Utilising $B \to \pi K$ decays at the High-Precision Frontier

Based on:
R. Fleischer, R. Jaarsma, E. Malami, and K. K. Vos; to appear
Introduction to \( B \rightarrow \pi K \) decays
Phenomenology

- Tree topologies suppressed by CKM element $V_{ub}$
- Leading contribution from QCD penguins
- CA EW penguins at same level as tree topologies
- QCD flavour symmetry to link topologies
Decays in the spotlight for over 2 decades

- Particular $B_d^0 \rightarrow \pi^0 K_S$ interesting: only channel with mixing-induced CP asymmetry

- Puzzling data in correlation between CP asymmetries

- Modified EWP sector $\rightarrow Z'$?
  Also affects rare semileptonic $B$ decays, which show anomalous data...

See also talks by:
$B \rightarrow \pi K$ decays

- What is the status of these decays?
- Little attention in recent years, neutral final state challenging for LHCb
- Excellent potential for upcoming Belle II experiment
- Difficult from theory side (QCD), but we can learn a lot!

We shall provide the state-of-the-art picture
$B \rightarrow \pi K$ decays in detail
Amplitudes

- **General parametrization:** [A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab (2004)]

\[
A(B^+ \to \pi^+ K^0) = - P' \left[ 1 + \rho_c e^{i\theta} e^{i\gamma} \right]
\]

\[
\sqrt{2}A(B^+ \to \pi^0 K^+) = P' \left[ 1 + \rho_c e^{i\theta} e^{i\gamma} - (e^{i\gamma} - qe^{i\phi} e^{i\omega})r_c e^{i\delta} \right]
\]

\[
A(B_d^0 \to \pi^- K^+) = P' \left[ 1 - re^{i\delta} e^{i\gamma} \right]
\]

\[
\sqrt{2}A(B_d^0 \to \pi^0 K^0) = - P' \left[ 1 + \rho_n e^{i\theta} e^{i\gamma} - qe^{i\phi} e^{i\omega} r_c e^{i\delta} \right]
\]

- **CP-conserving strong amplitude** \( P' = (1 - \lambda^2/2)A\lambda^2(P_t - P_c) \)

- **Amplitudes satisfy isospin relation**

\[
\sqrt{2}A(B_d^0 \to \pi^0 K^0) + A(B_d^0 \to \pi^- K^+) = \]

\[
\sqrt{2}A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0) = 3A_{3/2}
\]

\[
3A_{3/2} \equiv 3 \left| A_{3/2} \right| e^{i\phi_{3/2}} = -(\hat{T} + \hat{C}) (e^{i\gamma} - qe^{i\phi} e^{i\omega})
\]


Neglect small colour-suppressed EWP and annihilation

Parameters discussed on next slides

CKM parameters (Wolfenstein parametrization)
Amplitudes

- Hadronic parameters:

\[ r e^{i \delta} = \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \left[ \frac{T - (P_t - P_u)}{P_t - P_c} \right], \quad \rho_c e^{i \theta_c} = \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \left[ \frac{P_t - P_u}{P_t - P_c} \right] \approx 0, \]

\[ r_c e^{i \delta_c} = \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \left[ \frac{T + C}{P_t - P_c} \right], \quad \rho_n e^{i \theta_n} = \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \left[ \frac{C + (P_t - P_u)}{P_t - P_c} \right] = r_c e^{i \delta_c} - re^{i \delta} \]

- \( r_c e^{i \delta_c}, re^{i \delta} \) are non-perturbative, challenging to calculate from first principles

- Use \( B \to \pi\pi \) and SU(3) flavour symmetry [A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab (2004)]

\[ r_c e^{i \delta_c} = (0.17 \pm 0.06)e^{i(1.9\pm23.9)^\circ}, \]

\[ re^{i \delta} = (0.09 \pm 0.03)e^{i(28.6\pm21.4)^\circ}, \]

