Precision calculations for $h \rightarrow WW/ZZ \rightarrow 4\text{fermions}$ in the THDM with PROPHECY4F

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(in collaboration with L.Altenkamp and H.Rzehak; see arXiv:1704.02645 and arXiv:1710.07598)
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NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4$fermions

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Introduction
Some central LHC results from profiling the Higgs boson

Decay signal strength:

Fit of coupling modifiers:

\[ \mu = \frac{\Gamma_{\text{exp}}}{\Gamma_{\text{SM}}} \]

Compatibility with Standard Model

Reveal BSM effects with higher precision?

⇒ Precision calculations in BSM models necessary

→ THDM considered in this talk
Renormalization of the THDM
THDM Lagrangian and Higgs fields

**Lagrangian:** restriction to CP-conserving case!

\[ \mathcal{L}_{\text{Higgs}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2), \]

\[ D_\mu = \partial_\mu - ig_2 I^a W_\mu^a + ig_1 \frac{Y_W}{2} B_\mu \]

**Higgs potential:**

\[ V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right] \]

**Two complex scalar SU(2) doublets:** \( v_{1,2} = \text{vevs} \)

\[ \Phi_1 = \left( \frac{1}{\sqrt{2}} \left( \eta_1 + i \chi_1 + v_1 \right) \right), \quad \Phi_2 = \left( \frac{1}{\sqrt{2}} \left( \eta_2 + i \chi_2 + v_2 \right) \right), \quad Y_W(\Phi_{1,2}) = 1 \]
Transition to the “mass basis”:

CP-even neutral fields:
\[
\begin{pmatrix}
\eta_1 \\
\eta_2
\end{pmatrix}
= \begin{pmatrix} 
\cos \alpha & - \sin \alpha \\
\sin \alpha & \cos \alpha \end{pmatrix}
\begin{pmatrix}
H \\
h
\end{pmatrix}
\]

CP-odd neutral fields:
\[
\begin{pmatrix}
\chi_1 \\
\chi_2
\end{pmatrix}
= \begin{pmatrix} 
\cos \beta & - \sin \beta \\
\sin \beta & \cos \beta \end{pmatrix}
\begin{pmatrix}
G_0 \\
A_0
\end{pmatrix}, \quad \tan \beta = \frac{v_2}{v_1}
\]

charged fields:
\[
\begin{pmatrix}
\phi_1^\pm \\
\phi_2^\pm
\end{pmatrix}
= \begin{pmatrix} 
\cos \beta & - \sin \beta \\
\sin \beta & \cos \beta \end{pmatrix}
\begin{pmatrix}
G^\pm \\
H^\pm
\end{pmatrix}
\]

Higgs potential after diagonalization:
\[
V = -t_h h - t_H H + \frac{1}{2} M_h^2 h^2 + \frac{1}{2} M_H^2 H^2 + \frac{1}{2} M_{A_0}^2 A_0^2 + M_{H+}^2 H^+ H^- + \ldots
\]

tadpoles \rightarrow 0

Transformation of input parameters:

original set: \{\lambda_1, \ldots, \lambda_5, m_{11}^2, m_{22}^2, m_{12}^2, v_1, v_2, g_1, g_2\}

\downarrow

mass basis: \{M_H, M_h, M_{A_0}, M_{H+}, M_W, M_Z, e, \lambda_5, \alpha, \beta, t_H, t_h\}

renormalized on-shell \text{ (MS)}

2 ren. variants

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Renormalization  (see also Santos/Barroso ’97; Kanemura et al. ’04; Lopez-Val/Sola ’09; Degrande ’14)

→ follow on-shell renormalization as far as possible/reasonable
related work by Krause et al. ’16; Denner et al. ’16

On-shell renormalization:

• all particle masses: \( M_W, M_Z, M_h, M_H, \ldots \)
• matrix-valued renormalization for all fields:
  \[
  \begin{pmatrix}
  H_0 \\
  h_0
  \end{pmatrix} = \begin{pmatrix}
  1 + \frac{1}{2} \delta Z_H & \frac{1}{2} \delta Z_{Hh} \\
  \frac{1}{2} \delta Z_{hH} & 1 + \frac{1}{2} \delta Z_h
  \end{pmatrix}
  \begin{pmatrix}
  H \\
  h
  \end{pmatrix},
  \]
  etc.

