CHIRAL ANOMALIES AND NUMBER OF FAMILIES

- Anomalies
  - Definition
  - 4D and beyond
  - Constraints on fermion content

- Three gauge fermion families of Standard Model

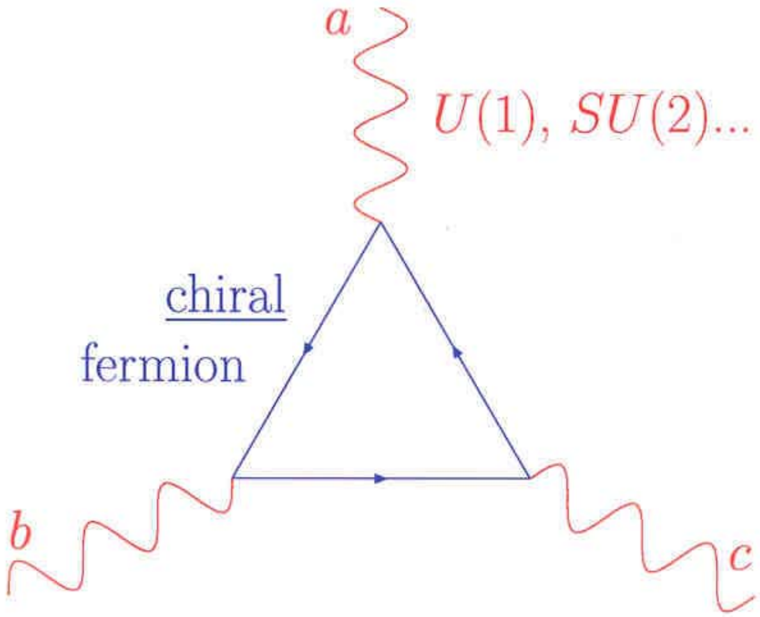
- Anomalies and families in 4D - SM

- Anomalies and families in 6D - $SU(5)$

Anomalies and fermion content of grand unified theories in extra dimensions
N.Borghini, M.Tytgat,Y.Gouverneur, Phys. Rev. D**65** (2002) 025017

Gauge anomalies in $D = 4$

Anomaly : $\underbrace{\text{classical symmetry}}_{\partial_\mu j^\mu = 0}$ spoilt by quantization $\quad \partial_\mu j^\mu_a \neq 0$



$$\partial_\mu j^\mu_a \propto D_{abc}\, \epsilon^{\nu\rho\sigma\tau} F^b_{\nu\rho} F^c_{\sigma\tau}$$

$$\overbrace{\sum_L \mathrm{STr}\,(T^a T^b T^c) - \sum_R \mathrm{STr}\,(T^a T^b T^c)}$$

If broken symmetry is *global* 

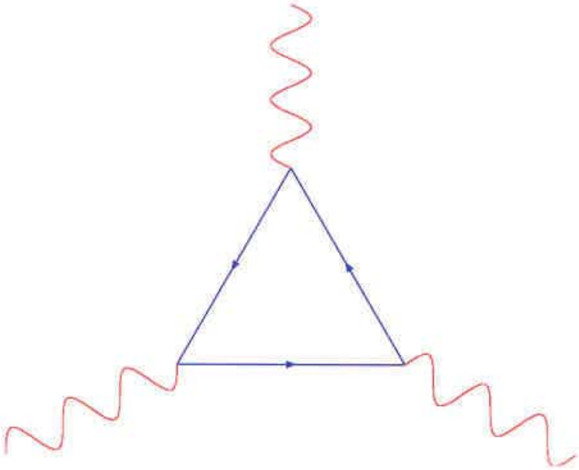
→ pion decay $\pi^0 \to \gamma\gamma$

If broken symmetry is **local** 

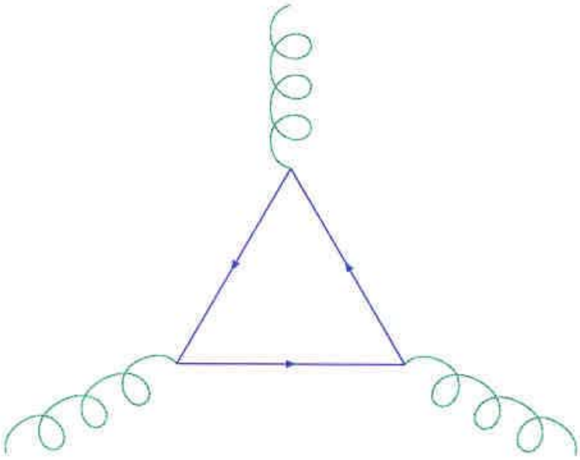
→ gauge invariance, unitarity and renormalizability

