Are Redshift-Space Distortions Actually a Probe of Growth of Structure?

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\[ c = \frac{\hbar}{M_G^2} = \frac{1}{8\pi G} = 1 \]
Contents

• Introduction
  Baryon, dark matter, dark energy

• Couplings of baryon and DM to DE
  Basic equations
  How to probe them observationally?
  Redshift space distortion

• Discussion and conclusions
Introduction
The (dominant) contents of our Universe

- The (dominant) nucleus (baryon)
- Dark matter
- Dark energy

Particle data group

(supernovae IA)

Perlmutter et al.

(rotational curve of our galaxy)

HST

- Dark energy might be responsible for the acceleration of the universe.
- Nuclear (baryon) fields
- Dynamical field φ

We do not know how these components (DE(φ), DM, baryon) couple to one another.

http://www2.astro.psu.edu/~mce/A001/lect19.html
How do baryons and dark matter couple to dark energy (& gravity) ?

In the same way ???

(See, e.g. B. Wang, E. Abdalla, F. Atrio-Barandela, D. Pavon 2016 for a review of an interaction between DM & DE)
Usual assumptions: the same coupling

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M^2_{\text{pl}}}{2} R[g] + \mathcal{L}_\phi [g, \phi] \right] + S_m. \]

Gravity & DE

\[ \mathcal{L}_\phi = -\frac{1}{2} (\partial \phi)^2 - V(\phi) \text{ for simplicity} \)

Matter

The same coupling of baryon and DM:

\[ S_m = \int d^4x \left[ \sqrt{-g} \mathcal{L}_b [g_{\mu\nu}, \psi_b] + \sqrt{-g} \mathcal{L}_c [g_{\mu\nu}, \psi_c] \right]. \]

The same coupling through \( g_{\mu\nu}. \)

But, is this true? or confirmed by observations?
Non-minimal (non-universal) couplings

\[ S_m = \int d^4 x \left[ \sqrt{-g} \mathcal{L}_b[g_{\mu\nu}, \phi_b] + \sqrt{-\bar{g}} \mathcal{L}_c[\bar{g}_{\mu\nu}, \phi_c] \right]. \]

New metric (invertible to original metric) :

\[ \bar{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu \phi \partial_\nu \phi, \quad X = -\frac{1}{2}g^{\mu\nu} \partial_\phi \partial_\phi \]

↑ conformal factor

↑ disformal factor

How to confirm these different couplings observationally ???

N.B. All baryons freely fall in the same way.
Basic equations

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} R[g] + \mathcal{L}_\phi[g, \phi] \right] + S_m. \]

\[ S_m = \int d^4 x \left[ \sqrt{-g} \mathcal{L}_b[g_{\mu\nu}, \psi_b] + \sqrt{-\bar{g}} \mathcal{L}_c[\bar{g}_{\mu\nu}, \psi_c] \right]. \]

\[ G_{\mu\nu} = 8\pi G \left( T^{(b)}_{\mu\nu} + T^{(c)}_{\mu\nu} + T^{(\phi)}_{\mu\nu} \right). \]

\[ T^{(b)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_b)}{\delta g^{\mu\nu}}, \]

\[ T^{(c)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_c)}{\delta g^{\mu\nu}}, \]

\[ T^{(\phi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_\phi)}{\delta g^{\mu\nu}}. \]

Energy momentum conservations:

\[ \nabla^\mu T^{(b)}_{\mu\nu} = 0, \]

Baryon is conserved individually.

\[ \nabla^\mu T^{(c)}_{\mu\nu} + \nabla^\mu T^{(\phi)}_{\mu\nu} = 0. \]

\[ \frac{\delta}{\delta \phi} \Box \phi - V_\phi \equiv Q \rightarrow \text{coupling between DM & } \phi(\text{DE}) \]
Concrete form of $Q$

\[\Box \phi - V_\phi = Q \quad \text{coupling between DM & } \varphi(\text{DE})\]

\[Q \equiv - \frac{1}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_c)}{\delta \phi} = \nabla_\mu W^\mu - Z.\]

\[
Z = \frac{1}{2A} \left\{ A_\phi + \frac{A_X X (A_\phi - 2B_\phi X)}{A - A_X X + 2B_X X^2} \right\} T_{(c)}^T + \left\{ B_\phi + \frac{B_X X (A_\phi - 2B_\phi X)}{A - A_X X + 2B_X X^2} \right\} T_{(c)}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,
\]

\[W^\mu = \frac{1}{2A} \left[ 2B T_{(c)}^{\mu\nu} \partial_\nu \phi - \frac{A - 2B X}{A - A_X X + 2B_X X^2} \right. \]

\[\times \left. \left( A_X T_{(c)} + B_X T_{(c)}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right) \partial_\mu \phi \right].\]

\[
\bar{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi, \quad X = -\frac{1}{2} g_{\mu\nu} \partial_\phi^\mu \partial_\phi^\nu.
\]
What kind of observations can probe the couplings?

