Power corrections to TMD factorization

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A typical factorization formula for production of a particle with a small transverse momentum in hadron-hadron collisions:

\[
\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp} D_f/A(x_A, b_\perp, \eta) D_f/B(x_B, b_\perp, \eta) \sigma(ff \to H) + \text{power corrections} + \text{Y-terms}
\]

When we increase transverse momentum \(q^2_\perp\) of the produced particle:

- At first the leading power TMD analysis with (nonperturbative) TMDs applies,
- then at some point power corrections kick in,
- and finally at \(q^2_\perp \sim Q^2\) they are transformed into so-called Y-term making smooth transition to collinear factorization formulas.

In this talk I try to answer the question about the first transition, namely at what \(q^2_\perp\) power corrections become significant.
Suppose we produce a scalar particle (Higgs) in a gluon-gluon fusion.  

For simplicity, assume the vertex is local:

\[ \mathcal{L}_\Phi = g_\Phi \int dz \, \Phi(z) F^2(z), \quad F^2 \equiv F^a_{\mu\nu}F^{\mu\nu}_a. \]

\[ \text{s} \gg Q^2 \gg q^2 \parallel \]

\[ q^2 = Q^2 = M_H^2 \]
Matrix element between hadron states \( \Rightarrow \sum_X = 1 \)

"Hadronic tensor"

\[
W(p_A, p_B, q) \overset{\text{def}}{=} \sum_X \int d^4x \ e^{-iqx} \langle p_A, p_B | F^2(x) | X \rangle \langle X | F^2(0) | p_A, p_B \rangle \\
= \int d^4x \ e^{-iqx} \langle p_A, p_B | F^2(x)F^2(0) | p_A, p_B \rangle
\]

Double functional integral for \( W \)

\[
W(p_A, p_B, q) = \sum_X \int d^4x \ e^{-iqx} \langle p_A, p_B | F^2(x) | X \rangle \langle X | F^2(0) | p_A, p_B \rangle
\]

\[
= \lim_{t_f \to \infty} \lim_{t_i \to -\infty} \int d^4x \ e^{-iqx} \int \tilde{A}(t_f) = A(t_f) D\tilde{A}_\mu D A_\mu \int \tilde{\psi}(t_f) = \psi(t_f) D\tilde{\psi} D\psi D\bar{\psi} D\bar{\psi} \Psi^*_p (\tilde{A}(t_i), \tilde{\psi}(t_i)) \\
\times \Psi_p (\tilde{A}(t_i), \tilde{\psi}(t_i)) e^{-iS_{\text{QCD}}(\tilde{A}, \tilde{\psi})} e^{iS_{\text{QCD}}(A, \psi)} \tilde{F}^2(x)F^2(y) \Psi_p (\tilde{A}(t_i), \psi(t_i)) \Psi_p (\tilde{A}(t_i), \psi(t_i))
\]

"Left" \( A, \psi \) fields correspond to the amplitude \( \langle X | F^2(0) | p_A, p_B \rangle \),

"right" fields \( \tilde{A}, \tilde{\psi} \) correspond to amplitude \( \langle p_A, p_B | F^2(x) | X \rangle \)

The boundary conditions \( \tilde{A}(t_f) = A(t_f) \) and \( \tilde{\psi}(t_f) = \psi(t_f) \) reflect the sum over intermediate states \( X \).
Rapidity factorization for particle production

Sudakov variables:

\[ p = \alpha p_1 + \beta p_2 + p_\perp, \quad p_1 \simeq p_A, \quad p_2 \simeq p_B, \quad p_1^2 = p_2^2 = 0 \]

\[ x_* \equiv p_2 \cdot x = \sqrt{\frac{s}{2}} x^+, \quad x_\cdot \equiv p_1 \cdot x = \sqrt{\frac{s}{2}} x^- \]

We integrate over “central” fields in the background of projectile and target fields.
$F_{\mu \nu}^2 (C)$ in the tree approximation

$F_{\mu \nu}^2 (C) = \text{sum of tree diagrams in external } A \text{ and } B \text{ fields}$
$F_{\mu\nu}(C)$ in the tree approximation

$F_{\mu\nu}(C) = \text{sum of tree diagrams in external } \tilde{A}, A \text{ and } \tilde{B}, B \text{ fields with sources } \tilde{J}_\mu = D^\mu F_{\mu\nu}(\tilde{A} + \tilde{B}) \text{ and } J_\mu = D^\mu F_{\mu\nu}(A + B)$
Approximations for projectile and target fields

At the tree level $\beta = 0$ for $A, \tilde{A}$ fields and $\alpha = 0$ for $B, \tilde{B}$ fields $\iff$

$A = A(x_\bullet, x_\perp)$, $\tilde{A} = \tilde{A}(x_\bullet, x_\perp)$ and $B = B(x_\star, x_\perp)$, $\tilde{B} = \tilde{B}(x_\star, x_\perp)$.