→ no mixing of external (on-shell) states
• elmg. coupling \( \alpha_{em} \) in the Thomson limit

\( \overline{\text{MS}} \) renormalization:

• mixing angles \( \alpha, \beta \)
  \( \leftrightarrow \) e.g. determined by Higgs mixing self-energies
• Higgs self-coupling \( \lambda_5 \)
  \( \leftrightarrow \) e.g. determined by \( HA_0A_0 \) vertex correction

⇒ Renormalization-scale-dependent parameters \( \alpha(\mu_r), \beta(\mu_r), \lambda_5(\mu_r) \)
Tadpole renormalization:

Note: No physical effect (just bookkeeping) if all parameters are fixed by “physical renormalization conditions”!

But: $\overline{\text{MS}}$ parameters in general depend on tadpole renormalization!

Two commonly used variants:

a) Vanishing renormalized tadpoles $t_s$: $t_{s,0} = t_s + \delta t_s = 0 + \delta t_s$
   - (explicit tadpole loops $\Gamma^S$) + $\delta t_s = 0 \implies$ explicit tadpoles can be ignored
   - (implicit) tadpole contributions $\delta t_s$ in counterterms
   - drawback: $t_{s,0} = \delta t_s$ enters relation between bare basic input parameters $\implies$ potentially gauge-dependent terms $\propto \delta t_s$ enter relations between renormalized parameters and predicted observables

b) Vanishing bare tadpoles $t_{s,0}$: $t_{s,0} = 0$ Fleischer/Jegerlehner '80; Actis et al. '06
   - explicit tadpole loops $\Gamma^S$ have to included everywhere, technical variant: remove $\Gamma^S$ from 2-point functions by shift $v_s \rightarrow v_s + \Delta v_s$
   - advantage: no gauge-dep. $\delta t_s$ terms in relations between bare parameters $\implies$ relation between ren. parameters and observables gauge independent
Different schemes employed in NLO calculation for $h \rightarrow 4f$:

- **$\overline{\text{MS}}(\alpha)$**: see also by Krause et al. ’16; Denner et al. ’16
  - input: $\beta, \lambda_5, \alpha$
  - tadpole treatment a): $t_S = 0$
  - gauge dependent: results tied to ’t Hooft–Feynman gauge

- **$\text{FJ}(\alpha)$**: see also by Krause et al. ’16; Denner et al. ’16
  - input: $\beta, \lambda_5, \alpha$
  - FJ tadpole treatment b): $t_{S,0} = 0$
  - gauge independent

- **$\overline{\text{MS}}(\lambda_3)$**:
  - as $\overline{\text{MS}}(\alpha)$, but $\alpha$ replaced by coupling $\lambda_3$ as input
  - gauge independent only in $R_\xi$ gauges at NLO

- **$\text{FJ}(\lambda_3)$**:
  - as $\text{FJ}(\alpha)$, but $\alpha$ replaced by coupling $\lambda_3$ as input
  - gauge independent

→ Study renormalization scheme and renormalization scale dependence of results
Running of $\overline{MS}$ parameters: (numerical solution of ren. group eqs.)

Example: $c_{\beta-\alpha}$ in a THDM low-mass scenario of Type I

Scenario A: $M_h = 125$ GeV, $c_{\beta-\alpha} = +0.1$ (Aa) or $c_{\beta-\alpha} = -0.1$ (Ab)

$M_H = 300$ GeV, $M_{A_0} = M_{H^+} = 460$ GeV, $\lambda_5 = -1.9$, $\tan \beta = 2$

default scale: $\mu_0 = \frac{1}{5} (M_h + M_H + M_{A_0} + 2M_{H^+}) = 361$ GeV

Strong dependence of running on renormalization scheme
Conversion between renormalization schemes:

Note: Values of ren. parameters of a model scenario depend on the ren. scheme!

