-meson decays
• High sensitivity to physics from beyond the Standard Model:

→ such as in NP models with leptoquarks, $Z'$ bosons ...

... also in SUSY + ... ???

→ particularly interesting: new (pseudo)-scalars: lift helicity suppression!?

[see also the talks by Gudrun Hiller, Andreas Crivellin, George Hou, ...]
Status of $B_{s,d}^0 \rightarrow \ell^+\ell^-$ Decays

- **Overview of branching ratio measurements:** $\rightarrow$ talk by Andrew Crocombe

![Graph showing branching ratios for B decays]

- **Comments:**
  - Only $B_{s}^0 \rightarrow \mu^+\mu^-$ has been observed: $\rightarrow$ highlight of LHC run 1.
  - First limits on $B_{s,d}^0 \rightarrow \tau^+\tau^-$: helicity suppression not very effective due to large $\tau$ mass but experimentally challenging due to $\tau$ reconstruction.
  - $B_{s,d} \rightarrow e^+e^-$ no attention (!?): huge helicity suppression in the SM!
Questions:

- Using the experimental $B_s^0 \rightarrow \mu^+\mu^-$ data obtained @ LHC as a guideline:
  - What are the constraints on New Physics, utilising new observables?
  - How large could be the $B^0_{s,d} \rightarrow \tau^+\tau^-$, $B^0_{s,d} \rightarrow e^+e^-$ branching ratios?
  - What is the impact of new sources of CP violation?

  $\rightarrow$ exploring $B^0_{s,d} \rightarrow \ell^+\ell^-$ at the high-precision frontier ...

- Discussion along the following two recent papers:

**Theoretical Framework**

- **Low-energy effective Hamiltonian for $\bar{B}_s^0 \rightarrow \ell^+\ell^-$:**

  \[ \mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2\pi}} V_{ts}^* V_{tb} \alpha \left[ C_{10}^{10} O_{10} + C_S^{\ell\ell} O_S + C_P^{\ell\ell} O_P + C_{10}^{\ell\ell'} O_{10}^* + C_S^{\ell\ell'} O_S^* + C_P^{\ell\ell'} O_P^* \right] \]

  \[ G_F: \text{Fermi's constant}, \ V_{tq}: \text{CKM matrix elements}, \ \alpha: \text{QED fine structure constant} \]

- **Four-fermion operators, with $P_{L,R} \equiv (1 \mp \gamma_5)/2$ and $b$-quark mass $m_b$:**

  \[
  \begin{align*}
  O_{10} &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell), \quad O_{10}^* &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \\
  O_S &= m_b (\bar{s}P_R b)(\bar{\ell}\ell), \quad O_S^* &= m_b (\bar{s}P_L b)(\bar{\ell}\ell) \\
  O_P &= m_b (\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell), \quad O_P^* &= m_b (\bar{s}P_L b)(\bar{\ell}\gamma_5 \ell)
  \end{align*}
  \]

  [Only operators with non-vanishing $\bar{B}_s^0 \rightarrow \mu^+\mu^-$ matrix elements are included]

- **The Wilson coefficients $C_{k}^{\ell\ell}, C_{k}^{\ell\ell'}$ encode the short-distance physics:**

  - **SM case:** only $C_{10}^{\ell\ell} \neq 0$, and is given by the *real* coefficient $C_{10}^{\text{SM}}$.
  
  - **Outstanding feature of $\bar{B}_s^0 \rightarrow \mu^+\mu^-$:** sensitivity to (pseudo)-scalar lepton densities $\rightarrow O_{(P)S}, O_{(P)S}^*$; WCs are still largely unconstrained.

[...; Altmannshofer, Niehoff and Straub (2017); Beneke, Bobeth and Szafron (2018); ...]
Impact of $B_s^0 - \bar{B}_s^0$ Mixing:

- **Quantum mechanics:** $\Rightarrow$ *time-dependent $B_s^0 - \bar{B}_s^0$ oscillations*:
  - Mass eigenstates $B^{(s)}_{H,L}$: $\Delta M_s \equiv M^{(s)}_H - M^{(s)}_L$, $\Delta \Gamma_s \equiv \Gamma^{(s)}_L - \Gamma^{(s)}_H$
  - CP-violating phase: $\phi_s = -2\delta\gamma + \phi^{\text{NP}}_s \sim -2^\circ + \phi^{\text{NP}}_s$
    $\rightarrow$ determined (in particular) from analyses of $B_s^0 \rightarrow J/\psi\phi$

