Recent Developments in the Theory of Large-scale Structure

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Motivation

We saw many talks on ongoing and future LSS surveys

- Tighter constraints on cosmological parameters
- Measurements of neutrino masses, non-Gaussianities, running…
- Possible surprises in the dark sector, modified gravity…

Analytical models are fundamental for modeling and data analysis
Outline

Perturbation theory

Modeling of the BAO peak — IR resummation

Reconstruction of the initial conditions
Part I

PT approach to LSS
PT approach to LSS

Matter behaves as a fluid on large scales
On large scales the density fluctuations are small

\[ \partial_\tau \delta + \nabla [(1 + \delta) v] = 0 \]
\[ \partial_\tau v + H v + \nabla \Phi + v \cdot \nabla v = \cdots \]
\[ \nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m \delta \]

One can find perturbative solutions

\[ \delta^{(n)}(k) = \int_{q_1, \ldots, q_n} F_n(q_1, \ldots, q_n) \delta^0(q_1) \cdots \delta^0(q_n) \]

SPT equations
Scoccimarro, Frieman (1996)
Bernardeau, Colombi, Scoccimarro (2002)
PT approach to LSS

The nonlinear power spectrum

\[ P_{NL}(k, \tau) = D^2(\tau) P_{\text{lin}}(k) + P_{1-\text{loop}}(k, \tau) + \cdots \]  

Scoccimarro, Frieman (1996)

\[ P_{22}(k) = 2 \int_q F_2^2(q, k - q) P_{\text{lin}}(q) P_{\text{lin}}(|k - q|) \]

\[ P_{13}(k) = 6 P_{\text{lin}}(k) \int_q F_3(q, -q, k) P_{\text{lin}}(q) \]
PT approach to LSS

A well known problem, SPT seems not to converge

In power-law cosmologies the loops may even be infinite
PT approach to LSS

Simulations and PT conserve mass and momentum

\[ \delta_{\text{NL}}^\text{Sim}(k) - \delta_{\text{NL}}^\text{PT}(k) \sim R^2 k^2 \delta_{\text{lin}}(k) + \cdots \]

Peebles (1980)

The scale \( R \) is not calculable from PT (we can only estimate it)
PT approach to LSS

Effects of short-scale fluctuations are encoded in counter-terms

Effective Field Theory approach to LSS

\[
\partial_\tau \delta + \nabla [(1 + \delta) v] = 0
\]
\[
\partial_\tau v + \mathcal{H} v + \nabla \Phi + v \cdot \nabla v = \frac{c_s^2}{\kappa} \nabla \delta + \cdots
\]
\[
\nabla^2 \Phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta
\]

EFT operators

Baumann, Nicolis, Senatore, Zaldarriaga (2010)
Carrasco, Hertzberg, Senatore (2012)

\[
P_{13}^{UV} (k) = -\frac{61}{630 \pi^2} P_{\text{lin}} (k) k^2 \int_0^\infty dq P_{\text{lin}} (q)
\]

Leading UV sensitivity

\[
P_{\text{count.}} (k) = -2 c_s^2 (\tau) k^2 P_{\text{lin}} (k)
\]
PT approach to LSS

Including counter-terms (a single free parameter at two loops)

\[
P_{\text{count.}}(k) \sim c_s^2(\tau)(2P_{13}^{q\to 0}(k) + 2P_{15}^{q\to 0}(k) + 2P_{24}^{q\to 0}(k) + P_{33-I I}^{q\to 0}(k))
\]

Baldauf, Mercolli, Zaldarriaga (2015)
PT approach to LSS

Bias expansion

\[ \delta^{(g)} = \mathcal{F}[\nabla_i \nabla_j \Phi] = b_1 \delta + b_2 \delta^2 + b_{s2} (\nabla_i \nabla_j \Phi)^2 + \tilde{b} \nabla^2 \delta \ldots \]

RSD and IR-resummation

Higher order statistics: bispectrum, trispectrum, covariance matrix…

Different flavors: Eulerian and Lagrangian EFT, TSPT,…

Fast methods for evaluation of loop integrals

Theoretical systematics and data analysis

Review:
Desjacques, Jeong, Schmidt (2016)
Part II

Modeling of the BAO peak
Modeling of the BAO peak

One well-known problem of the Eulerian PT

But PT should work at

$\ell_{BAO} \sim 100 \, h^{-1}\text{Mpc}$
Modeling of the BAO peak

Galaxies in free fall \( \delta x^i \sim \nabla^i \phi \sim \frac{\nabla^i}{\nabla^2} \delta \) (not Zel’dovich displacements)