The fields $A, \psi$ and $\tilde{A}, \tilde{\psi}$ do not depend on $x_\star \implies$

if they coincide at $x_\star = \infty \implies$ they coincide everywhere.

Similarly,

$B, \psi_b$ and $\tilde{B}, \tilde{\psi}_b$ do not depend on $x_\bullet \implies$

if they coincide at $x_\bullet = \infty$ they should be equal.

Since $\tilde{A} = A$ and $\tilde{B} = B$ the sources and background fields are the same to the left and to the right of the cut $\implies$

$F_{\mu\nu}(C)$ is a sum of diagrams with retarded Green functions

(F. Gelis, R. Venugopalan)
$F_{\mu\nu}(C)$ in the tree approximation

Feynman diagrams with retarded propagators $\Leftrightarrow$ perturbative solution of classical YM equations.
Classical solution

The sum of diagrams with retarded Green functions $\Leftrightarrow$ solution of classical YM equations

$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f t^a \gamma_\mu \psi^f, \quad (\not{p} + m_f)\psi^f = 0$$

with boundary conditions

$$A_\mu(x) \xrightarrow{x^* \to -\infty} \bar{A}_\mu(x_\bullet, x_\perp), \quad \psi(x) \xrightarrow{x^* \to -\infty} \psi_a(x_\bullet, x_\perp)$$

$$A_\mu(x) \xrightarrow{x_\bullet \to -\infty} \bar{B}_\mu(x_\bullet, x_\perp), \quad \psi(x) \xrightarrow{x_\bullet \to -\infty} \psi_b(x_\bullet, x_\perp)$$

following from $C_\mu, \psi_c \xrightarrow{t \to -\infty} 0$.

The projectile and target fields satisfy YM equations

$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f a t^a \gamma_\mu \psi^f, \quad (\not{p} + m_f)\psi^f_a = 0$$

$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f b t^a \gamma_\mu \psi^f, \quad (\not{p} + m_f)\psi^f_b = 0$$

Method of solution: start with $\bar{A}_\mu + \bar{B}_\mu$ and correct by computing Feynman diagrams (with retarded propagators) with a source $J_\nu = D^\mu F^{\mu\nu}(U + V)$
Convenient gauge: \( A_* = 0 \) for the projectile and \( A^\cdot = 0 \) for the target.

\[
U_i(x_\bullet, x_\perp) \sim m_\perp, \quad U^\cdot(x_\bullet, x_\perp) \sim m^2_\perp, \quad U_* = 0
\]
\[
V_i(x_\star, x_\perp) \sim m^2_\perp, \quad V^\star(x_\star, x_\perp) \sim m^2_\perp, \quad V^\cdot = 0
\]

and we have to solve

\[
D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f t^a \gamma_\mu \psi^f, \quad (\not{p} + m_f)\psi^f = 0
\]

with boundary conditions

\[
A_\mu(x) \xrightarrow{x_\star \to -\infty} U_\mu(x_\bullet, x_\perp), \quad \psi(x) \xrightarrow{x_\star \to -\infty} \Sigma_a(x_\bullet, x_\perp)
\]
\[
A_\mu(x) \xrightarrow{x^\bullet \to -\infty} V_\mu(x_\star, x_\perp), \quad \psi(x) \xrightarrow{x^\bullet \to -\infty} \Sigma_b(x_\star, x_\perp)
\]

Now we start with \( U_\mu + V_\mu \) and compute Feynman diagrams (with retarded propagators) with a source \( J_\nu = D^\mu F^{\mu\nu}(U + V) \sim m^3_\perp \)
Gluon fields in the leading order in $p^2_\perp/p^2_\parallel \sim q^2_\perp/Q^2$

The solution of YM equations in general case (scattering of two “color glass condensates”) is yet unsolved problem.