- **Interference effects (as in non-leptonic $B_s$ decays):**

[De Bruyn, R.F., Knegjens, Koppenburg, Merk, Pellegrino & Tuning (2012)]
→ convenient to go to the rest frame of the decaying $\bar{B}_s^0$ meson:

- **Distinguish between the $\ell_L^+\ell_L^-$ and $\ell_R^+\ell_R^-$ helicity configurations:**

  $$|\ell_L^+\ell_L^-\rangle_{\text{CP}} \equiv (CP)|\ell_L^+\ell_L^-\rangle = e^{i\phi_{\text{CP}(\mu\mu)}}|\ell_R^+\ell_R^-\rangle$$

  $[e^{i\phi_{\text{CP}(\ell\ell)}}$ is a convention-dependent phase factor $\rightarrow$ cancels in observables$]$  

- **General expression for the decay amplitude [$\eta_L = +1$, $\eta_R = -1$]:**

  $$A(\bar{B}_s^0 \rightarrow \ell^+_\lambda \ell^-_\lambda) = \langle \ell^-_\lambda \ell^+_\lambda | \mathcal{H}_{\text{eff}} | \bar{B}_s^0 \rangle = -\frac{G_F}{\sqrt{2}\pi} V_{ts}^* V_{tb} \alpha$$

  $$\times F_{B_s} M_{B_s} m_\ell C_{10}^{\text{SM}} e^{i\phi_{\text{CP}(\ell\ell)}(1-\eta_\lambda)/2} \left[ \eta_\lambda P_{\ell\ell} + S_{\ell\ell} \right]$$

- **Combination of Wilson coefficient functions [CP-violating phases $\varphi_{P,S}^{\ell\ell}$]:**

  $$P_{\ell\ell} \equiv \frac{C_{10}^{\ell\ell} - C_{10}^{\ell\ell'}}{C_{10}^{\text{SM}}} + \frac{M_{B_s}^2}{2 m_\ell} \left( \frac{m_b}{m_b + m_s} \right) \left[ \frac{C_{P}^{\ell\ell} - C_{P}^{\ell\ell'}}{C_{10}^{\text{SM}}} \right] \xrightarrow{\text{SM}} 1$$

  $$S_{\ell\ell} \equiv \sqrt{1 - 4 \frac{m_s^2}{M_{B_s}^2} \frac{m_s^2}{2 m_\ell} \left( \frac{m_b}{m_b + m_s} \right) \left[ \frac{C_{S}^{\ell\ell} - C_{S}^{\ell\ell'}}{C_{10}^{\text{SM}}} \right]} \xrightarrow{\text{SM}} 0$$

  $[F_{B_s}: B_s \text{ decay constant, } M_{B_s}: B_s \text{ mass, } m_\ell: \ell \text{ mass, } m_s: \text{ strange-quark mass}]$
• Key observable to calculate time-dependent decay rates:

\[
\xi_{\lambda}^{\ell \ell} \equiv -e^{-i\phi_s} \left[ e^{i\phi_{\text{CP}}(B_s)} \frac{A(B_s^0 \to \ell^+\ell^-)}{A(B_s^0 \to \ell^+\ell^-)} \right]
\]

\[
\Rightarrow A(B_s^0 \to \ell^+\ell^-) = \langle \ell^-\ell^+ | H_{\text{eff}} | B_s^0 \rangle is also needed ...
\]

• Using \((\mathcal{CP})^\dagger (\mathcal{CP}) = \hat{1}\) and \((\mathcal{CP}) | B_s^0 \rangle = e^{i\phi_{\text{CP}}(B_s)} | \bar{B}_s^0 \rangle\) yields:

\[
A(B_s^0 \to \ell^+\ell^-) = -\frac{G_F}{\sqrt{2\pi}} V_{ts} V_{tb}^* \alpha_f B_s M_{B_s} m_{\ell} C_{10}^{\text{SM}}
\]

\[
\times e^{i[\phi_{\text{CP}}(B_s) + \phi_{\text{CP}}(\ell\ell)(1-\eta_\lambda)/2]} \left[ -\eta_\lambda P_{\ell\ell}^* + S_{\ell\ell}^* \right]
\]