⇒ **contraints on the chiral fermion content**

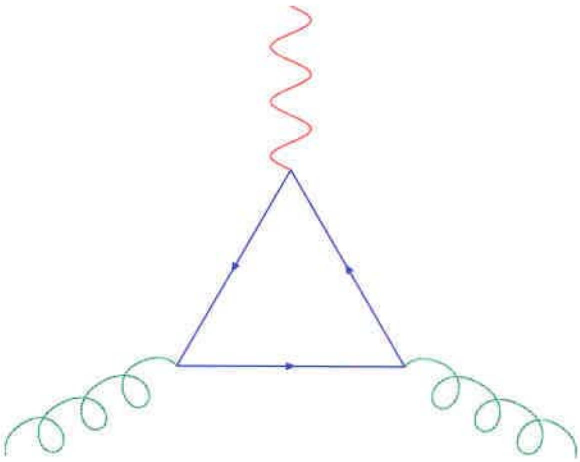
Perturbative anomalies in $D = 4$



Gauge anomalies

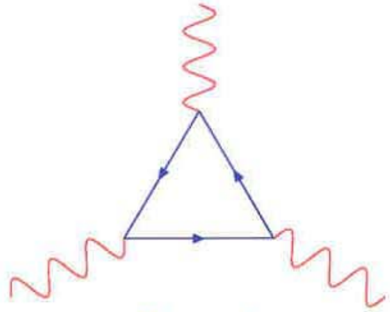


Gravitational anomalies

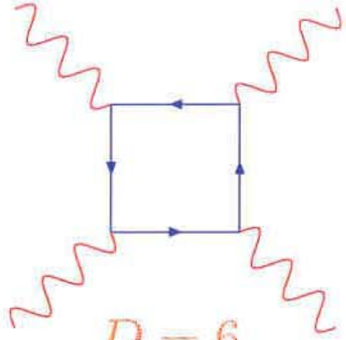


Mixed anomalies

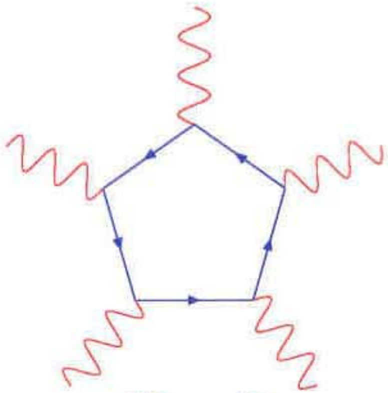
ANOMALIES IN ANY DIMENSION



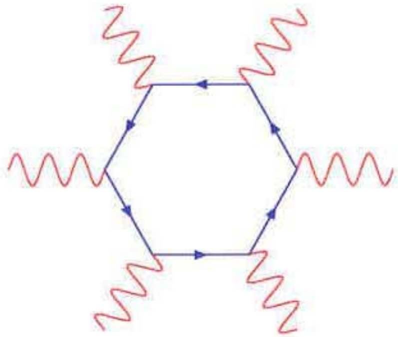
$D=4$



$D=6$



$D=8$



$D=10$

$$\text{Anomaly} \propto \sum_{L_D} \mathrm{STr}\,(T^1 ... T^{D/2+1}) - \sum_{R_D} \mathrm{STr}\,(T^1 ... T^{D/2+1})$$

CHIRAL ANOMALIES

- Only in even dimensions D -

- **Local gauge** : Traces + Green-Schwarz mechanism
  $\Rightarrow$ Irreducible anomaly (when GS exists) :

$$\left(\sum_{L_D - R_D} A_{D/2+1} \right) \mathrm{Tr}(t ... t^{D/2+1}) = 0$$

- **Global gauge** : Homotopy groups $\Pi_D(G) = Z_{m_D}$
  $\Rightarrow$

$$c_k^D [N(\mathbf{k}_{L_D}) - N(\mathbf{k}_{R_D})] = 0 \text{ mod } m_D$$

- **Mixed** : some are not reducible

- **Local gravitational** : Only in $D = 4k + 2$,

$$N_{L_D} - N_{R_D} = 0$$

if not $\Rightarrow$ chiral gauge singlets, ! GS !

Family structure of the Standard Model in $D = 4$

	3 fermion generations			$SU(3) \otimes SU(2) \otimes U(1)$		
$\mathcal{Q}$	$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}_L$	$\begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}_L$	$\begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}_L$	3	2	1/3
$\mathcal{U}$	$(\mathrm{u^c})_L$	$(\mathrm{c^c})_L$	$(\mathrm{t^c})_L$	$\bar{3}$	1	-4/3
$\mathcal{D}$	$(\mathrm{d^c})_L$	$(\mathrm{s^c})_L$	$(\mathrm{b^c})_L$	$\bar{3}$	1	2/3
$\mathcal{L}$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1	2	-1
$\mathcal{E}$	$(\mathrm{u^c})_L$	$(\mathrm{c^c})_L$	$(\mathrm{t^c})_L$	1	1	2