Conversion between schemes (1) and (2) via equality of bare parameters:

\[ p_0 = p^{(1)} + \delta p^{(1)}(p^{(1)}) = p^{(2)} + \delta p^{(2)}(p^{(2)}) \]
\[ \Rightarrow p^{(2)} = p^{(1)} + \delta p^{(1)}(p^{(1)}) - \delta p^{(2)}(p^{(2)}) \]
\[ \Rightarrow p^{(2)} = p^{(1)} + \delta p^{(1)}(p^{(1)}) - \delta p^{(2)}(p^{(1)}) + \ldots \]

Example: \( c_{\beta-\alpha} \) in low-mass scenario A

\[ c_{\beta-\alpha} \bigg|_{\text{Scenario A}} \]

\[ \mu_r = 361 \text{ GeV} \]

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NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4 \text{fermions}$
Survey of Feynman diagrams for NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4f$

Lowest order:

$$h \rightarrow VV \rightarrow ff = \sin(\beta - \alpha) M_{SM,LO}$$

Typical one-loop diagrams:

# diagrams = $\mathcal{O}(200-400)$

- pentagons
- boxes
- vertices
- self-energies
- counterterms
- tree graphs with real gluon or photons
Details of the NLO calculation

Virtual corrections

- model file generation with \textsc{feynrules}
- diagram generation with \textsc{feynarts}
- amplitude reduction with inhouse Mathematica routines or \textsc{formcalc}
- $W/Z$ resonances treated in the \textit{complex-mass scheme} \hfill Denner, S.D., Roth, Wieders '05
- loop integrals evaluated with \textsc{collier}

Real corrections and Monte Carlo integration

- all amplitudes from SM calculation via rescaling with factor $s^{\beta-\alpha}$ \hfill Catani, Seymour '96; S.D. '99; S.D. et al. '08
- IR singularities treated with dipole subtraction
- multi-channel Monte Carlo integration within \textsc{prophecy4f}

Two independent calculations of all ingredients
Details of the NLO calculation

Virtual corrections

- model file generation with FeynRules
- diagram generation with FeynArts
- amplitude reduction with inhouse Mathematica routines or FORM
- W/Z resonances treated in the complex-mass scheme

Denner, S.D., Roth, Wieders '05

- loop integrals evaluated with COLLIER Real corrections and Monte Carlo integration

- all amplitudes from SM calculation via rescaling with factors
- IR singularities treated with dipole subtraction

Catani, Seymour '96; S.D. '99; S.D. et al. '08

- multi-channel Monte Carlo integration within PROPHET

Two independent calculations of all ingredients

Collier – Hepforge

http://collier.hepforge.org/private/index.html

Collier is hosted by Hepforge, IPPP Durham

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Features of the library

COLLIER is a fortran library for the numerical evaluation of one-loop scalar and tensor integrals appearing in perturbative relativistic quantum field theory with the following features:

- scalar and tensor integrals for high particle multiplicities
- dimensional regularization for ultraviolet divergences
- dimensional regularization for soft infrared divergences
  (mass regularization for abelian soft divergences is supported as well)
- dimensional regularization or mass regularization for collinear mass singularities
- complex internal masses (for unstable particles) fully supported
  (external momenta and virtualities are expected to be real)
- numerically dangerous regions (small Gram or other kinematical determinants)
  cured by dedicated expansions
- two independent implementations of all basic building blocks allow for internal cross-checks
- cache system to speed up calculations

If you use Collier for a publication, please cite all the references listed here!
Details of the NLO calculation

Virtual corrections

• model file generation with \textsc{feynrules}
• diagram generation with \textsc{feynarts}
• amplitude reduction with inhouse Mathematica routines or \textsc{formcalc}
• W/Z resonances treated in the \textit{complex-mass scheme} \cite{Denner:2005fg,Denner:2001ia}
• loop integrals evaluated with \textsc{collier}

Real corrections and Monte Carlo integration

• all amplitudes from SM calculation via rescaling with factor $s_{\beta-\alpha}$
• IR singularities treated with dipole subtraction \cite{Catani:1996vz,Denner:1999gp,Denner:2008uj}
• multi-channel Monte Carlo integration within \textsc{prophecy4f}

Two independent calculations of all ingredients
NLO corrections to $h \rightarrow 4f$ in the THDM implemented in ...