• The convention-dependent phases cancel in \(\xi_{\lambda}^{\ell \ell} \ [\eta_L = +1, \eta_R = -1]\):

\[
\xi_{\lambda}^{\ell \ell} = -e^{-i\phi_{\text{NP}}^s} \left[ \frac{+\eta_\lambda P_{\ell\ell} + S_{\ell\ell}}{-\eta_\lambda P_{\ell\ell}^* + S_{\ell\ell}^*} \right] \Rightarrow \xi_{\lambda}^{\ell \ell} (\xi_{\lambda}^{\ell \ell})^* = \xi_{R}^{\ell \ell} (\xi_{L}^{\ell \ell})^* = 1
\]

[Note: analogous formalism for \(B_d \to \ell^+\ell^-\) decays; \(\Delta \Gamma_d / \Gamma_d\) is negligible.]
Application to

\[ B_s^0 \rightarrow \mu^+ \mu^- \]
Untagged $B_s^0 \to \mu^+ \mu^-$ Rate

• Interesting observable (well-known from studies of non-leptonic $B_s^0$ decays):

$$A_{\Delta \Gamma_s}^{\mu \mu, \lambda} \equiv \frac{2 \Re(\xi_{\lambda}^{\mu \mu})}{1 + |\xi_{\lambda}^{\mu \mu}|^2} = \frac{|P_{\mu \mu}|^2 \cos(2\varphi_{P}^{\mu \mu} - \phi_{sNP}^{\mu \mu}) - |S_{\mu \mu}|^2 \cos(2\varphi_{S}^{\mu \mu} - \phi_{sNP}^{\mu \mu})}{|P_{\mu \mu}|^2 + |S_{\mu \mu}|^2}$$

→ independent of the muon helicity $\lambda$: $A_{\Delta \Gamma_s}^{\mu \mu} \equiv A_{\Delta \Gamma_s}^{\mu \mu, \lambda}$

• Challenge to measure the muon helicity: $\rightarrow$ helicity-averaged rates:

$$\Gamma^{(\neg)}(B_s^0(t) \to \mu^+ \mu^-) \equiv \sum_{\lambda=L,R} \Gamma^{(\neg)}(B_s^0(t) \to \mu^+_\lambda \mu^-_\lambda)$$

• $B_s^0$ decay width difference $\Delta \Gamma_s$: $y_s \equiv \Delta \Gamma_s \tau_{B_s}/2 = 0.0645 \pm 0.0045$

$\Rightarrow$ access to $A_{\Delta \Gamma_s}^{\mu \mu}$ through the following untagged decay rate:

$$\langle \Gamma(B_s(t) \to \mu^+ \mu^-) \rangle \equiv \Gamma(B_s^0(t) \to \mu^+ \mu^-) + \Gamma(\bar{B}_s^0(t) \to \mu^+ \mu^-)$$

$$\propto e^{-t/\tau_{B_s}} \left[ \cosh(y_s t/\tau_{B_s}) + A_{\Delta \Gamma_s}^{\mu \mu} \sinh(y_s t/\tau_{B_s}) \right]$$
$B_s^0 \rightarrow \mu^+\mu^-$ Branching Ratio(s)

- First LHC measurement concerns the “experimental” branching ratio:

$$\rightarrow \text{time-integrated untagged rate:}$$

$$\overline{B}(B_s \rightarrow \mu^+\mu^-) \equiv \frac{1}{2} \int_0^\infty \langle \Gamma(B_s(t) \rightarrow \mu^+\mu^-) \rangle \, dt \quad \text{LHC} \equiv (3.0 \pm 0.5) \times 10^{-9}$$

- Relation to the “theoretical” branching ratio (referring to $t = 0$):

$$\overline{B}(B_s \rightarrow \mu^+\mu^-) = \left[ \frac{1 - y_s^2}{1 + \mathcal{A}_{\Delta \Gamma_s}^{\mu\mu} y_s} \right] \overline{B}(B_s \rightarrow \mu^+\mu^-)$$

- $\mathcal{A}_{\Delta \Gamma_s}^{\mu\mu}|_{\text{SM}} = +1$ gives a $\text{SM}$ reference value for the comparison with the time-integrated experimental branching ratio $\overline{B}(B_s \rightarrow \mu^+\mu^-)$:

$$\overline{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.57 \pm 0.16) \times 10^{-9}$$