\( q \ll 2\pi/\ell_{\text{BAO}} \)
no effect (exact squeezed limit)

\( 2\pi/\ell_{\text{BAO}} < q \ll 2\pi/\sigma \)
observable effect (spread of the peak)

\[
\tilde{P}(k) = P_{\text{lin}}^n(k) + P_{1-\text{loop}}^n(k) \\
+ e^{-\Sigma_\epsilon^2 k^2} (1 + \Sigma_\epsilon^2 k^2) P_{\text{lin}}^w(k) + e^{-\Sigma_\epsilon^2 k^2} P_{1-\text{loop}}^w(k)
\]

\[
\Sigma_\Lambda^2 \approx \frac{1}{6\pi^2} \int_0^\Lambda dq P_{\text{lin}}(q)[1-j_0(q\ell_{\text{BAO}})+2j_2(q\ell_{\text{BAO}})]
\]

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)
Modeling of the BAO peak

Different form the standard formula for the spread of the BAO peak

Crocc, Scoccimarro (2007)
Eisenstein, Seo, White (2007)

Only long-short shifts

Senatore, Zaldarriaga (2014)
Baldauf, Mirbabayi, MS, Zaldarriaga (2015)
Vlah, Seljak, Chu, Feng (2015)
Blas, Garny, Ivanov, Sibiryakov (2016)
Senatore, Trevisan (2017)

Parameter-free modeling of the BAO peak (including bias, RSD…)

FIG. 5.
Modeling of the BAO peak

Clear connection to higher-point correlation functions

\[ B_g^w \approx \frac{2\mu k}{b_1 q} \sin \left( \frac{x\mu}{2} \right) P_g(q) \cdot \ell_{BAO}^{-1} \frac{d}{dk} P_g^w(k) \quad \ell_{BAO}^{-1} < q \ll k \]

\[ x = q\ell_{BAO} \]

\[ \mu = \hat{k} \cdot \hat{q} \]

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)

A direct measurement of the linear bias
Modeling of the BAO peak

The BAO peak is a playground for modified gravity theories

Possible test for different models that violate the EP

The infrared structure of correlators for arbitrary small $q$

$$\langle \delta q \, \delta_{k_1} g^{A} \, \delta_{k_2} g^{B} \rangle'_{q \to 0} = -\lambda \frac{\vec{q} \cdot \vec{k_1}}{q^2} P\delta(q) \langle \delta_{k_1} g^{A} \, \delta_{k_2} g^{B} \rangle'$$

Creminelli, Gleyzes, Hui, MS, Vernizzi (2013)
Part III

Reconstruction of the initial conditions
Reconstruction of the initial conditions

Option 1: Measure all n-point functions

Option 2: Reconstruct the initial conditions and measure $P_{\text{lin}}(k)$

\[ \delta_{\text{in}} = F[\delta_{\text{NL}}, \psi_{\text{rec.}}] \approx 0 \]

Zhy, Pen, Chen (2016)
Baldauf, Schmittfull, Zaldarriaga (2017)
Reconstruction of the initial conditions

Baldauf, Schmittfull, Zaldarriaga (2017)

One algorithm: Iterative reconstruction

1. Smooth the density field on scale $R_n$
2. Calculate the displacement $\chi_n^{ZA}$
3. Move the particles
4. $n \to n + 1$

\[ \psi_{NL} \approx \chi = \chi_1^{ZA} + \chi_2^{ZA} + \cdots \]

\[ \hat{\delta}_0(k) = t_1(k)\hat{\delta}_\chi(k) + t_2(k) \int \frac{d^3 p_1}{(2\pi)^3} \left( 1 - \frac{(p_1 \cdot p_2)^2}{p_1 \cdot p_1 p_2 \cdot p_2} \right) \hat{\delta}_\chi(p_1)\hat{\delta}_\chi(p_2) \]
Reconstruction of the initial conditions

Can we do as well for haloes in redshift space?

Baldauf, Schmittfull, Zaldarriaga (2017)
Conclusions

Very good understanding of large-scale clustering of DM

More work needed for RSD and biased tracers

Much more work needed for higher order correlation functions

Reconstruction promising but largely unexplored