A Monte Carlo generator for a
Proper description of the
Higgs decay into 4 fermions

Prophecy4f

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Aleandar Mück

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Universität Freiburg, Germany
RWTH Aachen University, Germany

Former Authors
Axel Breidenstein
Marcus Weber

Prophecy4f is a Monte Carlo integrator for Higgs decays $H \rightarrow WW/ZZ \rightarrow 4$ fermions
It includes:
• all four-fermion final states
• NLO QCD and electroweak corrections
• all interferences at LO and NLO
• effects beyond NLO from heavy-Higgs effects
• alternatively an Improved Born Approximation (IBA) with leading effects of the corrections
• production of unweighted events for leptonic final states
• optional inclusion of a 4th fermion generation (w/ or w/o leading two-loop improvements)

← New PROPHECY4F version available on request (on hepforge soon)
Numerical results
Scale dependence of the $h \rightarrow 4f$ width in scenario A:

- Ren. scale dependence: reduction from LO $\rightarrow$ NLO in all schemes
  Note: scale $\mu_r = M_h$ inappropriate

- Ren. scheme dependence: reduction from LO $\rightarrow$ NLO
  Note: consistent parameter conversion mandatory!
$c_{\beta-\alpha}$ dependence of $h \to 4f$ width in scenario A:

\begin{align*}
\Gamma_{h \to 4f} \text{ [MeV]}:
\end{align*}

\begin{itemize}
  \item $\overline{\text{MS}}(\lambda_3)$ scheme used \quad \Rightarrow \quad \Gamma_{h \to 4f}^{\text{THDM,LO}} \bigg|_{\overline{\text{MS}}(\lambda_3)} = s_{\beta-\alpha}^2 \Gamma_{h \to 4f}^{\text{SM,LO}}$
  \item relative difference to SM: \quad \Delta_{\text{SM}} \lesssim 2\% (6\%) for $|c_{\beta-\alpha}| < 0.1 (0.2)$
\end{itemize}
Partial $h\to 4f$ widths in scenario Aa

<table>
<thead>
<tr>
<th>Final state</th>
<th>$\Gamma_{NLO}^{h\to 4f}$ [MeV]</th>
<th>$\delta_{EW}$ [%]</th>
<th>$\delta_{QCD}$ [%]</th>
<th>$\Delta_{SM}^{NLO}$ [%]</th>
<th>$\Delta_{SM}^{LO}$ [%]</th>
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</thead>
<tbody>
<tr>
<td>inclusive $h\to 4f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZZ</td>
<td>0.96730(7)</td>
<td>2.71(0)</td>
<td>4.96(1)</td>
<td>−1.05(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>WW</td>
<td>0.106126(6)</td>
<td>0.34(0)</td>
<td>4.88(0)</td>
<td>−1.13(1)</td>
<td>−1.00(0)</td>
</tr>
<tr>
<td>WW/ZZ int.</td>
<td>0.86630(8)</td>
<td>3.00(0)</td>
<td>5.01(1)</td>
<td>−1.04(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu e^+\mu^-\bar{\nu}_\mu$</td>
<td>−0.00513(5)</td>
<td>1.3(2)</td>
<td>12.0(8)</td>
<td>−1(1)</td>
<td>−1(1)</td>
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<td>$\nu e^+u\bar{d}$</td>
<td>0.010201(1)</td>
<td>3.03(0)</td>
<td>0.00</td>
<td>−1.04(1)</td>
<td>−1.00(1)</td>
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<tr>
<td>$\nu e^-\bar{u}d\bar{c}$</td>
<td>0.031719(4)</td>
<td>3.02(0)</td>
<td>3.76(1)</td>
<td>−1.04(2)</td>
<td>−1.00(1)</td>
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<td>$\nu e^+e^-\bar{\nu}_e$</td>
<td>0.09847(2)</td>
<td>2.97(0)</td>
<td>7.52(1)</td>
<td>−1.04(2)</td>
<td>−1.00(1)</td>
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<tr>
<td>$\nu e^-\bar{d}d\bar{u}$</td>
<td>0.010197(1)</td>
<td>3.12(0)</td>
<td>0.00</td>
<td>−1.04(1)</td>
<td>−1.00(1)</td>
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<td>$\nu e \nu_e \nu_\mu \bar{\nu}_\mu$</td>
<td>0.000949(0)</td>
<td>3.01(0)</td>
<td>0.00</td>
<td>−1.14(1)</td>
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<td>$e^-e^+\mu^-\mu^+$</td>
<td>0.000239(0)</td>
<td>1.30(1)</td>
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<td>$\nu e \bar{e} \nu_e \mu^-$</td>
<td>0.000477(0)</td>
<td>2.45(1)</td>
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<td>$e^-e^+e^-e^+$</td>
<td>0.000132(0)</td>
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<td>$\nu e \bar{e}u\bar{u}$</td>
<td>0.001679(0)</td>
<td>0.60(1)</td>
<td>3.76(1)</td>
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<td>−1.00(1)</td>
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<td>$\nu e \bar{e}d\bar{d}$</td>
<td>0.002177(1)</td>
<td>1.69(0)</td>
<td>3.76(1)</td>
<td>−1.12(2)</td>
<td>−1.00(1)</td>
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<tr>
<td>$e^-e^+u\bar{u}$</td>
<td>0.000845(0)</td>
<td>0.11(1)</td>
<td>3.76(1)</td>
<td>−1.12(2)</td>
<td>−1.00(1)</td>
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<tr>
<td>$e^-e^+d\bar{d}$</td>
<td>0.001088(0)</td>
<td>0.47(1)</td>
<td>3.76(1)</td>
<td>−1.12(2)</td>
<td>−1.00(1)</td>
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<tr>
<td>$u\bar{u}c\bar{c}$</td>
<td>0.002971(0)</td>
<td>−1.80(1)</td>
<td>7.51(1)</td>
<td>−1.11(2)</td>
<td>−1.00(1)</td>
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<tr>
<td>$d\bar{d}d\bar{d}$</td>
<td>0.002556(1)</td>
<td>−0.38(0)</td>
<td>4.38(2)</td>
<td>−1.21(3)</td>
<td>−1.00(1)</td>
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<tr>
<td>$d\bar{d}s\bar{s}$</td>
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<td>7.51(1)</td>
<td>−1.12(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$u\bar{u}s\bar{s}$</td>
<td>0.003852(1)</td>
<td>−0.66(1)</td>
<td>7.51(1)</td>
<td>−1.11(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$u\bar{u}u\bar{u}$</td>
<td>0.001506(0)</td>
<td>−1.92(1)</td>
<td>4.06(3)</td>
<td>−1.24(4)</td>
<td>−1.00(1)</td>
</tr>
</tbody>
</table>
NLO corrections to leptonic distributions in scenario A