- Assumes 20\% non-factorizable SU(3)-breaking corrections (guided by data)

Reminder:
- \( T \): colour-allowed (CA) tree
- \( C \): colour-suppressed (CS) tree
- \( P \): QCD penguin
Electroweak penguins

- The parameter $q e^{i\phi} e^{i\omega}$ describes EW penguin effects:

$$q e^{i\phi} e^{i\omega} \equiv - \left( \frac{\hat{P}_{EW} + \hat{P}^C_{EW}}{\hat{T} + \hat{C}} \right)$$

$$q e^{i\phi} e^{i\omega}^{\text{SM}} \equiv \frac{-3}{2\lambda^2 R_b} \left( \frac{C_9 + C_{10}}{C_1 + C_2} \right) R_q = (0.68 \pm 0.05) R_q$$

CP-violating phase
CP-conserving phase, vanishes in $SU(3)$ limit
$SU(3)$-breaking corrections $R_q = 1.0 \pm 0.3$
Short-distance coefficients

[See e.g. R. Fleischer (1995); A. J. Buras, R. Fleischer (1998); M. Neubert, J. L. Rosner (1998)]
$B \rightarrow \pi K$

observables
Branching ratios

❖ First observables:

\[ R_c \equiv 2 \left( \frac{\mathbb{B}r(B^\pm \rightarrow \pi^0 K^\pm)}{\mathbb{B}r(B^\pm \rightarrow \pi^\pm K)} \right) = 1 - 2r_c \cos \delta_c (\cos \gamma - q \cos \phi) + \mathcal{O}(r_c^2), \]

\[ R_n \equiv \frac{1}{2} \left( \frac{\mathbb{B}r(B_d \rightarrow \pi^\mp K^\pm)}{\mathbb{B}r(B_d \rightarrow \pi^0 K)} \right) = 1 - 2r_c \cos \delta_c (\cos \gamma - q \cos \phi) + \mathcal{O}(r_c^2), \]

\[ R \equiv \left[ \frac{\mathbb{B}r(B_d \rightarrow \pi^\mp K^\pm)}{\mathbb{B}r(B^\pm \rightarrow \pi^\pm K)} \right] \frac{\tau_{B^\pm}}{\tau_{B_d}} = 1 - 2r \cos \delta \cos \gamma + 2r_c \tilde{a}_C q \cos \phi + \mathcal{O}(r_c^2) \]

Colour-suppressed (CS) EWP parameter \( \tilde{a}_C \equiv a_C \cos(\Delta_C + \delta_c) \)

❖ We obtain the relation: \( R_c - R_n = 0 + \mathcal{O}(r_c^2) \)

❖ Is satisfied experimentally at the 1σ level

Experiment:
\[ R_c = 1.09 \pm 0.06, \quad R_n = 0.99 \pm 0.06, \quad R = 0.89 \pm 0.04 \]

[PDG (2016)]
Direct CP asymmetries

- Interference of penguin and tree  ➞ direct CP asymmetry $A_f^{\text{CP}}$
- Proportional to $r_{(c)} \sin \delta_{(c)}$  ➞ values at $\mathcal{O}(10\%)$ level

Direct CP asymmetries and branching ratios satisfy sum rule:

\[
\Delta_{\text{SR}} \equiv \left[ A_{\text{CP}}^{\pi^+ K^0} \frac{\mathcal{B} r(\pi^+ K^0)}{\mathcal{B} r(\pi^- K^+)} - A_{\text{CP}}^{\pi^0 K^+} \frac{2 \mathcal{B} r(\pi^0 K^+)}{\mathcal{B} r(\pi^- K^+)} \right] \frac{\tau_{B_d}}{\tau_{B^\pm}} + A_{\text{CP}}^{\pi^- K^+} - A_{\text{CP}}^{\pi^0 K^0} \frac{2 \mathcal{B} r(\pi^0 K^0)}{\mathcal{B} r(\pi^- K^+)} = 0 + \mathcal{O}(r_{(c)}^2)
\]

