Measuring Gravitational Mass of Antihydrogen via Interferometry of Gravitational Quantum States

A. Voronin in collaboration with Gbar
Plan of the talk

• Gravitational states of antihydrogen? Yes, it is possible!
• Spectroscopy and Interferometry of GraviAtom
• Gravitational mass measurement
Gravitational quantum states?

State of motion of a quantum particle, which is localized near reflecting surface in a gravitational field of the Earth.

\[ \text{Nature 415, 297 (2002)} \]

\[ \text{TABLE I. The eigenvalues, gravitational energies, and classical turning points of a quantum bouncer with the mass of (anti)hydrogen in the Earth's gravitational field.} \]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \lambda_n^0 )</th>
<th>( E_n^0 ) (peV)</th>
<th>( z_n^0 ) (( \mu \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.338</td>
<td>1.407</td>
<td>13.726</td>
</tr>
<tr>
<td>2</td>
<td>4.088</td>
<td>2.461</td>
<td>24.001</td>
</tr>
<tr>
<td>3</td>
<td>5.521</td>
<td>3.324</td>
<td>32.414</td>
</tr>
<tr>
<td>4</td>
<td>6.787</td>
<td>4.086</td>
<td>39.846</td>
</tr>
<tr>
<td>5</td>
<td>7.944</td>
<td>4.782</td>
<td>46.639</td>
</tr>
<tr>
<td>6</td>
<td>9.023</td>
<td>5.431</td>
<td>52.974</td>
</tr>
<tr>
<td>7</td>
<td>10.040</td>
<td>6.044</td>
<td>58.945</td>
</tr>
</tbody>
</table>
Quantum reflection from material surface

Ultracold antihydrogen is reflected from material surfaces and can survive on the surface up to few seconds.

<table>
<thead>
<tr>
<th>d (thickness)</th>
<th>t (s), Silicon</th>
<th>t (s), Silica</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 nm</td>
<td>0.34</td>
<td>0.61</td>
</tr>
<tr>
<td>2 nm</td>
<td>0.25</td>
<td>0.46</td>
</tr>
<tr>
<td>5 nm</td>
<td>0.19</td>
<td>0.33</td>
</tr>
<tr>
<td>10 nm</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td>20 nm</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>50 nm</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>∞</td>
<td>0.14</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Liquid He bulk – lifetime 1.3 s!
Correction by Casimir-Polder potential + annihilation:

\[ |a|/l_0 \approx 0.005 \]

\[ \Psi(x) = 0 \]

\[ \frac{\Psi(0)}{\Psi'(0)} = -\frac{a_{CP}}{l_g} \approx i0.005 \]

Correction by Casimir-Polder and annihilation:

\[ \tilde{\lambda}_n = \lambda_n + a/l_g \]

\[ \varepsilon_n = \varepsilon_0(\lambda_n + \text{Re} \, a/l_g) \quad \Gamma = 2\varepsilon_0 |\text{Im} \, a|/l_g \]

\[ \tau = \frac{l_g}{\varepsilon_g} \frac{\hbar}{2|\text{Im} \, a|} = \frac{\hbar}{2Mg|\text{Im} \, a|} > 0.1s \]

All states have equal shift and lifetime \( \Rightarrow \)

No surface effects in transition frequencies

Time and Spatial Resolving of Gravitational States

Momentum distribution of gravitational state can be mapped into measurable time or spatial distribution.

\[ H = 10 \text{cm} \]

\[ H_a = 15 \text{ \(\mu\)m} \]
**Mapping of momentum distribution**

\[
\Psi(z, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ipz/h} G(p, t, p') F_0(p') dpdp'
\]

\[
G(p, t, p') = \exp\left[-\frac{it}{2m\hbar} (p^2 - Mgpt + M^2 g^2 t^2 / 3) \right] \delta(p - Mgt - p')
\]

\[
\Psi(z, t) \approx \sqrt{\frac{m}{t}} e^{im^2 z^2 / 2\hbar + it^2 M^2 g^2 / 2m\hbar} F_0(p_0 - Mgt); \ p_0 = (z + \frac{gt^2}{2}) \frac{m}{t}
\]

\[
|\Psi(z, t)|^2 \approx \frac{m}{t} |F_0(k)|^2
\]

1) \(z = z_0 : k = mg(t - t_0), \ t_0 = \sqrt{2g / z_0}\)

2) \(t = t_0 = L / v : k = \frac{m(z - z_0)}{t_0}\)
Mapping of momentum distribution into time distribution

\[ l_g = \frac{\delta(n)\hbar}{\Delta k_n} \]

1 state velocity distribution

2 state velocity distribution

Time-of-fall distribution \( H = 10 \text{ cm} \)