Fortunately, for our case of particle production with $q_\perp/Q \ll 1$ we can use this small parameter and construct the approximate solution as a series in $q_\perp/Q$:

$$A = A^{(-1)} + A^{(0)} + A^{(1)} + ...$$

NB: After the expansion

$$\frac{1}{p^2 + i\epsilon p_0} = \frac{1}{p^2_{\parallel} - p^2_\perp + i\epsilon p_0} = \frac{1}{p^2_{\parallel}} - \frac{1}{p^2_{\parallel} + i\epsilon p_0} \frac{p^2_\perp}{p^2_{\parallel} + i\epsilon p_0} + ...$$

the dynamics in transverse space is trivial.

Gluon fields:

$$F^{(-1)}_{\bullet i} = V_{\bullet i}, \quad F^{(-1)}_{*i} = U_{*i}, \quad F^{(-1)}_{*\bullet} = U_{*\bullet} + V_{*\bullet} - \frac{is}{2} U^{ab} V^{bj}$$

$$F^{(0)a}_{\bullet i} = U^{a}_{\bullet i} - iU^{ab} V^{b}_{i} - \frac{i}{2(\alpha + i\epsilon)} \tilde{L}^{(0)}_{i} - \mathcal{D}^{ab}_{i} V^{bc}_{j} \frac{1}{2(\alpha + i\epsilon)} U^{cj},$$

$$F^{(0)a}_{*i} = V^{a}_{*i} - iV^{ab}_{*} U^{b}_{i} - \frac{i}{2(\beta + i\epsilon)} \tilde{L}^{(0)}_{i} - \mathcal{D}^{ab}_{i} U^{bc}_{j} \frac{1}{2(\beta + i\epsilon)} V^{cj},$$

$$F^{(0)}_{ik} = U_{ik} + V_{ik} - i[U_{i}, V_{k}] - i[V_{i}, U_{k}],$$

We integrate over $\alpha$ without cutoff $\alpha > \sigma$ since the contour over $\alpha$ can be removed from the pole to the region of large $\alpha$ (if there is no pinch). Similarly, we integrate over all $\beta$'s. (Different from SCET where they keep the cutoffs $\alpha > \sigma_b$ and $\beta > \sigma_a$).
**Gluon fields in the leading order in \( p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2 \)**

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\[
\frac{1}{p^2 + i\epsilon_0} = \frac{1}{p_\parallel^2 - p_\perp^2 + i\epsilon_0} = \frac{1}{p_\parallel^2} - \frac{1}{p_\parallel^2 + i\epsilon_0} \frac{p_\perp^2}{p_\parallel^2 + i\epsilon_0} + \ldots
\]

the dynamics in transverse space is trivial.

**Gluon fields:**

\[
\begin{align*}
F^{(-1)}_{\bullet i} & = V_{\bullet i}, \quad F^{(-1)}_{*i} = U_{*i}, \quad F^{(-1)}_{*\bullet} = U_{*\bullet} + V_{*\bullet} - \frac{iS}{2} U^{ab} V^{bj} \\
F^{(0)a}_{\bullet i} & = U^{a}_{\bullet i} - i U^{ab} V^{b}_{i} - \frac{i}{2(\alpha + i\epsilon)} \tilde{L}^{(0)}_{i} - \mathcal{D}^{ab}_{i} V^{bc}_{j} \frac{1}{2(\alpha + i\epsilon)} U^{cj}, \\
F^{(0)a}_{*i} & = V^{a}_{*i} - i V^{ab}_{*} U^{b}_{i} - \frac{i}{2(\beta + i\epsilon)} \tilde{L}^{(0)}_{i} - \mathcal{D}^{ab}_{i} U^{bc}_{j} \frac{1}{2(\beta + i\epsilon)} V^{cj}, \\
F^{(0)}_{ik} & = U_{ik} + V_{ik} - i[U_{i}, V_{k}] - i[V_{i}, U_{k}],
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\]

We integrate over \( \alpha \) without cutoff \( \alpha > \sigma \) since the contour over \( \alpha \) can be removed from the pole to the region of large \( \alpha \) (if there is no pinch). Similarly, we integrate over all \( \beta \)’s.