- Satisfied experimentally at $1\sigma$ level but uncertainty large due to $A_{\text{CP}}^{\pi^0 K^0}$

**Experimental uncertainty at Belle II ➞ ±0.04**  [Belle-II Collaboration, arXiv:1011.0352]

- Prediction from sum rule: $A_{\text{CP}}^{\pi^0 K^0} = -0.14 \pm 0.03$
Mixing-induced CP asymmetry

- $B_d^0 \to \pi^0 K^0$ is special \(\Rightarrow\) only channel with mixing-induced CP asymmetry
- Arises from interference between $B_d^0 - \bar{B}_d^0$ mixing and decay
- Just like $A_{CP}^{\pi^0 K^0}$, also difficult for LHCb \(\Rightarrow\) large uncertainty
- Also great prospects for Belle II, but not in the focus (?!)

\[
S_{CP}^{\pi^0 K_S} = 0.58 \pm 0.17 \quad [\text{PDG (2016)}]
\]
Mixing-induced CP asymmetry

- Follows from time-dependent rate asymmetry:
  \[
  \frac{\Gamma(\bar{B}_d^0(t) \to \pi^0 K_S) - \Gamma(B_d^0(t) \to \pi^0 K_S)}{\Gamma(\bar{B}_d^0(t) \to \pi^0 K_S) + \Gamma(B_d^0(t) \to \pi^0 K_S)} = A_{CP}^{\pi^0 K_S} \cos(\Delta M_d t) + S_{CP}^{\pi^0 K_S} \sin(\Delta M_d t)
  \]
  
  with \( \Delta M_d \) mass difference \( B_d \) mass eigenstates

\[S_{CP}^{\pi^0 K_S} = \sin(\phi_d - \phi_{00}) \sqrt{1 - (A_{CP}^{\pi^0 K_S})^2}\]

Measured in \( B_d^0 \to J/\psi K_S \) \( \phi_{00} \equiv \arg(\bar{A}_{00} A_{00}^*) \)

- Angle given by
  \[
  \tan \phi_{00} = 2 \left( r \cos \delta - r_c \cos \delta_c \right) \sin \gamma + 2 r_c \left( \cos \delta_c - 2 \tilde{a}_c / 3 \right) q \sin \phi + O(r_c^2)
  \]

What is the best way to calculate \( \phi_{00} \)?
We may use the isospin relation:

\[
\sqrt{2}A(B_d^0 \to \pi^0 K^0) + A(B_d^0 \to \pi^- K^+) \equiv 3A_{3/2}
\]

3\(A_{3/2}\) \(\equiv 3 |A_{3/2}| e^{i\phi_{3/2}} = -(\hat{T} + \hat{C})(e^{i\gamma} - qe^{i\phi}e^{i\omega})\)

\(\phi_{00}\) follows from amplitude triangles

If \(q\) and \(\phi\) are known, only \(SU(3)\) input for:

\[
|\hat{T} + \hat{C}| = R_{T+C} \left| \frac{V_{us}}{V_{ud}} \right| \sqrt{2} |A(B^+ \to \pi^+ \pi^0)|
\]

\[
R_{T+C} \approx f_K/f_\pi = 1.2 \pm 0.2
\]

Minimal hadronic input
Correlation between CP asymmetries

- We may now use

\[ S_{CP}^{\pi^0K_S} = \sin(\phi_d - \phi_{00}) \sqrt{1 - (A_{CP}^{\pi^0K_S})^2} \]

to obtain a correlation between \( S_{CP}^{\pi^0K_S} \) and \( A_{CP}^{\pi^0K_S} \)

- Discrepancy with SM in 2008

What is the current status?
Correlation between CP asymmetries

Sharper inputs ($\gamma$) $\rightarrow$ discrepancy stronger!
Puzzling patterns

New aspect: $\phi_{\pm} = \arg(\bar{A}_\pm A^*_\pm)$,

$$\phi_{\pm} \bigg|_{\phi=0} = 2 r \cos \delta \sin \gamma + \mathcal{O}(r^2) = (8.7 \pm 3.5)°$$

Also the correlation is inconsistent!