Altenkamp et al. ’17

\[ \frac{d\Gamma}{dM_{\mu\mu}} \quad h \rightarrow \mu^- \mu^+ e^- e^+ \quad \text{SM} \]

\[ \frac{d\Gamma}{d\phi} \left[ \frac{10^{-7} \text{MeV}}{\text{deg}} \right] \quad h \rightarrow \mu^- \mu^+ e^- e^+ \quad \text{MS}(\lambda_3) \]

\[ \delta_{\text{NLO}} \% \]

\[ \delta_{\text{NLO}} \% \]

\[ \frac{d\Gamma}{dM_{\mu\mu}} \quad h \rightarrow \mu^- \mu^+ e^- e^+ \]

\[ \frac{d\Gamma}{d\phi} \left[ \frac{10^{-7} \text{MeV}}{\text{deg}} \right] \quad h \rightarrow \mu^- \mu^+ e^- e^+ \]

\[ \delta_{\text{NLO}} \% \]

\[ \delta_{\text{NLO}} \% \]

\[ \delta_{\text{THDM}} \approx \delta_{\text{SM}} + \text{const.} \]

mainly due to external \( hH \) mixing
Conclusions
NLO corrections in the THDM

in principle straightforward, but involves issues:

• choice of input parameters, which ones in $\overline{\text{MS}}$?
• gauge dependences, perturbative stability, etc.

$\rightarrow$ several schemes proposed and applied in recent literature

$h \rightarrow WW/ZZ \rightarrow 4f$ at NLO in the THDM

• results presented for a low-mass scenario ($M_{H,A_0,H^+} \sim 300-460$ GeV)
  ◦ $|\text{THDM} - \text{SM}| \lesssim 5\%$ for viable THDM parameters $c_\beta - c_\alpha$
  ◦ significant reduction in ren. scale and scheme dependence for LO $\rightarrow$ NLO
  ◦ no further distortion of distributions in SM $\rightarrow$ THDM at NLO
  ◦ no sensitivity of $h \rightarrow 4f$ to the type of THDM

• results for large $M_{H,A_0,H^+}$ in recent publication
  ◦ results generically similar
  ◦ but: pathologies for scenarios near exp. exclusion and theoretical bounds

$\rightarrow$ study of ren. scale and scheme dependence crucial for solid predictions

Outlook:

• Similar studies recently carried out for a singlet extension of the SM
• Construction of “universally well-behaved” ren. schemes in progress
Backup slides
Yukawa couplings:

Avoid FCNC at tree level!