(Different from SCET where they keep the cutoffs \( \alpha > \sigma_b \) and \( \beta > \sigma_a \).)
In the region $s \gg Q^2 \gg Q_{\perp}^2$ at the tree level

\[
W(p_A, p_B, q) = \frac{64/s^2}{N_c^2 - 1} \int d^2x_\perp e^{i(q,x)_{\perp}2/s} \int dx_\bullet dx_\bullet^* e^{-i\alpha q x_\bullet - i\beta q x_\bullet^*}
\times \left\{ \langle p_A|U_{\bullet i}^m(x_\bullet, x_\perp)U_{\bullet j}^m(0)|p_A\rangle \langle p_B|V_{\bullet i}^n(x_\bullet, x_\perp)V_{\bullet j}^n(0)|p_B\rangle 
- \frac{N_c^2}{N_c^2 - 4} \frac{\Delta_{ij,kl}}{Q^2} \int_{-\infty}^{x_\bullet} d^2x_\bullet' \frac{2}{s} d^abc \langle p_A|U_{\bullet i}^a(x_\bullet, x_\perp)U_{\bullet j}^b(x_\bullet', x_\perp)U_{\bullet r}^c(0)|p_A\rangle 
\times \int_{-\infty}^{x_\bullet} d^2x_\bullet^* \frac{2}{s} d^{mnl} \langle p_B|V_{\bullet k}^m(x_\bullet, x_\perp)V_{\bullet l}^n(x_\bullet', x_\perp)V_{\bullet r}^n(0)|p_B\rangle + x \leftrightarrow 0 \right\}
\]

\[
\Delta_{ij,kl} = g^{ij}g^{kl} - g^{ik}g^{jl} - g^{il}g^{jk}
\]

The correction is $\sim \frac{q_{\perp}^2}{Q^2}$.
In the region $s \gg Q^2 \gg Q^2_\perp$ at the tree level

$$W(p_A, p_B, q) = \frac{64/s^2}{N_c^2 - 1} \int d^2 x_\perp e^{i(q, x)_\perp} \frac{2}{s} \int dx_\bullet dx_\ast e^{-i\alpha q x_\bullet - i\beta q x_\ast}$$

$$\times \left\{ \langle p_A | U_{m_i}^* (x_\bullet, x_\perp) U_{m_j}^* (0) | p_A \rangle \langle p_B | V_{n_i}^* (x_\bullet, x_\perp) V_{n_j}^* (0) | p_B \rangle - \frac{N_c^2}{N_c^2 - 4} \frac{\Delta_{ij, kl}}{Q^2} \int_{-\infty}^{x_\bullet} \int_{-\infty}^{x'_\bullet} \frac{2}{s} d^2 x'_\bullet d^2 x_\ast d^{abc} \langle p_A | U_{a_i}^* (x_\bullet, x_\perp) U_{a_j}^* (x'_\bullet, x_\perp) U_{a_r}^* (0) | p_A \rangle$$

$$\times \int_{-\infty}^{x_\bullet} \int_{-\infty}^{x'_\bullet} \frac{2}{s} d^2 x'_\ast d^{mnl} \langle p_B | V_{m_k}^* (x_\bullet, x_\perp) V_{m_l}^* (x'_\bullet, x_\perp) V_{m_r}^* (0) | p_B \rangle + x \leftrightarrow 0 \right\}$$

$$\Delta_{ij, kl} \equiv g_{ij} g^{kl} - g_{ik} g^{jl} - g^{il} g^{jk}$$

The correction is $\sim \frac{q^2_\perp}{Q^2}$.

Unfortunately, even at $x_\perp = 0$, not much is known about matrix elements of twist-3 gluon operators.
Z-boson production in $pp$ scattering

$$\frac{d\sigma}{dQ^2 dy dq^2_{\perp}} = \frac{e^2 Q^2}{192 s s_W c_W^2} \frac{1 - 4 s_W^2 + 8 s_W^4}{(m_Z^2 - Q^2)^2 + \Gamma_Z^2 m_Z^2} \left[-W_Z(p_A, p_B, q)\right],$$

$$W_Z(p_A, p_B, q) \equiv \frac{1}{(2\pi)^4} \sum_X \int d^4 x \, e^{-i q x} \langle p_A, p_B | J_\mu(x) | X \rangle \langle X | J_\mu^\mu(0) | p_A, p_B \rangle.$$
Same story: factorization + integration over central fields

\[ \beta < \sigma_a \]