Isospin relation
Current status

State-of-the-art analysis of $S_{CP}^{\pi^0 K_S}$:

- Problem with measurements? Discrepancy could be solved if
  - CP asymmetries $B_d^0 \rightarrow \pi^0 K_S$ move by $\sim 1\sigma$
  - $Br(B_d \rightarrow \pi^0 K^0)$ moves by $\sim 2.5\sigma$
- Or is it New Physics? → Study possibility of a modified EWP sector

With upcoming Belle II data the situation should be resolved
Determinaton of $q$ and $\phi$

- Use the amplitude triangles in a different way: convert $S^{\pi^0 K_s}_{\text{CP}}$ into $q$ and $\phi$

- The isospin relation holds also for neutral as well as charged decays:

\[
\sqrt{2} A(B^+_d \to \pi^0 K^0) + A(B^+_d \to \pi^- K^+) = \sqrt{2} A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0) = 3A_{3/2}
\]

\[
3A_{3/2} \equiv 3 |A_{3/2}| e^{i\phi/3} = -(\hat{T} + \hat{C})(e^{i\gamma} - qe^{i\phi}e^{i\omega})
\]

- Current data is better for charged decays, but the method works for both.

- Derive a set of equations for contours in $q, \phi$-plane

\[
q = \sqrt{N^2 - 2c \cos \gamma - 2s \sin \gamma + 1},
\]

\[
\tan \phi = \frac{\sin \gamma - s}{\cos \gamma - c}, \quad q \sin \phi = \sin \gamma - s,
\]

- where

\[
c \equiv \pm N \cos(\Delta \phi_{3/2}/2), \quad s \equiv \pm N \sin(\Delta \phi_{3/2}/2),
\]

\[
N \equiv 3 |A_{3/2}| / |\hat{T} + \hat{C}|, \quad \Delta \phi_{3/2} \equiv \phi_{3/2} - \bar{\phi}_{3/2}
\]
Determination of $q$ and $\phi$

- This method requires minimal $SU(3)$ input, only from

$$|\hat{T} + \hat{C}| = R_{T+C} \left| \frac{V_{us}}{V_{ud}} \right| \sqrt{2} |A(B^+ \to \pi^+\pi^0)|$$

$$R_{T+C} \approx f_K/f_{\pi} = 1.2 \pm 0.2$$

No topologies have to be neglected

- Need to fix relative orientation triangles:

$$\phi_{+0} = \arg(\bar{A}_{+0}A_{+0}^*) \approx 0 \text{  (charged) or  } S_{\text{CP}}^{\pi^0K_s} \text{  (neutral)}$$
Results of the new strategy
Results for current data

Apply method to charged data as current uncertainty $S_{\pi^0 K_S}^{CP}$ still large

Potential to implement also for neutral data at Belle II!
Results for current data

Complement analysis with:

\[ R_c = 1 - 2 r_c \cos \delta_c (\cos \gamma - q \cos \phi) + \mathcal{O}(r_c^2) \]

CS EWPs only at \( \mathcal{O}(r_c^2) \)

\( \Rightarrow \) contour in \( q, \phi \)-plane

Excellent agreement

Further input needed to determine the value of \( q \) and \( \phi \)
Additional contour from $S_{CP}^{\pi^0 K_S}$