[arXiv:1703.10160 [hep-ph]]

[De Bruyn, R.F., Knegjens, Koppenburg, Merk, Pellegrino & Tuning (2012)]
Effective $B_s^0 \rightarrow \mu^+\mu^-$ Lifetime

Collecting more and more data + include decay time information ⇒

- The effective $B_s \rightarrow \mu^+\mu^-$ lifetime can be measured:

$$\tau_{\mu\mu} \equiv \frac{\int_0^\infty t \langle \Gamma(B_s(t) \rightarrow \mu^+\mu^-) \rangle \, dt}{\int_0^\infty \langle \Gamma(B_s(t) \rightarrow \mu^+\mu^-) \rangle \, dt}$$

- Pioneering LHCb result: [arXiv:1703.05747 [hep-ex]]

$$\tau_{\mu\mu}^s = [2.04 \pm 0.44(\text{stat}) \pm 0.05(\text{syst})] \text{ ps}$$

- $A^\mu\mu_{\Delta\Gamma_s}$ can be extracted from the effective lifetime $\tau_{\mu\mu}^s$:

$$A^\mu\mu_{\Delta\Gamma_s} = \frac{1}{y_s} \left[ \frac{(1 - y_s^2)\tau_{\mu\mu} - (1 + y_s^2)\tau_{B_s}}{2\tau_{B_s} - (1 - y_s^2)\tau_{\mu\mu}} \right] \xrightarrow{\text{LHCb}} 8.24 \pm 10.72$$

⇒ LHCb upgrade era and beyond...
Probing New Physics through $B_s^0 \rightarrow \mu^+\mu^-$

- Useful to introduce the following ratio:

$$R_{\mu\mu}^s \equiv \frac{\mathcal{B}(B_s \rightarrow \mu^+\mu^-)}{\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}}} \xrightarrow{\text{SM}} 1$$

$$= \left[ \frac{1 + y_s \cos(2\varphi_{P}^{\mu\mu} - \phi_{NP}^{s})}{1 + y_s} \right] |P_{\mu\mu}|^2 + \left[ \frac{1 - y_s \cos(2\varphi_{S}^{\mu\mu} - \phi_{NP}^{s})}{1 + y_s} \right] |S_{\mu\mu}|^2$$

- Result following from current LHC data:

$$\overline{R}_{\mu\mu}^s = 0.84 \pm 0.16$$

- $\overline{R}_{\mu\mu}^s$ does not allow a separation of the $P_{\mu\mu}$ and $S_{\mu\mu}$ contributions:

$$\Rightarrow$$ sizeable NP could still be present ... 

[See also Buras, R.F., Girrbach & Knegjens (2013)]
• Further information comes from the measurement of $A_{\Delta \Gamma_s}^{\mu\mu}$:

$$|S_{\mu\mu}| = |P_{\mu\mu}| \sqrt{\frac{\cos(2\phi_P^{\mu\mu} - \phi_{NP}^s) - A_{\Delta \Gamma_s}^{\mu\mu}}{\cos(2\phi_S^{\mu\mu} - \phi_{NP}^s) + A_{\Delta \Gamma_s}^{\mu\mu}}}$$

• Constraints in the $P_{\mu\mu}$–$S_{\mu\mu}$ plane following from current data:

→ assume real coefficients (e.g. MFV without flavour-blind phases):

[CP-violating phases $\phi_P^{\mu\mu}, \phi_S^{\mu\mu} \neq 0^\circ, 180^\circ$ are discussed below]
Mapping Out
Further Decays:

\[ B_d \rightarrow \mu^+ \mu^- \]
\[ B_{s,d} \rightarrow \tau^+ \tau^- \]
\[ B_{s,d} \rightarrow e^+ e^- \]

New Physics Scenario

- We assume flavour-universal New Physics contributions:

\[ C_{10}^{\ell\ell'}, C_P^{\ell\ell'}, C_S^{\ell\ell'} \] do not depend on flavour labels:

\[
P_{\ell\ell}^q = \frac{C_{10} - C_{10}'}{C_{10}^{SM}} + \frac{M_{Bq}^2}{2m_\ell} \left( \frac{m_b}{m_b + m_q} \right) \left[ \frac{C_P - C'_P}{C_{10}^{SM}} \right]
\]

\[
S_{\ell\ell}^q \equiv \sqrt{1 - 4 \frac{m_\ell^2}{M_{Bq}^2}} \frac{M_{Bq}^2}{2m_\ell} \left( \frac{m_b}{m_b + m_q} \right) \left[ \frac{C_S - C'_S}{C_{10}^{SM}} \right]
\]