Answer

Observations of large scale structure

- Redshift space distortion
- Growth rate of matter perturbations
Background equations

- **Einstein equations:**

\[
\begin{align*}
H^2 &= \frac{8\pi G}{3} \left( \rho_c + \rho_b + \frac{1}{2} \dot{\phi}^2 + V \right), \\
3H^2 + 2\dot{H} &= 8\pi G \left( -\frac{1}{2} \dot{\phi}^2 + V \right).
\end{align*}
\]

- **EOMs for baryon (b), DM (c), and φ:**

\[
\begin{align*}
\dot{\rho}_b + 3H \rho_b &= 0, \\
\dot{\rho}_c + 3H \rho_c &= Q_0 \dot{\phi}, \\
\ddot{\phi} + 3H \dot{\phi} + V_\phi &= -Q_0.
\end{align*}
\]

Non-minimal coupling between DM & φ

A, Ax, B, and Bx are evaluated at background fields.
Metric and matter (perturbations)

- **Metric** (perturbations in Newtonian gauge):

\[
 ds^2 = -[1 + 2\Phi(t, x)]dt^2 + a^2(t)[1 - 2\Psi(t, x)]dx^2. 
\]

  \[\Phi\] (gravitational potential)

  \[\Psi\] (curvature perturbation)

- **Matter energy momenta** (perturbations in Newtonian gauge):

\[
 \begin{align*}
 T^{(I)0}_0 &= -\rho_I(t)\left[1 + \delta_I(t, x)\right], \\
 T^{(I)0}_i &= -\rho_I(t)\partial_i v_I(t, x), \\
 T^{(I)ij} &= 0.
\end{align*}
\]

  \[\delta_I\] (density perturbations)

  \[v_I\] (velocity perturbations)

\(I = c\) (DM) or \(b\) (baryon) or \(m\) (total matter)

\[
\begin{align*}
 \rho_m(t) &= \rho_c(t) + \rho_b(t) \\
 \rho_m\delta_m &= \rho_c\delta_c + \rho_b\delta_b, \\
 \rho_m v_m &= \rho_c v_c + \rho_b v_b,
\end{align*}
\]
Perturbed equations in quasi static approximations

we pay attention to “subhorizon” dynamics, \( H^2 \ll \frac{k^2}{a^2} \)

- **Einstein equations**: 
  \[
  \frac{k^2}{a^2} \psi = \frac{k^2}{a^2} \Phi \simeq -4\pi G \left( \rho_c \delta_c + \rho_b \delta_b \right).
  \]

- **EOMs for baryon (b), DM (c), and \( \phi \)**:

  **baryon**: \( \dot{\delta}_b + \frac{k^2}{a^2} \nu_b \simeq 0 \), \( \dot{\nu}_b - \Phi \simeq 0 \),

  **DM**: \( \dot{\delta}_c + \frac{k^2}{a^2} \nu_c \simeq \frac{\dot{\phi}}{\rho_c} \left( \delta Q - Q_0 \delta_c \right) \), \( \dot{\nu}_c - \Phi \simeq \frac{Q_0}{\rho_c} \left( \delta \phi - \dot{\phi} \nu_c \right) \),

  **\( \phi \)**: \( -\frac{k^2}{a^2} \delta \phi \simeq \delta Q \equiv Q_0 \delta_c + (R_1 + R_2) \dot{\phi} \delta_c + R_1 \frac{k^2}{a^2} \nu_c + R_2 \frac{k^2}{a^2} \delta \phi \),

  \[
  R_1 = \frac{B \rho_c}{A}, \quad R_2 = -\frac{(A - B \dot{\phi}^2)(A_X - B_X \dot{\phi}^2) \rho_c}{A(2A - A_X \dot{\phi}^2 + B_X \dot{\phi}^4)}.
  \]
Modified continuity equation for DM

\[(1 - \gamma_1) \left( \delta_c + \frac{k^2}{a^2} \nu_c \right) = \gamma_2 \left( \delta_c - \frac{Q_0}{\phi} \delta_c \right), \]

\[\gamma_1 = \frac{\dot{\phi}^2}{\rho_c} \frac{R_1}{1 + R_2}, \quad \gamma_2 = \frac{\dot{\phi}^2}{\rho_c} \frac{R_2}{1 + R_2} \quad \left( R_1 = \frac{B \rho_c}{A}, \quad R_2 = -\frac{(A - B \dot{\phi}^2)(A_X - B_X \dot{\phi}^2)\rho_c}{A(2A - A_X \dot{\phi}^2 + B_X \dot{\phi}^4)} \right) \]

\[\bar{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi,\]