“Projectile” fields

\[ \alpha < \sigma_b \]

“Target” fields

“Central” fields
Classical solution = sum of perturbative diagrams with retarded propagators in the background of projectile and target fields.
Expansion of quark fields

Expanding it in powers of $p_{\perp}^2 / p_{\parallel}^2$ as for gluons we get:

$$\Psi(x) = \Psi_A^{(0)} + \Psi_B^{(0)} + \Psi_A^{(1)} + \Psi_B^{(1)} + \ldots,$$

where

$$\Psi_A^{(0)} = \psi_A + \Xi_{2A}, \quad \Xi_{2A} = -\frac{g \gamma^i}{s} \frac{1}{\alpha + i\epsilon} \psi_A,$$

$$\bar{\Psi}_A^{(0)} = \bar{\psi}_A + \bar{\Xi}_{2A}, \quad \bar{\Xi}_{2A} = - \left( \bar{\psi}_A \frac{1}{\alpha - i\epsilon} \right) \gamma^i \frac{g \gamma^i}{s},$$

$$\Psi_B^{(0)} = \psi_B + \Xi_{1B}, \quad \Xi_{1B} = -\frac{g \gamma^i}{s} \frac{1}{\beta + i\epsilon} \psi_B,$$

$$\bar{\Psi}_B^{(0)} = \bar{\psi}_B + \bar{\Xi}_{1B}, \quad \bar{\Xi}_{1B} = - \left( \bar{\psi}_B \frac{1}{\beta - i\epsilon} \right) \gamma^i \frac{g \gamma^i}{s}.$$
Leading-$N_c$ power corrections

Power corrections are $\sim$ leading twist $\times \frac{q^2}{Q^2} \times (1 + \frac{1}{N_c} + \frac{1}{N_c^2})$.

(Pleasant) surprise: terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist terms due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s} \int dx_\bullet d^2 x_\perp e^{-i\alpha x_\bullet + i(k,x)_\perp} \langle A|\hat{\psi}_f(x_\bullet, x_\perp) \not{p}_2 \hat{\psi}_f(0)|A\rangle = f_{1f}(\alpha, k^2_\perp)$$

Power correction:

$$\frac{g}{8\pi^3 s} \int dx_\bullet dx_\perp e^{-i\alpha_q x_\bullet + i(k,x)_\perp}$$

$$\times \langle A|\hat{\psi}_f(x_\bullet, x_\perp) \not{p}_2 [\hat{U}_i(x_\bullet, x_\perp) - i\gamma_5 \hat{U}_i(x_\bullet, x_\perp)] \hat{\psi}_f(0)|A\rangle$$

$$= -k_i f_{1f}(\alpha_q, k^2_\perp) + O(\alpha_q).$$

(Mulders & Tangerman, 1996)
Result:

\[
W_Z(p_A, p_B, q) = - \frac{e^2}{8 s_W c_W N_c} \int d^2 k_\perp \frac{e^2}{8 s_W c_W N_c} \int d^2 k_\perp \\
\times \left\{ (1 + a_u^2) \left[ 1 - 2 \frac{(k, q - k)_\perp}{Q^2} \right] f_{1u}(\alpha_z, k_\perp) \bar{f}_{1u}(\beta_z, q_\perp - k_\perp) \right\} \\
+ 2(a_u^2 - 1) \frac{k^2_{\perp} (q - k)_{\perp}^2}{m_N^2 Q^2} h_{1u}^{1u}(\alpha_z, k_\perp) \bar{h}_{1u}^{1u}(\beta_z, q_\perp - k_\perp) + (\alpha_z \leftrightarrow \beta_z) \\
+ \left\{ u \leftrightarrow c \right\} + \left\{ u \leftrightarrow d \right\} + \left\{ u \leftrightarrow s \right\} \left( 1 + O\left( \frac{1}{N_c} \right) \right).
\]

\[
a_{u,c} = (1 - \frac{8}{3} s_W^2), \quad a_{d,s} = (1 - \frac{4}{3} s_W^2)
\]

