H+jet production at NLO
including full top quark mass dependence

Matthias Kerner
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In collaboration with
Stephen Jones, Gionata Luisoni

arXiv:1802.00349
HEFT vs. full theory

Many calculations in Higgs physics done in the $m_t \to \infty$ limit (Higgs EFT)

HEFT is only good approximation if $m_T$ largest scale of process!
→ valid for inclusive $gg \to H$ production
→ possibly poor description of $gg \to H+X$ production

H+jet production at large $p_T$

Particle in loop gets resolved
→ top mass effects can be important
→ possible effects due to BSM physics
Content

• Overview
  - previously known results
  - LO predictions
  - NLO corrections

• Approximated results @ NLO

• HJ @ NLO with full top quark mass dependence
  - Details of Computation
  - Results

• Outlook & Conclusion
1) LO (full $m_t$ dependence)
   [Ellis, Hinchliffe, Soldate, van der Bij 87]
   [Baur, Glover 89]

2) NLO
   • HEFT
     [de Florian, Grazzini, Kunszt 99; Glosser, Schmidt 02; Ravindran, Smith, van Neerven 02]
   • approximated $m_t$ dependence
     [Harlander, Neumann, Ozeren, Wiesemann 12]
     [Neumann, Wiesemann 14] [Frederix, Frixione, Vryonidou, Wiesemann 16] [Neumann, Williams 16]
     [Caola, Forte, Marzani, Muselliand, Vita 16]
     [Braaten, Zhang, Zhang 17]
     [Lindert, Kudashkin, Melnikov, Wever 18] [Neumann 18]
   • top-bottom interference
     [(Lindert,) Melnikov, Tancredi, Wever 16, 17]

3) NNLO (HEFT)
   $K \approx 1.8$
   [Boughezal, Caola, Melnikov, Petriello, Schulze 13, 14]
   [Chen, (Martinez,) Gehrmann, Glover, Jaquier 14, 16]
   [Boughezal, Focke, Giele, Liu, Petriello 15]
   
   $K \approx 1.2$
Leading Order

HJ production in gluon-fusion is loop induced $\rightarrow$ LO = 1-loop

$\sqrt{s} = 13$ TeV  
$m_H = 125$ GeV  
$m_t = 173.05$ GeV  
PDF4LHC15  
$\mu_R = \mu_F = m_H$

<table>
<thead>
<tr>
<th>Process</th>
<th>$p_T &gt; 30$ GeV</th>
<th>$p_T &gt; 300$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow Hg$</td>
<td>5836 fb (73%)</td>
<td>53.1 fb (62.5%)</td>
</tr>
<tr>
<td>$qg \rightarrow Hq, \bar{q}g \rightarrow H\bar{q}$</td>
<td>2096 fb (26%)</td>
<td>31.0 fb (36.5%)</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow Hg$</td>
<td>38 fb (0.5%)</td>
<td>0.8 fb (1%)</td>
</tr>
</tbody>
</table>
NLO corrections — Feynman Diagrams

**virtual corrections**

2-loop 4-point diagrams with internal mass

- computation very challenging
- analytic results for some integrals

[Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]

- only planar diagrams
- in Euclidean region \( s = m_{Hj}^2 < 0 \)
  analytic continuation to \( s > 0 \) not known

- known as expansion in
  - \( 1/m_t^2 \) [Neumann, Wiesemann, (Harlander, Ozeren) 12,14]
  - \( m_q^2/Q^2 \) [Mueller, Öztürk 15], [Melnikov, Tancredi, Wever 16, 17], [Kudashkin, Melnikov, Wever 17]

**we compute all 2-loop integrals numerically**

**real corrections**

1-loop 5-point integrals

need to be evaluated in infrared limit
→ can cause numerical instabilities
Approximated Results

large $m_t$ expansion up to $\mathcal{O}(1/m_t^4)$

<table>
<thead>
<tr>
<th></th>
<th>full asymptotic</th>
<th>real improved</th>
<th>NLO*</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_t expansion order</td>
<td>$1/m_t^0$</td>
<td>$1/m_t^2$</td>
<td>$1/m_t^4$</td>
</tr>
</tbody>
</table>

Figure 2: Higgs inclusive $p_T$ spectrum for three different approximations, each taking into account higher orders of an asymptotic expansion in $1/m_t$. The upper panel shows the absolute distribution, while the lower two panels display the ratio to the LO distribution and the NLO$^\ast$ approximation, respectively.