$\rightarrow$ Couple each fermion flavour only to one $\Phi_n$ ($\mathbb{Z}_2$ symmetry)

$$\mathcal{L}_{\text{Yukawa}} = -\bar{L}'^L Y^l l'^R \Phi_{n_1} - \bar{Q}'^L Y^u u'^R \Phi_{n_2} - \bar{Q}'^L Y^d d'^R \Phi_{n_3} + h.c.$$
Generic diagrams for $hh$, $hH$, $HH$ self-energies

$\leftrightarrow$ external wave-function renormalization + $hH$ mixing

$S = h, H, A_0, H^\pm, G_0, G^\pm$

Generic diagrams with internal heavy Higgs bosons $H, A_0, H^\pm$
Classification of QCD corrections

Possible Born diagrams:

(1) \[ hVV \]
\[ f_a \]
\[ \bar{f}_b \]
\[ V \]
\[ f_c \]
\[ \bar{f}_d \]

(2) \[ hV'V' \]
\[ f_a \]
\[ \bar{f}_b \]
\[ V' \]
\[ f_c \]
\[ \bar{f}_d \]

diagrams (2) only for \( f\bar{f}f\bar{f} \) and \( f\bar{f}f'\bar{f}' \) channels
\((f' = \) weak-isospin partner of \( f \))

Classification of QCD corrections into four categories: (typical diagrams shown)

(a)

(d) only QCD correction without universal scaling \( \propto s^{\beta - \alpha} \) from \( \mathcal{M}_{SM} \)

(b,c,d) = corrections to interferences (only for \( q\bar{q}q\bar{q} \) and \( q\bar{q}q'\bar{q}' \) channels)
### Partial $h \to 4f$ widths in scenario Ab

Altenkamp et al. '17

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<td>ZZ</td>
<td>0.95980(7)</td>
<td>1.87(0)</td>
<td>4.97(1)</td>
<td>−1.82(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>WW</td>
<td>1.05546(5)</td>
<td>−0.34(0)</td>
<td>4.90(0)</td>
<td>−1.75(1)</td>
<td>−1.00(0)</td>
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<tr>
<td>WW/ZZ int.</td>
<td>0.85938(8)</td>
<td>2.14(0)</td>
<td>5.01(1)</td>
<td>−1.83(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e e^+\mu^-\bar{\nu}_\mu$</td>
<td>−0.00504(5)</td>
<td>0.5(1)</td>
<td>10.7(8)</td>
<td>−2(1)</td>
<td>−1(1)</td>
</tr>
<tr>
<td>$\nu_e e^+u\bar{d}$</td>
<td>0.010116(1)</td>
<td>2.17(1)</td>
<td>0.00</td>
<td>−1.87(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e e^-\nu_e$</td>
<td>0.031463(4)</td>
<td>2.16(0)</td>
<td>3.76(1)</td>
<td>−1.84(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{d}s\bar{c}$</td>
<td>0.09770(2)</td>
<td>2.11(0)</td>
<td>7.52(1)</td>
<td>−1.81(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}e^+\nu_e$</td>
<td>0.010112(1)</td>
<td>2.27(1)</td>
<td>0.00</td>
<td>−1.87(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{d}d\bar{u}$</td>
<td>0.09972(2)</td>
<td>1.99(0)</td>
<td>7.38(2)</td>
<td>−1.80(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}\nu_e\nu_e\bar{\nu}_\mu$</td>
<td>0.000943(0)</td>
<td>2.34(0)</td>
<td>0.00</td>
<td>−1.78(1)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$e^- e^+\mu^-\mu^+$</td>
<td>0.000237(0)</td>
<td>0.62(1)</td>
<td>0.00</td>
<td>−1.79(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}e\mu^-\mu^+$</td>
<td>0.000474(0)</td>
<td>1.78(1)</td>
<td>0.00</td>
<td>−1.78(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}\nu_e\nu_e\nu_e$</td>
<td>0.000565(0)</td>
<td>2.23(0)</td>
<td>0.00</td>
<td>−1.79(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$e^- e^+e^- e^+$</td>
<td>0.000131(0)</td>
<td>0.45(1)</td>
<td>0.00</td>
<td>−1.78(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}e\bar{u}\bar{u}$</td>
<td>0.001668(0)</td>
<td>−0.08(1)</td>
<td>3.76(1)</td>
<td>−1.76(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$\nu_e \bar{e}d\bar{d}$</td>
<td>0.002163(0)</td>
<td>1.02(0)</td>
<td>3.76(1)</td>
<td>−1.76(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$e^- e^+u\bar{u}$</td>
<td>0.000840(0)</td>
<td>−0.57(1)</td>
<td>3.76(1)</td>
<td>−1.77(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$e^- e^+d\bar{d}$</td>
<td>0.001081(0)</td>
<td>−0.21(1)</td>
<td>3.76(1)</td>
<td>−1.76(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$u\bar{u}c\bar{c}$</td>
<td>0.002952(0)</td>
<td>−2.48(1)</td>
<td>7.51(1)</td>
<td>−1.75(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$d\bar{d}d\bar{d}$</td>
<td>0.002545(1)</td>
<td>−1.06(0)</td>
<td>4.57(2)</td>
<td>−1.67(3)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$d\bar{d}s\bar{s}$</td>
<td>0.004925(1)</td>
<td>−1.04(0)</td>
<td>7.51(1)</td>
<td>−1.74(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$u\bar{u}s\bar{s}$</td>
<td>0.003828(1)</td>
<td>−1.35(1)</td>
<td>7.51(1)</td>
<td>−1.74(2)</td>
<td>−1.00(1)</td>
</tr>
<tr>
<td>$u\bar{u}u\bar{u}$</td>
<td>0.001500(0)</td>
<td>−2.60(1)</td>
<td>4.31(2)</td>
<td>−1.65(3)</td>
<td>−1.00(1)</td>
</tr>
</tbody>
</table>
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\[ \frac{d\Gamma}{dM_{\nu\mu\mu}} \quad h \rightarrow \nu_\mu \mu^+ e^- \nu_e \]