- Data for \( B \to K^{(*)}\ell^+\ell^- \) decays: \( C_{10} \equiv \frac{C_{10} - C_{10}'}{C_{10}^{SM}} \)

  - Use \( C_{10} = 1 \) as the working assumption \( \Rightarrow NP \) in (pseudo)-scalars.

- No new sources of CP violation: \( \Rightarrow \) real Wilson coefficients.
Linking $B_s^0 \rightarrow \mu^+ \mu^-$ with $B_q^0 \rightarrow \ell^+ \ell^-$ Decays

- Conversion of $R_{\mu \mu}^s$ and $A_{\Delta \Gamma_s}^{\mu \mu}$ into Wilson coefficients:

\[
|P_{\mu \mu}^s| = \sqrt{\frac{1}{2} (1 + y_s) \left[ \frac{1 + A_{\Delta \Gamma_s}^{\mu \mu}}{1 + y_s A_{\Delta \Gamma_s}^{\mu \mu}} \right]} R_{\mu \mu}^s \quad \Rightarrow \quad \left| \frac{C_P - C'_P}{C_{10}^{SM}} \right|
\]

\[
|S_{\mu \mu}^s| = \sqrt{\frac{1}{2} (1 + y_s) \left[ \frac{1 - A_{\Delta \Gamma_s}^{\mu \mu}}{1 + y_s A_{\Delta \Gamma_s}^{\mu \mu}} \right]} R_{\mu \mu}^s \quad \Rightarrow \quad \left| \frac{C_S - C'_S}{C_{10}^{SM}} \right|
\]
• Constraints for the Wilson coefficients from $B_s^0 \rightarrow \mu^+ \mu^-$:
Predictions for $B^0_d \to \mu^+\mu^-$

$$\frac{\mathcal{B}(B_d \to \mu^+\mu^-)}{\mathcal{B}(B_s \to \mu^+\mu^-)} \propto \left[ \frac{\left| P_{\mu\mu}^d \right|^2 + \left| S_{\mu\mu}^d \right|^2}{\left| P_{\mu\mu}^s \right|^2 + \left| S_{\mu\mu}^s \right|^2} \right] \left( \frac{f_{B_d}}{f_{B_s}} \right)^2 \left| \frac{V_{td}}{V_{ts}} \right|^2$$

- **Unitarity Triangle analysis:** $\Rightarrow |V_{td}/V_{ts}| = 0.220 \pm 0.010$

- **Flavour Universal New Physics Scenario:**

![Graphs showing predictions for $B_d^0 \to \mu^+\mu^-$ and $B_s \to \mu^+\mu^-$](image)
Predictions for $B^0_s \rightarrow \tau^+ \tau^-$ and $B^0_d \rightarrow \tau^+ \tau^-$

- Standard Model predictions and experimental LHCb upper bounds (2017):

$$\overline{B}(B_s \rightarrow \tau^+ \tau^-)_{\text{SM}} = (7.56 \pm 0.35) \times 10^{-7} < 6.8 \times 10^{-3} \ (95\% \ \text{C.L.})$$

$$\overline{B}(B_d \rightarrow \tau^+ \tau^-)_{\text{SM}} = (2.14 \pm 0.12) \times 10^{-8} < 2.1 \times 10^{-3} \ (95\% \ \text{C.L.})$$

$$R^s_{\tau\tau} \equiv \frac{\overline{B}(B_s \rightarrow \tau^+ \tau^-)}{\overline{B}(B_s \rightarrow \tau^+ \tau^-)_{\text{SM}}} \rightarrow 1$$