Spatial distribution time-of-flight \( T = 0.1 \text{s} \)
Spectroscopy - to induce transitions between gravitational states with alternating magnetic field

Developed for neutrons by V. Nesvizhevsky, S. Baessler, G. Pignol, K. Protassov, A.Voronin

\[ \omega_{12} = 254.54 \text{ Hz} \]
\[ \omega_{13} = 463.77 \text{ Hz} \]
\[ \omega_{14} = 648.12 \text{ Hz} \]
\[ \omega_{15} = 815.20 \text{ Hz} \]
\[ \omega_{16} = 972.11 \text{ Hz} \]
\[ \omega_{17} = 1119.99 \text{ Hz} \]

\[ z_1 = 13.7 \mu m \]
\[ z_2 = 24.0 \mu m \]
\[ z_3 = 32.4 \mu m \]
\[ z_4 = 39.8 \mu m \]
\[ z_5 = 46.6 \mu m \]
\[ z_6 = 52.9 \mu m \]
\[ z_7 = 58.9 \mu m \]
Possible scheme of flow-throw experiment

1-source of ultracold antihydrogen, 2-mirror, 3-absorber, 4-magnetic field, 5-detector

\[ v \approx 1 \text{m/s}, H_a = 15 \mu m, H_d = 25 \mu m, B_0 \approx 10 \text{Gs}, \beta \approx 10 \text{Gs/m}, L = 30 \text{cm} \]
Transition probability

\[ P_{ik} = \frac{1}{2} \frac{\Omega_{ik}^2}{\Omega_{ik}^2 + \hbar^2 \Delta^2} \sin^2 \left( \frac{t}{\hbar} \sqrt{\Omega_{ik}^2 + \hbar^2 \Delta^2} \right) \exp(-\Gamma t) \]

\[ \Omega_{ik} = \frac{(\mu_B + \mu_{\overline{p}}) \beta l_g^3}{\hbar(z_k - z_i)^2} \]

Transition probability as a function of frequency. Transition 1->6

\[ f_{res} = 972.459 \text{ Hz} \quad \Delta = -0.002 \text{Hz} \quad \text{Time of observation} \quad t=1 \text{ s} \]
Free-fall of superposition of states
Observation of Interference+ Resonance Transition

1->3 resonance transition

1->6 resonance transition
Gravitational states and Gravitational mass

Classical: \( m\ddot{z} = Mg \rightarrow \ddot{z} = g \rightarrow T = \sqrt{2H / g} \rightarrow M / m = 1 \)

Quantum: \[
\left[-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + Mgz - E\right] \Psi(z) = 0 \quad \Rightarrow \quad \left[-\frac{d^2}{dx^2} + x - \lambda_n\right] F(x) = 0
\]

\[\varepsilon_g = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} = 0.61 \times 10^{-12} \text{eV}; \quad l_g = \frac{\sqrt[3]{\hbar^2}}{2Mmg} = 5.87 \times 10^{-6} m\]

\[E_n = \varepsilon_g \lambda_n \]

\[z = l_g x \quad \Psi_n(z) = \frac{1}{\sqrt{l_g}} F\left(z / l_g - E_n / \varepsilon_g\right)\]

\[m = \frac{\hbar^2}{2\varepsilon_g l_g^2}; \quad M = \frac{\varepsilon_g}{g l_g}\]

\[M = m \Rightarrow \frac{\hbar}{\varepsilon_g} = \sqrt{\frac{2l_g}{g}} \quad \text{or} \quad T = \sqrt{\frac{2H}{g}}\]

Gravitational states are all about energy and spatial scales
Gravitational mass

\[ \varepsilon_g = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} = 0.61 \times 10^{-12} \text{eV}; \quad l_g = \sqrt[3]{\frac{\hbar^2}{2Mmg}} = 5.87 \times 10^{-6} m \]

\[ M = \sqrt[3]{\frac{2m \hbar k_m^3}{g^2(t_m - t_0)^3}}. \]

\[ \hbar \omega_{ik} = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} (\lambda_k - \lambda_i) \Rightarrow M = \sqrt[3]{\frac{2m \hbar \omega_{ik}^3}{g^2 (\lambda_k - \lambda_i)^3}} \]

\[ \frac{\Delta M}{m} \sim 10^{-4} \quad N = 10^3 \]
Conclusions

• Gravitational states of Antihydrogen: simplest bound antimatter quantum system, determined by gravity. Effects of surface are canceled out in the first order ($10^{-6}$ accuracy)

• Gravitational states of Antihydrogen- metastable and long-living, easy to study due to annihilation signal

• Gravitational states- a way to precision measurement of the gravitational mass $M$

• Gravitational states are most sensitive to short-range additional to gravity interactions within micrometer range