\[ \frac{d\Gamma}{d\phi_{\mu e, T}} \quad h \rightarrow \nu_\mu \mu^+ e^- \nu_e \]

\[ \delta_{\text{NLO}} \quad [\%] \]

\[ M_{\nu\mu\mu} \quad [\text{GeV}] \]

\[ \phi_{\mu e, T} \quad [\text{deg}] \]

Correction \( \delta_{\text{THDM}} \approx \delta_{\text{SM}} + \text{const.} \)

mainly due to external \( hH \) mixing

\( \phi_{T, \mu e} = \angle(\mu, e) \) in a fixed plane \( \approx \) (plane \( \perp \) beams)
NLO corrections to semileptonic distributions in scenario A

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\[ \frac{d\Gamma}{dM_{qq}} \quad h \rightarrow \bar{d}d e^{-} e^{+} \]

\[ \frac{d\Gamma}{d|\cos \phi|}[\text{MeV}] \quad h \rightarrow \bar{d}d e^{-} e^{+} \]

\[ \text{MS}(\lambda_3) \]

\[ \delta_{\text{NLO}} \% \]

\[ \delta_{\text{NLO}} \% \]

\[ M_{qq}[\text{GeV}] \]

\[ | \cos \phi | \]
NLO corrections to semileptonic distributions in scenario A

\[
\frac{d\Gamma}{dM_{qq}} \quad h \rightarrow \nu_e e^+ d\bar{u}
\]

\[
\frac{d\Gamma}{d\cos \phi_{eW}} [\text{MeV}] \quad h \rightarrow \nu_e e^+ d\bar{u}
\]

\[
\delta_{NLO} [\%]
\]

\[
M_{qq}[\text{GeV}]
\]

\[
\cos \phi_{eW}
\]

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\[\overline{\text{MS}}(\lambda_3)\]

Stefan Dittmaier, Precision calculations for \(h \rightarrow WW/ZZ \rightarrow 4f\) ... relationships.
Scale dependence of the $h \to 4f$ width in large-mass scenario B1:

$M_H = 600 \text{ GeV}, \quad M_{A_0} = M_{H^+} = 690 \text{ GeV}, \quad \lambda_5 = -1.9, \quad \tan\beta = 4.5$

Ren. scale and scheme dependence in LO → NLO:

- stabilization degrades when $\cos(\beta - \alpha)$ increases (getting away from the decoupling limit)

- good stability for $\overline{\text{MS}}(\alpha)$ and $\overline{\text{MS}}(\lambda_3)$ schemes

- FJ schemes degrade earlier due to large tadpole terms