NLO cross section expanded in $1/m_t$

full $m_t$ dependence in real radiation

$2\text{Re}(\mathcal{M}^\ast_{\text{full}} \cdot \mathcal{M}^\text{expand})$
**Approximated Results**

Large $m_t$ expansion up to $O(1/m_t^4)$

<table>
<thead>
<tr>
<th>$m_t$ expansion order</th>
<th>full asymptotic</th>
<th>real improved</th>
<th>NLO*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/m_t^0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/m_t^1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/m_t^2$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$1/m_t^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/m_t^6$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Delta \sigma/\Delta m_{H,j}$ [fb/GeV]

$\Delta \sigma/\Delta p_{T,H}$ [fb/GeV]

$\Delta p_{T,H}$ [GeV]

$m_{H+hard jet}$ [GeV]

Ratio to LO $1/m_t^0$

Ratio to NLO* $1/m_t^0$

Expansion cannot describe behavior above the top threshold

Figure 2: Higgs inclusive $p_T$ spectrum for three different approximations, each taking into account higher orders of an asymptotic expansion in $1/m_t$. The upper panel shows the absolute distribution, while the lower two panels display the ratio to the LO distribution and the NLO* $1/m_t$ approximation, respectively.

Figure 3: Invariant mass spectrum of the Higgs plus hardest jet system at LO. The upper part displays the absolute distribution, while the lower part displays the ratio to the EFT result.
Approximated Results

small m_t expansion up to $O(m_t^2/p_T^2)$  [Kudashkin, Melnikov, Wever 17]

[Lindert, Kudashkin, Melnikov, Wever 18]

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>33.8$^{+44%}_{-29%}$</td>
<td>61.4$^{+20%}_{-19%}$</td>
<td>1.82</td>
<td>12.4$^{+44%}_{-29%}$</td>
<td>23.6$^{+24%}_{-21%}$</td>
<td>1.90</td>
</tr>
<tr>
<td>$p_\perp &gt;$ 450 GeV</td>
<td>22.0$^{+45%}_{-29%}$</td>
<td>39.9$^{+20%}_{-19%}$</td>
<td>1.81</td>
<td>6.75$^{+45%}_{-29%}$</td>
<td>12.9$^{+24%}_{-21%}$</td>
<td>1.91</td>
</tr>
<tr>
<td>$p_\perp &gt;$ 500 GeV</td>
<td>14.7$^{+44%}_{-28%}$</td>
<td>26.7$^{+20%}_{-19%}$</td>
<td>1.81</td>
<td>3.80$^{+45%}_{-29%}$</td>
<td>7.28$^{+24%}_{-21%}$</td>
<td>1.91</td>
</tr>
<tr>
<td>$p_\perp &gt;$ 1000 GeV</td>
<td>0.628$^{+46%}_{-30%}$</td>
<td>1.14$^{+21%}_{-19%}$</td>
<td>1.81</td>
<td>0.0417$^{+47%}_{-30%}$</td>
<td>0.0797$^{+24%}_{-21%}$</td>
<td>1.91</td>
</tr>
</tbody>
</table>

$K_{SM}^{HEFT} = 1.04 \ldots 1.06$

Approximated results predict top mass effects at the few percent level

[Neumann 18]
Interference of top and bottom quark contributions relevant at low $p_T$

obtained in the limit $m_t \to \infty$, $m_b \to 0$

[Lindert, Melnikov, Tancredi, Wever 17]

\[
d\sigma_{tb}^{\text{virt}} \sim \text{Re} \left[ A_t^{\text{LO}} A_b^{\text{LO}*} + \frac{\alpha_s}{2\pi} (A_t^{\text{NLO}} A_b^{\text{LO}*} + A_t^{\text{LO}} A_b^{\text{NLO}*}) \right]
\]