- **minimal coupling case (A=1, B=0)**: \[Q_0 = R_1 = R_2 = 0\]

  \[\implies \gamma_1 = \gamma_2 = 0.\]

- **conformal factor, A = A(\phi), disformal factor, B = B(\phi) case:**

  \[Q_0 \neq 0, \quad R_1 \neq 0, \quad R_2 = 0\]

  \[\implies \gamma_1 \neq 0, \quad \gamma_2 = 0.\]

The continuity equation is unchanged up to the overall factor.

But, in general, \[R_2 \neq 0 \quad \Rightarrow \quad \text{modification of continuity eq. for DM}.\]
Modified continuity equation for total matter

\[ \dot{\delta}_m + \frac{k^2}{a^2} \nu_m = \frac{\dot{\phi}}{\rho_m} [\delta Q - Q_0 (\omega_b \delta_b + \omega_c \delta_c)] \]

\[ = \omega_c \frac{\gamma_2}{1 - \gamma_1} \left( \frac{\dot{\delta}_c}{\dot{\phi}} - \frac{Q_0}{\phi} \delta_c \right) \]

\[ + \omega_b \frac{Q_0 \dot{\phi}}{\rho_m} (\delta_c - \delta_b) , \]

Two sources of the violation of continuity eq. for total matter:

- violation of continuity eq. for CDM associated with R2
- deviation of the background dynamics associated with Q0

\[ \left( \delta_m = \frac{\rho_c}{\rho_m} \delta_c + \frac{\rho_b}{\rho_m} \delta_b, \quad v_m = \frac{\rho_c}{\rho_m} v_c + \frac{\rho_b}{\rho_m} v_b \right) \]
How to probe the difference of couplings?

(See also Marcondes, Landim, Costa, Wang, Abdalla 2016 for similar idea)
Strategy

- If continuity equation \( \dot{\delta} + \frac{k^2}{a^2}v \approx 0 \) is satisfied,

  \[ \dot{\delta} : \text{growth rate} \quad \leftrightarrow \quad v : \text{(peculiar) velocity} \]

  \( \text{(Kaiser formula)} \)

  \[ \text{Redshift (space distortion)} \]

- If continuity equation \( \dot{\delta} + \frac{k^2}{a^2}v \approx 0 \) is violated due to the (non-minimal) coupling,

  \[ \dot{\delta} : \text{growth rate} \quad \& \quad \text{coupling} \quad \leftrightarrow \quad v : \text{(peculiar) velocity} \]

  \[ \text{Redshift (space distortion)} \]
Growth rate

Combining all the perturbed equations to **eliminate velocities** as usual

> two coupled second-order differential equations for \( \delta c \) & \( \delta b \).

\[
\delta_I(t, k) = D_I(t) \delta_0(k) \quad (I = c \text{ or } b)
\]

\[
\text{growing mode}
\]

the (normalized) \( k \)-independent linear growth factors

- **growth rate**:
  \[
  f_I(t) \equiv \frac{d \ln D_I}{d \ln a} = \frac{\dot{\delta}_I}{\delta_I H}
  \]

- **effective growth rate**:
  \[
  \nu_I(t, k) = -\frac{a^2 H}{k^2} f_I^{\text{eff}}(t) \delta_I(t, k).
  \]

N.B. if **continuity eq. applies**, \( f_I^{\text{eff}}(t) = f_I(t) \), otherwise not.

\[
\nu_m(t, k) = -\frac{a^2 H}{k^2} f_m^{\text{eff}}(t) \delta_m(t, k).
\]

\[
\begin{align*}
  f_m^{\text{eff}} &= f_m + \omega_c \frac{D_c}{D_m} \Delta f_c - \omega_b \frac{Q_0 \phi}{H \rho_m} \frac{D_c - D_b}{D_m} \\
  \Delta f_c &= f_c^{\text{eff}} - f_c = \frac{\tau_2}{1 - \tau_1} \left( f_c - \frac{Q_0}{H \phi} \right)
\end{align*}
\]

\[e.g.\]
Redshift space distortion

In survey, the distance is measured by redshift, \( z \).