- Flavour Universal New Physics Scenario:

$$P^s_{\tau\tau} = \left(1 - \frac{m_\mu}{m_\tau}\right) C_{10} + \frac{m_\mu}{m_\tau} P^s_{\mu\mu}, \quad S^s_{\tau\tau} = \frac{m_\mu}{m_\tau} \sqrt{\frac{1 - 4 \frac{m^2_\tau}{M^2_{B_s}}}{1 - 4 \frac{m^2_\mu}{M^2_{B_s}}}} S^s_{\mu\mu}$$

$$\Rightarrow \ NP \ \text{effects strongly suppressed by } m_\mu/m_\tau \sim 0.06:$$

$$0.8 \leq R^s_{\tau\tau} \leq 1.0, \quad 0.995 \leq A^\tau_{\Delta\Gamma_s} \leq 1.000$$
Predictions for $B_s^0 \to e^+e^-$ and $B_d^0 \to e^+e^-$

- SM predictions and experimental CDF upper bounds (2009):

$$\overline{\mathcal{B}}(B_s \to e^+e^-)_{\text{SM}} = (8.35 \pm 0.39) \times 10^{-14} < 2.8 \times 10^{-7} \ (90\% \ \text{C.L.})$$

$$\overline{\mathcal{B}}(B_d \to e^+e^-)_{\text{SM}} = (2.39 \pm 0.14) \times 10^{-15} < 8.3 \times 10^{-8} \ (90\% \ \text{C.L.})$$

- Flavour Universal New Physics Scenario:

$$P_{ee}^s = \left(1 - \frac{m_\mu}{m_e}\right) C_{10} + \frac{m_\mu}{m_e} P_{\mu\mu}^s, \quad S_{ee}^s = \frac{m_\mu}{m_e} \sqrt{\frac{1 - 4\frac{m_\mu^2}{M_{B_s}^2}}{1 - 4\frac{m_\mu^2}{M_{B_s}^2}}} S_{\mu\mu}^s$$

$\Rightarrow$ NP effects hugely amplified by $m_\mu/m_e \sim 207$:

$$R_{e\mu,\mu\mu}^{ee} \equiv \frac{\overline{\mathcal{B}}(B_s \to e^+e^-)}{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)} \approx \frac{(C_{10} - P_{\mu\mu}^s)^2 + (S_{\mu\mu}^s)^2}{(P_{\mu\mu}^s)^2 + (S_{\mu\mu}^s)^2}$$
(Pseudo)-scalar New Physics contributions lift in this scenario the helicity suppression of the extremely small Standard Model branching ratio:

$$R_{ee}^S = \frac{\overline{B}(B_s \to e^+e^-)}{\overline{B}(B_s \to \mu^+\mu^-)}$$

$$R_{\mu\mu}^S = \frac{\overline{B}(B_s \to \mu^+\mu^-)}{\overline{B}(B_s \to \mu^+\mu^-)_{SM}}$$

[red: $P_{\mu\mu}^S < 0$, green: $P_{\mu\mu}^S > 0$]

$$\Rightarrow 0 \leq R_{ee}^S \leq 1.7 \times 10^5, \quad 0 \leq \overline{B}(B_s \to e^+e^-) \leq 1.4 \times 10^{-8}$$

Similar situation for $B_d \to e^+e^-$: \[0 \leq \overline{B}(B_d \to e^+e^-) \leq 4.0 \times 10^{-10}\]
⇒ search for $B_{s(d)} \rightarrow e^+ e^-$: may give an unambiguous NP signal!

◊ looking forward to the first LHC result for $\bar{B}(B_{s,d} \rightarrow e^+ e^-)$ ...
Impact of \textit{CP}-violating Phases

$\rightarrow$ focus on $B_s \rightarrow \mu^+\mu^-$:

General $B_s \rightarrow \mu^+ \mu^-$ Branching Ratio Constraints

- **Observable:** \( \overline{R}_{\mu\mu} \equiv \overline{R} = 0.84 \pm 0.16 \rightarrow \) useful quantity:

\[
\overline{R} = \overline{R} = \left| \frac{1 + y_s}{1 + A_{\Delta \Gamma_s}^\mu y_s} \right| \overline{R} = |P_{\mu\mu}|^2 + |S_{\mu\mu}|^2
\]

- **Constraints on** \(|P| \equiv |P_{\mu\mu}|, |S| \equiv |S_{\mu\mu}| \) **in the presence of unconstrained CP-violating phases** \( \varphi_P \equiv \varphi_P^{\mu\mu}, \varphi_S \equiv \varphi_S^{\mu\mu} \) **yielding** \(-1 \leq A_{\Delta \Gamma_s}^\mu \leq +1:\)

\[
\Rightarrow \text{how to narrow down (pseudo)-scalar NP contributions?}
\]
**CP Asymmetries of** $B_s \rightarrow \mu^+ \mu^-$ **Decays**