From SU(5) point of view

$SU(5)$	$SU(3) \otimes SU(2) \otimes U(1)$
$\bar{5}$	$\mathcal{D} + \mathcal{L}$
10	$\mathcal{Q} + \mathcal{U} + \mathcal{E}$

ANOMALIES IN SM $D = 4$

all fermions taken left-handed

- **Mixed Anomaly** :

$$\sum_L \mathrm{Tr} Y = 6\times\frac{1}{3}+3\times\left(\frac{-4}{3}\right)+3\times\frac{2}{3}+2\times(-1)+1\times2 = 0$$

- **Global gauge anomaly :** $SU(2)$:

$$N(2_L) = 3 + 1 = 4 = 0 \bmod 2$$

- **Local gauge anomaly** $\sum_L \mathrm{STr}\,(T^a T^b T^c)$

  - $SU(3)^3$: $\sum_L \mathrm{STr}\,(T^a T^b T^c) = (2-1-1)\times \mathrm{STr}\,(t^a t^b t^c)$
  - $SU(2)^3$: $\mathrm{Tr}\,(\sigma^a\{\sigma^b,\sigma^c\}) = 2\delta^{bc}\,\mathrm{Tr}\,\sigma^a = 0$
  - $U(1)^3$: $\sum_L Y^3 = 6\left(\frac{1}{3}\right)^3 + 3\left(\frac{-4}{3}\right)^3 + 3\left(\frac{2}{3}\right)^3 + 2(-1)^3 + 2^3 = 0$ need quarks and leptons
  - $SU(3)^2\,U(1)$ $\sum_{\mathbf{3},\bar{\mathbf{3}}} Y = 2\times\frac{1}{3} + \frac{-4}{3} + \frac{2}{3} = 0$
  - $SU(2)^2\,U(1)$ $\sum_{\mathrm{doublets}} Y = 3\times\frac{1}{3} + (-1) = 0$

$$\Rightarrow$$

SM anomaly free generation by generation !

BUT

Cancellations "magical" + Why 3 families ?

$SU(5)$ IN $D = 6$

Local gauge anomaly

- $\bar{\mathbf{5}}$: $\quad \mathrm{STr}\,(T^4) = \boxed{1} \times \mathrm{STr}\,(t^4)$
- $\mathbf{10}$: $\quad \mathrm{STr}\,(T^4) = \boxed{-3} \times \mathrm{STr}\,(t^4) + 3 \times \underbrace{\mathrm{STr}\,(t^2)^2}_{\text{reducible}}$

anomaly cancellation requirement

$\Rightarrow$

♠ Vector-like solution $\bar{\mathbf{5}}_L$, $\mathbf{10}_L$, $\bar{\mathbf{5}}_R$, $\mathbf{10}_R$...

♡ Economical chiral solution $\boxed{\bar{\mathbf{5}}_L,\ \bar{\mathbf{5}}_L,\ \bar{\mathbf{5}}_L,\ \mathbf{10}_L}$

but 45 fermions $\in$ SM !

$\Rightarrow$

♡ Add $\boxed{\mathbf{10}_L, \mathbf{10}_R}$

Mixed Anomaly : killed by Green-Schwarz mechanism

Gravitational Anomaly : one adds 3 singlets

Global anomaly : $\Pi_6(SU(5)) = 0$

anomaly cancellation is nice

$SU(5)$ AND MORE IN $D = 6$

Anomaly free solution :

$$\begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_L \end{pmatrix}, \quad \begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_L \end{pmatrix}, \quad \begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_R \end{pmatrix}$$

and 3 singlets under $SU(5)$ ("sterile neutrinos" ?)

+ Green & Schwarz tensor!

3 generations !!!

Other GUTs ?

- 3rd generation cannot $\in$ 16 of $SO(10)$
- Economical solution i.e. 3 incomplete generations "can" $\in$ 27 of E_6

Phenomenology $D = 6$

Mass and Yukawa couplings in $D = 6$?

$$\begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_L \end{pmatrix}, \quad \begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_L \end{pmatrix}, \quad \begin{pmatrix} \bar{\mathbf{5}}_L \\ \mathbf{10}_R \end{pmatrix}$$

- Lorenz invariance $\rightarrow$ only $R + L$ because $D = 6$
- $SU(5)$ invariance and Higgs mechanism

  $\rightarrow \mathbf{10}_L + \mathbf{10}_R \Rightarrow m_t$

  $\rightarrow \bar{\mathbf{5}}_L + \mathbf{10}_R \Rightarrow m_b$
- Yukawa are complex $\Rightarrow$ CP violation ?

Need quantum corrections and dimensional reduction

Strong CP problem - Green-Schwarz tensor

Proton stability 

...

CONCLUSION

Could one find a GUT group G and a dimension D such that the anomaly cancellation condition(s) impose **3 generations** - as defined in the SM ?

The answer is "unique" : $SU(5)$ in 6D

Comments : but result is true only modulo 3 and vectorial generations, why generations, why these representations... and what is the phenomenology ?