Power correction is \( \sim \frac{q_\perp^2}{Q^2} \).
Result:

\[
W_Z(p_A, p_B, q) = -\frac{e^2}{8s_W^2 c_W^2 N_c} \int d^2 k_\perp \\
\times \left\{ (1 + a_u^2) \left[ 1 - 2 \frac{(k, q - k)_{\perp}}{Q^2} \right] f_{1u}(\alpha_z, k_{\perp}) \bar{f}_{1u}(\beta_z, q_{\perp} - k_{\perp}) \right. \\
+ 2(a_u^2 - 1) \frac{k_\perp^2 (q - k)_\perp^2}{m_N^2 Q^2} h^+_{1u}(\alpha_z, k_{\perp}) \bar{h}^+_{1u}(\beta_z, q_{\perp} - k_{\perp}) + (\alpha_z \leftrightarrow \beta_z) \left\} \\
+ \left\{ u \leftrightarrow c \right\} + \left\{ u \leftrightarrow d \right\} + \left\{ u \leftrightarrow s \right\} \left( 1 + O\left(\frac{1}{N_c}\right) \right). \\
\]

\[
a_{u,c} = (1 - \frac{8}{3}s_W^2), \quad a_{d,s} = (1 - \frac{4}{3}s_W^2) \\
\]

Power correction is \( \sim \frac{q_{\perp}^2}{Q^2} \).

( \frac{1}{N_c} \) and \( \frac{1}{N_c^2} \) terms involve twist-3 quark-quark-gluon TMDs which do not reduce to leading-twist distributions).
Estimate of power corrections

If $Q^2 \gg k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(\alpha_z, k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^+(\alpha_z, k_\perp^2) \simeq \frac{m_N^2 h(\alpha_z)}{k_\perp^4}$$

$$\Rightarrow W_Z(p_A, p_B, q) \simeq -\frac{e^2}{8s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k_\perp^2 (q-k_\perp)^2} \left[ 1 - 2 \frac{(k, q-k_\perp)_\perp}{Q^2} \right]$$

$$\times \sum_f (1 + a_f^2) \left[ f_f(\alpha_z) \bar{f}_f(\beta_z) + \bar{f}_f(\alpha_z) f_f(\beta_z) \right]$$
Estimate of power corrections

If $Q^2 \gg k^2_\perp \gg m_N^2$, we can approximate

$$f_1(\alpha_z, k^2_\perp) \simeq \frac{f(\alpha_z)}{k^2_\perp}, \quad h_1^+(\alpha_z, k^2_\perp) \simeq \frac{m_N^2 h(\alpha_z)}{k^4_\perp}$$

$$\Rightarrow W_Z(p_A, p_B, q) \simeq -\frac{e^2}{8s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k^2_\perp (q-k)^2_\perp} \left[ 1 - 2 \frac{(k, q-k)_\perp}{Q^2} \right]$$

$$\times \sum_f (1 + a_f^2) \left[ f_f(\alpha_z)\bar{f}_f(\beta_z) + \bar{f}_f(\alpha_z)f_f(\beta_z) \right]$$

With logarithmic accuracy

$$W_Z(p_A, p_B, q) = -\frac{\pi e^2}{4s_W^2 c_W^2 N_c} \left[ \frac{1}{q^2_\perp} \ln \frac{q^2_\perp}{m_N^2} + \frac{1}{Q^2} \ln \frac{Q^2}{q^2_\perp} \right]$$

$$\times \sum_f (1 + a_f^2) \left[ f_f(\alpha_z)\bar{f}_u(\beta_z) + \bar{f}_f(\alpha_z)f_f(\beta_z) \right]$$

$$\Rightarrow \text{power correction reaches 10\% level at } q_\perp \sim \frac{1}{4} Q \sim 20 \text{ GeV}$$
Conclusions

Higher-twist power correction to $H$ and $Z$ production at $s \gg q^2 \gg q_{\perp}^2$ are calculated. The estimate gives 10% corrections at $q_{\perp} \sim \frac{1}{4} Q$.

Outlook

- Power corrections to $W_{\mu\nu}$ for Drell-Yan and SIDIS
- Factorization at the one-loop level (and match to evolution equations for TMDs).
Conclusions

1 Conclusions
   - Higher-twist power correction to $H$ and $Z$ production at $s \gg q^2 \gg q^2_\perp$ are calculated. The estimate gives 10% corrections at $q_\perp \sim \frac{1}{4}Q$.

2 Outlook
   - Power corrections to $W_{\mu\nu}$ for Drell-Yan and SIDIS
   - Factorization at the one-loop level (and match to evolution equations for TMDs).

Thank you for attention!