- **Time-dependent rate asymmetry:** requires tagging of $B^0_s$ and $\bar{B}^0_s$:

\[
\frac{\Gamma(B^0_s(t) \rightarrow \mu^+ \mu^-) - \Gamma(\bar{B}^0_s(t) \rightarrow \mu^+ \mu^-)}{\Gamma(B^0_s(t) \rightarrow \mu^+ \mu^-) + \Gamma(\bar{B}^0_s(t) \rightarrow \mu^+ \mu^-)} = \frac{C^\lambda_{\mu\mu} \cos(\Delta M_s t) + S^\lambda \sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B^0_s}) + A^{\mu\mu,\lambda}_{\Delta \Gamma_s} \sinh(y_s t/\tau_{B^0_s})}
\]

- **Observables:** theoretically clean (no dependence on $F_{B_s}$): $\text{SM} \rightarrow 0$

\[
C^\lambda_{\mu\mu} \equiv \frac{1 - |\xi^\lambda|^2}{1 + |\xi^\lambda|^2} = -\eta^\lambda \left[ \frac{2|PS| \cos(\varphi_P - \varphi_S)}{|P|^2 + |S|^2} \right] \equiv -\eta^\lambda C_{\mu\mu}
\]

\[
S^\lambda_{\mu\mu} \equiv \frac{2 \Im(\xi^\lambda)}{1 + |\xi^\lambda|^2} = \frac{|P|^2 \sin(2\varphi_P - \phi^{NP}_s) - |S|^2 \sin(2\varphi_S - \phi^{NP}_s)}{|P|^2 + |S|^2} \equiv S_{\mu\mu}
\]

\[
A^{\mu\mu,\lambda}_{\Delta \Gamma_s} \equiv \frac{2 \Re(\xi^\lambda)}{1 + |\xi^\lambda|^2} = \frac{|P_{\mu\mu}|^2 \cos(2\varphi^{\mu\mu}_P - \phi^{NP}_s) - |S_{\mu\mu}|^2 \cos(2\varphi^{\mu\mu}_S - \phi^{NP}_s)}{|P_{\mu\mu}|^2 + |S_{\mu\mu}|^2}
\]

- **Note:** $C_{\mu\mu}, S_{\mu\mu} \equiv S^\lambda_{\mu\mu}, A^{\mu\mu,\lambda}_{\Delta \Gamma_s} \equiv A^{\mu\mu,\lambda}_{\Delta \Gamma_s}$ are independent of muon helicity $\lambda$. 
• Helicity-averaged decay rates, as for the branching ratio discussion:

\[ \Gamma(B_s^0(t) \to \mu^+\mu^-) \equiv \sum_{\lambda=L,R} \Gamma(B_s^0(t) \to \mu_\lambda^+\mu_\lambda^-) \]

\[ \Rightarrow C_{\lambda}^{\mu\mu} \propto \eta_{\lambda}^{\mu\mu} \text{ terms cancel in the following } CP \text{ asymmetry:} \]

\[ \frac{\Gamma(B_s^0(t) \to \mu^+\mu^-) - \Gamma(\bar{B}_s^0(t) \to \mu^+\mu^-)}{\Gamma(B_s^0(t) \to \mu^+\mu^-) + \Gamma(\bar{B}_s^0(t) \to \mu^+\mu^-)} = \frac{S_{\mu\mu} \sin(\Delta M_{st})}{\cosh(y_{st}/\tau_{B_s}) + A_{\Delta \Gamma_s}^{\mu\mu} \sinh(y_{st}/\tau_{B_s})} \]

• Observables are not independent from one another:

\[ (C_{\mu\mu})^2 + (S_{\mu\mu})^2 + (A_{\Delta \Gamma_s}^{\mu\mu})^2 = 1 \]

• Four NP parameters: \( |P_{\mu\mu}|, |S_{\mu\mu}|, \varphi_P^{\mu\mu}, \varphi_S^{\mu\mu} \)

... while three independent observables ...
Probing $P_{\mu\mu}$ and $S_{\mu\mu}$: General Case