**Peculiar velocity** must be taken into account in measuring actual distance and significantly modifies shape.

\[
 z_{\text{obs}} = H_0 x + v_g \cdot \hat{x}
\]

- \( z_{\text{obs}} \): observed redshift
- \( H_0 \): Hubble parameter
- \( x \): real space position
- \( v_g \): peculiar velocity
- \( \hat{x} \): unit vector of line-of-sight

\[
 s \equiv \frac{z_{\text{obs}}}{H_0}
\]

\[
 s = x + \frac{v_{g,x}}{H_0} \hat{x}
\]

- \( s \): redshift space position
- \( v_{g,x} \): projection of peculiar velocity
- \( H_0 \): Hubble parameter

\[
 v_g(t, k) = v_m(t, k) = -\frac{a^2 H}{k^2} f_m(t) \delta_m(t, k),
\]

The observed numbers (of galaxies) are the same in both spaces.
Standard (linear) Kaiser formula (Kaiser 1987)

\[ P_{g,s}(k) = (1 + \beta \mu^2)^2 P_g(k), \quad P_g(k) = b^2 P_m(k), \quad \beta = \frac{f_m}{b} \]

\[ \mu = \hat{k} \cdot \hat{x} \]

\[ \text{(linear) bias factor} \]

**Real Space**

**Redshift Space**

**isotropic**

**directional dependence**

\[ \text{Squeezed in the line-of-sight direction (x) (perturbations are enhanced)} \]

**Figure 9.12.** A hundred galaxies in real space squashed in redshift space due to linear velocities. The apparent overdensity in redshift space is much larger near the center than it is in real space. We, the observers, are sitting at the bottom of the page. (taken from Dodelson’s textbook)

\[ \frac{P_{g,s}^{(2)}(k)}{P_{g,s}^{(0)}(k)} = \frac{5 \int_{-1}^{1} \frac{d\mu}{2} P_2(\mu) P_{g,s}(k)}{\int_{-1}^{1} \frac{d\mu}{2} P_0(\mu) P_{g,s}(k)} = \frac{\frac{4}{3} \beta + \frac{4}{7} \beta^2}{1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2} \]

\[ \beta (\text{fm}) \text{ can be probed by comparing monopole & quadrupole components for example.} \]

\[ \beta = \frac{f_m}{b} \simeq \frac{\Omega_m^{0.55}}{b} \text{ cross-correlation between clustering of galaxies and weak lensing} \]
Modified Kaiser formula

\[ v_m(k) = -\frac{H_0}{k^2} f_m \delta_m(k) \quad \text{at} \quad t = t_0. \quad \left( f_m = \frac{d \log D_m}{d \log a} \right) \]

\[ v_m(t, k) = -\frac{a^2 H}{k^2} f_{m}^{\text{eff}}(t) \delta_m(t, k). \]

\[
\begin{align*}
 f_{m}^{\text{eff}} & = \frac{\omega_c D_c f_c^{\text{eff}} + \omega_b D_b f_b}{\omega_c D_c + \omega_b D_b} \\
 & = f_m + \omega_c \frac{D_c}{D_m} \Delta f_c - \omega_b \frac{Q_0 \dot{\phi}}{H \rho_m} \frac{D_c - D_b}{D_m} \neq f_m
\end{align*}
\]

\[ P_{g,s}(k; t) = \left( 1 + \beta_{\text{eff}}(t) \mu^2 \right)^2 P_g(k; t), \quad P_g = b^2 P_m \]

\[
\begin{align*}
 \beta_{\text{eff}} & \equiv \frac{f_{m}^{\text{eff}}}{b} \\
 & = \beta + \frac{1}{b D_m} \left[ \omega_c D_c \Delta f_c - \omega_b \frac{Q_0 \dot{\phi}}{H \rho_m} (D_c - D_b) \right] \neq \beta.
\end{align*}
\]

Single RSD measurement cannot determine \( f_m \) !! (even if \( b \) is fixed)
Strategy

● If continuity equation \( \dot{\delta} + \frac{k^2}{a^2}v \approx 0 \) is satisfied,

\[ \dot{\delta} : \text{growth rate} \quad \leftrightarrow \quad v : \text{(peculiar) velocity} \]

(Kaiser formula)

Redshift (space distortion)

● If continuity equation \( \dot{\delta} + \frac{k^2}{a^2}v \approx 0 \) is violated due to the (non-minimal) coupling,

\[ \dot{\delta} : \text{growth rate} \quad \& \quad \text{coupling} \quad \leftrightarrow \quad v : \text{(peculiar) velocity} \]

Measure through lensing at each \( z \)  

Redshift (space distortion)
Summary

- We have addressed the question how differently baryons and dark matter can couple to dark energy.
- We have found that, in the presence of X-dependence of conformal and/or disformal couplings, continuity and Euler equations are significantly modified.
- With different couplings, the (effective) linear growth rate, which is measured by the peculiar velocities of the distributed galaxies, no longer corresponds to the time derivative of the density perturbations and is rather characterized by the couplings as well.