$-8\%$ at $p_T = 20$ GeV

$+2\%$ at $p_T = 100$ GeV

size of interference effects nearly identical at LO and NLO
Details of Computation with full $m_t$ Dependence

• implemented in Powheg-Box framework, except the virtual contributions
• methods previously applied for $gg \rightarrow HH$ production
  [Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]; [Heinrich, Jones, MK, Luisoni, Vryonidou 17]

LO contributions: using analytic calculation [Baur, Glover 90]
Real radiation: matrix elements generated using GoSam [Cullen et. al.]

Virtual amplitude — code generation:

1. generate Feynman diagrams, apply Feynman rules
   → 3767 integrals: quite complicated, divergent in $d=4$ space-time dimensions

2. apply IBP relations to integrals using Reduze [von Manteuffel, Studerus]
   → 458 integrals: linearly independent, simpler

3. use SecDec [Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke]
   applies sector decomposition to extract divergent terms, expand in $\varepsilon = (4 - d)/2$
   → 22 675 integrals: simpler, finite
   → integrals can be evaluated using Monte-Carlo methods

Code size for
• integrals: ~400 MB
• coefficients: ~350 MB

$O(\text{weeks})$ compile time
Virtual amplitude — evaluation of loop integrals

- all integrals evaluated using Quasi-Monte-Carlo integration
  - generating vector
    - constructed component-by-component [Nuyens 07]
    - minimizing worst-case error
    - for fixed lattice sizes
  - $O(n^{-1})$ scaling of integration error
- parallelization on gpu
- many further optimizations ...

**QMC rank-1 lattice rule**

$$ I = \int \! d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k}) $$

$$ \vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \Delta_k \right\} $$

$\{\ldots\} =$ fractional part

$\vec{g} =$ generating vector

$\Delta_k =$ randomized shift

$m$ different estimates $I_1 \ldots I_m$

$\rightarrow$ error estimate

[Li, Wang, Yan, Zhao 16]

Review: [Dick, Kuo, Sloan]

**time / phase space point:**

<table>
<thead>
<tr>
<th></th>
<th>GPU time</th>
<th>accuracy reached</th>
</tr>
</thead>
<tbody>
<tr>
<td>low $p_T$</td>
<td>~12 h</td>
<td>&lt; 0.1%</td>
</tr>
<tr>
<td>very high $p_T</td>
<td>~48 h</td>
<td>10% - 100%</td>
</tr>
</tbody>
</table>

**phase space integration:**

- based on unweighted LO events
- additional $p_T$ dependent reweighting

$\rightarrow$ obtained $p_T$-distribution

using only ~2000 PS points
Results — Total Cross Section

<table>
<thead>
<tr>
<th>THEORY</th>
<th>LO [pb]</th>
<th>NLO [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEFT:</td>
<td>$\sigma_{LO} = 8.22^{+3.17}_{-2.15}$</td>
<td>$\sigma_{NLO} = 14.63^{+3.30}_{-2.54}$</td>
</tr>
<tr>
<td>$F{T}_{approx}$:</td>
<td>$\sigma_{LO} = 8.57^{+3.31}_{-2.24}$</td>
<td>$\sigma_{NLO} = 15.07^{+2.89}_{-2.54}$</td>
</tr>
<tr>
<td>Full:</td>
<td>$\sigma_{LO} = 8.57^{+3.31}_{-2.24}$</td>
<td>$\sigma_{NLO} = 16.01^{+1.59}_{-3.73}$</td>
</tr>
</tbody>
</table>

$F{T}_{approx}$:

$$d\sigma_{NLO}^{FT_{approx}} = \int dPS_2 \left( d\sigma_{B}^{Full} + \frac{d\sigma_{B}^{Full}}{d\sigma_{B}^{HEFT}} d\sigma_{V}^{HEFT} \right) + \int dPS_3 d\sigma_{R}^{Full}$$

