Sound (and related) anomalies near the QCD phase transition

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Near $T_c$ quark matter is expected to exhibit several spectacular phenomena.

1. Trace anomaly
   \[ \Theta/T_c = \frac{\varepsilon - 3p}{T_c^4} \]

2. Speed of sound
   \[ T = T_c \rightarrow \text{the softest EoS point} \]

3. $\frac{\varepsilon}{S}$ divergence
   \[ P = \rho \text{visc} - S \frac{\varepsilon}{S} \]

4. Sound attenuation anomaly

5. Electrical conductivity growth

6. Excess of soft photon emission
Physics depends on the \((T, \mu)\) values - QCD phase diagram

("Theorist's science fiction" - Owe Philipsen)

Focus of this talk - finite density

\(\mu_q \sim 300-400\) MeV, \(T\) approaching

\(T_c\) from above, \(T \to T_c (\sim 40\) MeV)

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We show that the following phenomena

- sound attenuation anomaly
- bulk viscosity divergence
- electrical conductivity growth
- enhanced soft photon emission

have the same dynamical origin - collective soft mode of the diquark field
Pre-critical fluctuations — inherent in the II-nd or weak I-st order phase transitions

Wide fluctuation region \( G_i = \frac{ST}{T_c} \approx 10^8 \left( \frac{ST}{T_c} \right)_{BCS} \)

What are we calculating and how?

\( \sigma, \omega \frac{d\rho}{d\rho} \)

\( \langle j^e_m(t, \vec{x}), j^e_{y}(0, \vec{0}) \rangle \)

\( \langle T^i_{\gamma}(t, \vec{x}), T^i_{\gamma}(0, \vec{0}) \rangle \)

\( \gamma \)-sound absorption

\( \eta \)-bulk viscosity

\( \Pi^R_{\mu\nu} \) — retarded response operator, Wightman correlator, polarization tensor. Not calculable beyond perturbation theory
\[ \Pi_{\mu \nu}^R - \text{one loop contribution, Matsubara formalism} \]

- Photon-Drude-Lorentz conductivity

\[ \frac{\delta F}{\delta e_{\mu}} = \pi e^2 \sum_{e_m} (\hat{q}, \omega) = T \sum_{\tilde{\omega}_n} \frac{d\tilde{\omega}_n}{(2\pi)^3} \left[ G(\tilde{p}, \tilde{\omega}_n) \frac{1}{\omega - i\gamma} \right] x G(\tilde{p}, \tilde{\omega}_n + \omega) \frac{1}{\omega - i\gamma} = \]

\[ = \frac{1}{3} \frac{\tilde{\omega}_n^2}{2\pi} \sum_{\omega_k} \frac{d\omega_k}{4\pi} \frac{1}{1 + |\omega_k|^2 + i\tilde{\omega}_n^2} \rightarrow \text{Hard density loop, } q \ll p \]

\[ \tilde{\omega}_n = \frac{\Gamma T}{2} (2n + 1) + \frac{4}{2\pi} \text{sgn} \tilde{\omega}_n, \Gamma - \text{momentum relaxation time}, T \equiv 0.3 \text{ fm} \]

\[ \Gamma = \frac{M_\alpha F}{2T} \text{ replaces } g^2 T^2 \text{ in HTL} \]

\[ \Gamma \text{ regulates collinear singularities} \]

\[ \text{vs. HTL} \quad \Pi_{\nu \nu}(\hat{q}, \omega) = m^2 D \omega \sum_{\omega_k} \frac{d\omega_k}{4\pi} \frac{q^2 \omega_k}{\omega - i\gamma + \omega_k} \sim g^2 T^2 \]

\( \xi_1 \) (one-loop) is temperature independent, single-particle effect, no dependence on temperature Fermi surface blurring
Near $T_c$ slow relaxation collective mode dominates

**Fluctuating quark pair field**

\[
\begin{align*}
T < T_c & \quad \text{gap } \Delta \\
\hline
\text{color quark condensate } \langle \psi \psi \rangle & \quad \text{fluctuating } \langle \psi \psi \rangle \\
\end{align*}
\]

\[
T > T_c, T \to T_c^+ \\
\begin{align*}
\text{fluctuating } \langle \psi \psi \rangle & \quad T_{GL} = \frac{Jt}{8(T-T_c)} \\
\end{align*}
\]

Wide fluctuation region, $G_i = \frac{ST}{T_c} \sim 10^{8-10} \left( \frac{ST}{T_c} \right)_{BCS}$

\[\text{quark propagator } G(s,0)\]

\[\text{fluctuation propagator } L(s,0)\]

The famous Aslamazov-Larkin diagram
Derivation of $L(\tilde{\eta}, \omega)$

- Dyson Equation

\[ L(\tilde{\eta}, \omega) = \begin{array}{c}
\text{cross}\quad \rightarrow\quad \text{cross plus loop}
\end{array} \]