- Determination of $|P| \equiv |P_{\mu\mu}|$, $|S| \equiv |S_{\mu\mu}|$ as functions of $\varphi_S \equiv \varphi_{\mu\mu}^S$:

- Illustration: $\overline{R} = 0.84$, $A_{\Delta\Gamma_s}^{\mu\mu} = 0.37$, $S_{\mu\mu} = 0.71$, $C_{\mu\mu} = 0.60$

$\Rightarrow$ would establish non-vanishing (pseudo)-scalar NP contribution!
Using More Information/Assumptions

• Relations in “SM Effective Field Theory” (SMEFT): [arXiv:1407.7044 [hep-ph]]

\[ C_P = -C_S, \quad C'_P = C'_S \]

• New parametrisation:

\[ x \equiv |x|e^{i\Delta} \equiv \left| \frac{C'_S}{C_S} \right| e^{i(\varphi'_S - \varphi_S)}, \quad P \equiv |P|e^{i\varphi_P} = C_{10} - \left[ \frac{1 + |x|e^{i\Delta}}{1 - |x|e^{i\Delta}} \right] |S|e^{i\varphi_S} \]

• Determination of NP parameters:
• Various patterns of observables for different SMEFT assumptions:

\[ x, \Delta \]
\[ A_{\Delta \Gamma_s}^{\mu \mu}(\varphi_S) \]
\[ C_{10} \]
\[ S_{\mu \mu}(\varphi_S) \]
\[ C_{\mu \mu}(\varphi_S) \]

Further experimental data

|S|, ϕS

Sign information

• Studies of different scenarios: → interesting playground ...
Experimental Aspects

• **Future timeline:**

```
\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \xrightarrow{\Delta \Gamma_s} \mathcal{A}_{\mu\mu} \xrightarrow{\text{Tagging}} S_{\mu\mu} \xrightarrow{\mu\mu} \Delta \Gamma_s
```

Time information Tagging Muon helicity

```
t
2014 2017 ? ?
```

• **NP scenario:** \( x = 0 \) with the following “measured” observables:

\[
\mathcal{A}_{\Delta \Gamma_s}^{\mu\mu} = 0.58 \pm 0.20, \quad S_{\mu\mu} = -0.80 \pm 0.20, \quad C_{\mu\mu} = 0.16 \pm 0.20.
\]

\( \Rightarrow \) degeneracy with \( |x| \rightarrow \infty \): \( \Rightarrow \) could be resolved through sign of \( C_{\mu\mu} \):

\[
\rightarrow |S| = 0.43^{+0.07}_{-0.08}, \quad \varphi_S = (54^{+6}_{-7})^\circ
\]

\( \rightarrow \) Perform detailed feasibility studies for LHCb upgrade and beyond!
Conclusions and Outlook
Towards New Frontiers with $B_{s,d} \rightarrow \ell^+\ell^-$ Decays

- **Highlight of LHC run 1:** $B_s^0 \rightarrow \mu^+\mu^-$ has been observed by CMS/LHCb.

- $\Delta\Gamma_s$ provides access to another (theoretically clean) observable $A_{\Delta\Gamma_s}^{\mu\mu}$:

  $\rightarrow$ pioneering LHCb measurement $\Rightarrow$ fully exploit in the future!

- Implications of $B_s^0 \rightarrow \mu^+\mu^-$ measurement for other $B_{s,d} \rightarrow \ell^+\ell^-$ decays:

  $\rightarrow$ assume flavour-universal New Physics:

  - $B_d \rightarrow \mu^+\mu^-$: moderately suppressed with respect to the SM.
  - $B_{s,d} \rightarrow \tau^+\tau^-$: NP effects strongly suppressed by $m_\mu/m_\tau \sim 0.06$.
  - $B_{s,d} \rightarrow e^+e^-$: NP effects hugely amplified by $m_\mu/m_e \sim 207$:

    $\Rightarrow \mathcal{B}(B_s \rightarrow e^+e^-)$ could be as large as $\mathcal{O}(\mathcal{B}(B_s \rightarrow \mu^+\mu^-))$

    $\rightarrow$ search for $B_{s,d} \rightarrow e^+e^-$ at the LHC $\rightarrow$ would give clear NP signal!

- **CP violating effects very interesting:** explore further, feasibility studies...

  $\Rightarrow$ new degrees of freedom for NP searches @ LHC upgrade and beyond!