- LHC @ 13 TeV
- $p_{T,j} > 30$ GeV, $R = 0.4$, anti-$k_T$
- scale: $\frac{H_T}{2} = \frac{1}{2} \left( \sqrt{m_H^2 + p_{t,H}^2} + \sum |p_{t,i}| \right)$
- PDF4LHC15
- $m_H = 125$ GeV
  - $m_t = \sqrt{23/12} m_H \approx 173.05$ GeV

**top-quark mass effects:**

+4.3% at LO
+9% at NLO (+6% compared to $F{T}_{approx}$)
mass effects compared to HEFT

HEFT and full theory predict different scaling of $d\sigma/dp_T^2$:

$\sim p_T^{-2}$ in HEFT

$\sim p_T^{-4}$ in full theory

[Caola, Forte, Marzani, Muselli, Vita, 15,16]

confirmed at NLO

nearly constant K-factor in full theory
mass effects compared to $\text{FT}_{\text{approx}}$

$\text{FT}_{\text{approx}}$ and full theory predict same shape of $p_T$ distribution

nearly constant increase of $\sim 8\%$ due to top mass in virtual contribution
Results — Different scale choices

comparison of central scales $H_T/2$ and $m_H$

choosing $\mu_R, \mu_F = m_H$ leads to

- different shape of LO distribution
- $\text{FT}_{\text{approx}}$
  - good agreement at low $p_T$
  - overestimates the tail
- full result in very good agreement with results with $\mu_R, \mu_F = H_T/2$

→ top-quark mass effects only small for $\mu_R, \mu_F = H_T/2$
we plan to **provide code** to calculate Hj@NLO in form of

- grid interpolation of virtual amplitude
  - avoids evaluation of 2-loop integrals → fast
  - useful for theorists
- Implementation in Powheg-Box (based on grid)
  - everything except the virtual amplitude already implemented
  - can be combined with parton shower
  - useful for experimentalists

**many possibilities for further studies:** (though, some are very challenging)

- more differential results
- matching to parton shower
  - include resummation effects
  - include bottom quark
  - combination with NNLO in HEFT
- …
Summary

H+jet production at NLO

- retaining full top-quark mass dependence
- virtual 2-loop amplitude calculated numerically
- plan to release code in form of grid & Powheg-Box implementation
- top mass effects increase NLO by \( \sim 9\% \)
- only small \( p_T \) dependence of corrections for \( \mu_R, \mu_F = H_T/2 \)

Thank you for your attention!
Backup
HH Amplitude Evaluation — Example

\[ \sqrt{s} = 327.25 \text{ GeV}, \quad \sqrt{-t} = 170.05 \text{ GeV}, \quad M^2 = s/4 \]

contributing integrals:

<table>
<thead>
<tr>
<th>integral</th>
<th>value</th>
<th>error</th>
<th>time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1_011111110_ord0</td>
<td>(0.484, 4.96e-05)</td>
<td>(4.40e-05, 4.23e-05)</td>
<td>11.8459</td>
</tr>
<tr>
<td>N3_111111110_k1p2k2p2_ord0</td>
<td>(0.0929, -0.224)</td>
<td>(6.32e-05, 5.93e-05)</td>
<td>235.412</td>
</tr>
<tr>
<td>N3_1111111100_1_ord0</td>
<td>(-0.0282, 0.179)</td>
<td>(8.01e-05, 9.18e-05)</td>
<td>265.896</td>
</tr>
<tr>
<td>N3_1111111100_k1p2k1p2_ord0</td>
<td>(0.0245, 0.0888)</td>
<td>(5.06e-05, 5.31e-05)</td>
<td>282.794</td>
</tr>
<tr>
<td>N3_1111111100_k1p2_ord0</td>
<td>(-0.00692, -0.108)</td>
<td>(3.05e-05, 3.05e-05)</td>
<td>433.342</td>
</tr>
</tbody>
</table>