- Time dependent Ginzburg-Landau with Langevin force

\[ F[\varphi] = (\pm |\Psi|^2 + \frac{\pi}{8\tilde{T}_c} \varphi \partial_{\varphi} |\Psi|^2) \rightarrow \gamma (\pm |\Psi|^2 + \frac{\pi}{8\tilde{T}_c} \varphi \partial_{\varphi}^2) \]

\[ t = \frac{T-T_c}{T_c}, \quad \varphi - \text{diffusion coefficient} \]

\[ -\chi \frac{\partial \varphi(\tilde{\eta}, t)}{\partial \tilde{\eta}} = \frac{s F}{s \varphi} + \bar{\zeta}(\tilde{\eta}, t), \quad \bar{\zeta} - \text{Langevin forces, thermodynamic fluctuations} \]

\[ \hat{L} = -\left[ \chi \frac{\partial^2}{\partial \tilde{\eta}^2} + \gamma (\pm + \frac{\pi}{8\tilde{T}_c} \partial_{\tilde{\eta}}^2) \right]^{-1} \rightarrow -\frac{1}{\gamma} \frac{1}{\frac{1}{T} + \frac{\pi}{8\tilde{T}_c} (-i\omega + \partial_{\tilde{\eta}}^2)} \]

At $T \rightarrow T_c$, $\omega, \varphi -$ small, $\hat{L}$ arbitrary large and rapidly varying \( \Rightarrow \) All diagram dominates (Ornstein-Zernike)
Aslamazov-Larkin Diagram (AL)

\[ \Pi(\tilde{q}, \omega) \sim \text{Diagram} \]

\[ \text{em, vector} \quad \text{sound, scalar} \]

\[ B \to \sim \sum_{\text{even}} \frac{d\tilde{p}}{(2\pi)^3} \]

AL diagram is twice singular at \( T_c \) due to two \( \uparrow \)
Electrical Conductivity and Soft Photon Emission

\[ \sigma_{el} = -\frac{1}{\omega_{K}} y m \Gamma_R \gamma_{\mu} \], \quad \omega \frac{dR_{x}}{d^3p} = -\frac{2}{(2\pi)^3} \frac{1}{e^{\omega T} - 1} y m \Gamma_R \gamma_{\mu} \]

\[ \simeq 1 \omega \ll T \rightarrow \frac{2}{(2\pi)^3} T \sigma_{el} \]

Very close to \( T_c \), the rise is suppressed by the interaction between fluctuations.

Results for \( G_{1} = t = \frac{T - T_c}{T_c} = 10^{-2} \)

\( \sigma_{el} (AL) = 0.08 \text{ fm}^{-1} \)

\( \sigma_{el} (\text{Karsch et al}) = 0.02 - 0.06 \text{ fm}^{-1} \)

\( \omega \frac{dR_{x}}{d^3p} = 1.3 \times 10^{-2} \text{ fm}^{-4} \text{ GeV}^{-2} \)

\( \omega \frac{dR_{x}}{d^3p} (\text{Karsch}) = (0.3 - 1.0) \text{ fm}^{-4} \text{ GeV}^{-2} \)

An excess of low invariant mass dielectrons has been observed by CERES, PHENIX, ALICE. Significant enhancement of soft photons is expected in future experiments at NICA.
Sound Absorption Anomaly and Bulk Viscosity Divergence

AL diagram with scalar vertices $g$ (instead of $\epsilon \chi_e$) and two fluctuation propagators $\tilde{L}$ and energy-dependent density of states near the Fermi surface.

$$A(x) = A_0 e^{-\gamma x}, \quad \gamma \text{ - attenuation constant}$$

$\gamma$ from Dyson equation for phonon:

$$D^{-1}(\omega, k) = D_0^{-1}(\omega, k) - i L^R(\omega, k), \quad D_0(\omega, k) = \frac{\omega^2}{\omega^2 - \omega_0^2 + i 0}$$

$$\gamma m i L^R = \omega^2 g^2 R t^{-\frac{3}{2}} \rightarrow \gamma = \omega^2 g^2 R t^{-\frac{3}{2}}$$

$$\gamma \sim \omega^2 g \rightarrow s \sim t^{-\frac{3}{2}}$$

Near $T_c$, sound absorption is anomalously large and bulk viscosity diverges as $t^{-\frac{3}{2}}$ (vs. $t^{-\frac{1}{2}}$ for $\xi_d$ and $\omega d\xi_d / dp$).
Bulk viscosity near $T_c$ in other approaches

A difficult problem. Not yet completely solved.

- Ising-like universality
- Mode coupling theory
- Renormalization group

$d = 4 - \delta$ regularization

$\gamma, \delta \sim \xi^{2-4\gamma}$, $\frac{\delta}{\mu} = |t|^{-\gamma} \sim |t|^{-0.61}$

$\gamma, \delta \sim |t|^{-1.68}$ vs. $t^{-3/2}$ here
SUMMARY

- The properties of quark matter in finite-density pre-critical region are determined by the soft mode.
- The soft mode is described by the fluctuation propagator singular at the critical temperature.
- Fluctuation propagator brings a dominant contribution into transport coefficients and soft photon emission around the transition line.
- Sound attenuation constant and bulk viscosity diverge as $t^{-3/2}$ at $T \rightarrow T_c$.
- Electrical conductivity and soft photon emission are enhanced as $t^{-1/2}$.

Thank you for attention!