- Convert measurement of $S_{CP}^{\pi^0 K_S}$ in value of $\phi_{00}$
- Obtain contour from
  \[
  \tan \phi_{00} = 2 (r \cos \delta - r_c \cos \delta_c) \sin \gamma + 2 r_c (\cos \delta_c - 2 \tilde{a}_C / 3) q \sin \phi + \mathcal{O}(r_{(c)}^2)
  \]
- CS EWP parameter $\tilde{a}_C \equiv a_C \cos(\Delta_C + \delta_c)$ is determined from
  \[
  R = 1 - 2 r \cos \delta \cos \gamma + 2 r_c \tilde{a}_C q \cos \phi + \mathcal{O}(r_{(c)}^2)
  \]
- Consider 3 different scenarios for measurements of $S_{CP}^{\pi^0 K_S}$ at Belle II

Cosines of small phases $\rightarrow$ low sensitivity to variations
Future scenarios

Experimental uncertainty (small band):
\[ \Delta S_{\text{CP}}^{\pi^0 K_S} \big|_{\text{exp}} \sim 0.04 \]
[Belle-II Collaboration, arXiv:1011.0352]

Consistent with \( R_c \)

Theoretical uncertainty (wide band):
20% non-factorizable \( SU(3) \)-breaking corrections on the hadronic parameters

We can match the experimental precision with theory!

Future theory errors
Future scenarios

- Precision depends on region in parameter space
- Potential for discovery of NP at Belle II!

\[ S_{\text{CP}}^{\pi^0K_s} = 0.91 \]

\[ S_{\text{CP}}^{\pi^0K_s} = 0.33 \]

\[ q \]

\[ \phi \text {[deg]} \]
Resolution of $B \rightarrow \pi K$ puzzle

- Can we now resolve the $B \rightarrow \pi K$ puzzle?

- Consider 2 scenarios:

  
  \[ (q, \phi) = (0.70, -54^\circ) \]

  \[ (q, \phi) = (0.94, 48^\circ) \]
What about the sum rule?

❖ Belle II performed feasibility study of the sum rule [Belle-II Collaboration, arXiv:1011.0352]

\[
\Delta_{SR} \equiv \left[ A_{CP}^{\pi^+ K^0} \frac{Br(\pi^+ K^0)}{Br(\pi^- K^+)} - A_{CP}^{\pi^0 K^+} \frac{2Br(\pi^0 K^+)}{Br(\pi^- K^+)} \right] \frac{\tau_{B_d}}{\tau_{B^\pm}}
\]

+ \left[ A_{CP}^{\pi^- K^+} - A_{CP}^{\pi^0 K^0} \frac{2Br(\pi^0 K^0)}{Br(\pi^- K^+)} \right] = 0 + \mathcal{O}(r_c^2)

❖ Could it reveal $q$ and $\phi$?

The resolution is not sufficient for $q < 3$

[R. Fleischer, RJ, E. Malami, K. K. Vos; to appear]
Conclusions

- Data from $B_d^0 \rightarrow \pi^0 K_S$ have shown puzzling patterns in the past
- We have performed a state-of-the-art analysis:
  - Discrepancy became stronger $\Rightarrow$ something has to happen
- Data move to eventually confirm the Standard Model?
- Is it New Physics?
- We have presented a new strategy to pin down the EWP parameters
- We look forward to data from Belle II and LHCb
Backup slides
Prediction for $\phi = 0$

- We can define

$$\left(\sin 2\beta\right)_{\pi^0K_S} \equiv \frac{S_{CP}^{\pi^0K_S}}{\sqrt{1 - (A_{CP}^{\pi^0K_S})^2}} = \sin(\phi_d - \phi_{00})$$

- In the SM we have $\phi = 0$, yielding

$$\tan \phi_{00} = 2 (r \cos \delta - r_c \cos \delta_c) \sin \gamma + \mathcal{O}(r_c^2)$$

- From the $B \to \pi\pi$ data we then find

$$\left(\sin 2\beta\right)_{\pi^0K_S} = 0.80 \pm 0.06$$

Only CS EWPs in higher-order corrections

Includes 20% $SU(3)$-breaking and higher-order corrections