\[
I(s, t, m_t^2, m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\varepsilon)M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)
\]

sector decomposition

<table>
<thead>
<tr>
<th>sector</th>
<th>integral value</th>
<th>error</th>
<th>time [s]</th>
<th>#points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(-1.34e-03, 2.00e-07)</td>
<td>(2.38e-07, 2.69e-07)</td>
<td>0.255</td>
<td>1310420</td>
</tr>
<tr>
<td>6</td>
<td>(-1.58e-03, -9.23e-05)</td>
<td>(7.44e-07, 5.34e-07)</td>
<td>0.266</td>
<td>1310420</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>(0.179, -0.856)</td>
<td>(1.10e-05, 1.22e-05)</td>
<td>29.484</td>
<td>79952820</td>
</tr>
<tr>
<td>42</td>
<td>(0.359, -1.308)</td>
<td>(1.40e-06, 1.58e-06)</td>
<td>80.24</td>
<td>211436900</td>
</tr>
<tr>
<td>44</td>
<td>(0.0752, -1.185)</td>
<td>(5.44e-07, 6.76e-07)</td>
<td>99.301</td>
<td>282904860</td>
</tr>
</tbody>
</table>

\approx 700 \text{ integrals}

slide:

MK, L&L 2016
contribution integrals:

<table>
<thead>
<tr>
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<th>value</th>
<th>error</th>
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<tr>
<td>N3_111111100_k1p2</td>
<td></td>
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<td>N3_111111100_k1p2</td>
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\[ I(s, t, m_t^2, m_h^2) = - \left( \sqrt{s} = 327.25 \text{ GeV}, \sqrt{-t} = 170.05 \text{ GeV}, M^2 = s/4 \right) \]

\[ \approx 700 \text{ integrals} \]

\[ \sqrt{s} = \frac{25}{3} \text{ GeV}, \quad \sqrt{-t} = 170.05 \text{ GeV} \]

\[ \text{sector integral value error time [s]} \]

\[ \text{contributing integrals:} \]

\[ \text{sector in} \]

\[ \begin{array}{c|c}
5 & 10^{-1} \\
6 & 10^{-2} \\
41 & 10^{-3} \\
42 & 10^{-4} \\
44 & 10^{-6} \\
\end{array} \]

\[ \begin{array}{c|c|c|c}
<table>
<thead>
<tr>
<th>n (Function Evaluations/m)</th>
<th>QMC</th>
<th>1/\sqrt{n}</th>
<th>1/n</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^5</td>
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<tr>
<td>10^6</td>
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<tr>
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<tr>
<td>10^8</td>
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</tr>
<tr>
<td>10^9</td>
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</tr>
<tr>
<td>10^{10}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\end{array} \]

slide: MK, L&L 2016
GPU performance

- **R1SL: Implementation Performance**
  - Accuracy limited primarily by number of function evaluations.
  - Implemented in OpenCL 1.1 for CPU & GPU, generate points on GPU/CPU core, sum blocks of points (reduce memory usage/transfers).

<table>
<thead>
<tr>
<th>CPU Configuration</th>
<th>CPU (s)</th>
<th>GPU (s)</th>
<th>C/G</th>
</tr>
</thead>
<tbody>
<tr>
<td>655357</td>
<td>6.63</td>
<td>1.60</td>
<td>4.1</td>
</tr>
<tr>
<td>7208951</td>
<td>72.3</td>
<td>16.4</td>
<td>4.4</td>
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<tr>
<td>67264993</td>
<td>674.2</td>
<td>152.2</td>
<td>4.4</td>
</tr>
</tbody>
</table>

- **2 CPUs (20 Cores + HT)**
- **1 GPU**

- **Million Function Evaluations / Second**
- **Cores**
  - 2 x Xeon E5-2680v2 (CPU)
  - 1 x Tesla K20Xm (GPU)

- **Ideal Linear Scaling**

- **Hyperthreading**

- **Plot:** Stephen Jones