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Charginos and neutralinos at e^+e^- Linear Colliders

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Outline:

1. Motivation
2. Chargino and neutralino sectors
3. Production processes at e^+e^- LCs
4. Extracting M_1, M_2, μ and $\tan\beta$
5. Conclusions

based on:
S.Y.Choi, JK, G. Moortgat-Pick, P.M. Zerwas, Eur. Phys. J. C 22 (2001) 563
and *Addendum*, EPJC, in press
Choi, Guchait, J.K., Zerwas, Phys.Lett.B 479 (2000) 235
Choi, Djouadi, Guchait, J.K., Song, Zerwas, Eur.Phys.J.C 14 (2000) 535

related work:
V. Barger et al., hep-ph/0101106; Phys. Lett. **B475** (2000) 342
J. L. Kneur and G. Moultaka, ,Phys. Rev. D **61** (2000) 095003
J.L. Feng et al., 1995, 1997
A. Bartl, H. Fraas, W. Majerotto, Nucl. Phys. **B278** (1986) 1
Vienna-Würzburg group, 1986–2001

Motivation

- in the MSSM many new parameters:
  - ⊕ gauge couplings: $g_{q\tilde{q}\tilde{g}}$, $g_{\tilde{W}e\tilde{\nu}}$, ...
  - ⊕ SUSY breaking parameters: M_i, A_i, $m_{\tilde{q}}$, $m_{\tilde{l}}$, ...
- after SUSY discovered:
  - ⊕ measure parameters in a model–independent way
  - ⊕ verify SUSY relations
  - ⊕ check for relations among them $\longrightarrow$ learn about SUSY breaking mechanism
- e^+e^- linear colliders ideal: tunable energy, polarized beams, clean environment etc.
- charginos and neutralinos may copiously be produced – a good starting for the determination of fundamental parameters
- question; if only light light $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_{1,2}^0$ produced at an early stage of LC:

$$\text{Can we determine } M_1,\ M_2,\ \mu \text{ and } \tan\beta?$$

- tree level to reconstruct the basic structure of charginos and neutralinos
- at the very end include loop corrections
  M.A. Diaz, S.F. King, D.A. Ross 1998,
  M.A. Diaz 1999,
  T. Blank, W. Hollik 2000,
  T. Blank 2000

Chargino sector

- chargino mass matrix in the $(\tilde{W}^-, \tilde{H}^-)$ basis

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & |\mu| e^{i\Phi_\mu} \end{pmatrix}$$

- to diagonalize, two unitary matrices needed

$$U_L = \begin{pmatrix} c_L & s_L^* \\ -s_L & c_L \end{pmatrix}, \qquad U_R = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} c_R & s_R^* \\ -s_R & c_R \end{pmatrix}$$

  where $c_{L,R} = \cos\phi_{L,R}, \quad s_{L,R} = e^{i\beta_{L,R}} \sin\phi_{L,R}$

  i.e. two mixing angles $\phi_{L,R}$ and three phases $\beta_{L,R}$ and $\gamma_1 - \gamma_2$

- the chargino masses

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\left[M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta_C\right]$$

$$\Delta_C = [(M_2^2 - |\mu|^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2(M_2^2 + |\mu|^2) + 8m_W^2 M_2 |\mu| \sin 2\beta \cos\Phi_\mu]^{1/2}$$

- the mixing angles

$$\cos 2\phi_{L,R} = -\left[M_2^2 - |\mu|^2 \mp 2m_W^2 \cos 2\beta\right] / \Delta_C$$

Inverting

From the set $\{m_{\tilde{\chi}^\pm_{1,2}}, \cos 2\phi_{L,R}\} \longrightarrow \{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$

- the mass parameters

$$M_2 = [(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m_W^2)/2 - \Delta_C(c_{2L} + c_{2R})/4]^{1/2}$$

$$|\mu| = [(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m_W^2)/2 + \Delta_C(c_{2L} + c_{2R})/4]^{1/2}$$

- the phase of μ (the sign of μ in CP–invariant cases)

$$\cos\Phi_\mu = \frac{\Delta_C^2(2 - c_{2L}^2 - c_{2R}^2) - 8m_W^2(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m_W^2)}{\sqrt{[16m_W^4 - \Delta_C^2(c_{2L} - c_{2R})^2]\,[4(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m_W^2)^2 - \Delta_C^2(c_{2L} + c_{2R})^2]}}$$

- and $\tan\beta$

$$\tan\beta = \sqrt{\frac{4m_W^2 - \Delta_C(\cos 2\phi_L - \cos 2\phi_R)}{4m_W^2 + \Delta_C(\cos 2\phi_L - \cos 2\phi_R)}}$$

Neutralino sector

- The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

$$M_1 = |M_1|\, e^{i\Phi_1} \qquad \text{and} \qquad \mu = |\mu|\, e^{i\Phi_\mu} \qquad [0 \leq \Phi_1, \Phi_\mu < 2\pi]$$

- the diagonalization matrix N, $\mathcal{M}_{diag} = N^* \mathcal{M} N^\dagger$,

  is parametrized by 6 angles and 10 phases, e.g.

$$N = \mathsf{M}\,\mathsf{D} = \mathsf{diag}\left\{e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4}\right\}\mathsf{D}$$

  where

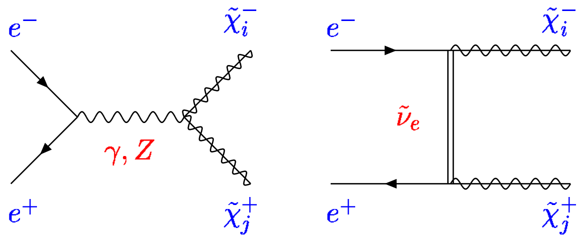
$$\mathsf{D} = \mathsf{R}_{34}\,\mathsf{R}_{24}\,\mathsf{R}_{14}\,\mathsf{R}_{23}\,\mathsf{R}_{13}\,\mathsf{R}_{12}$$

  and, for example,

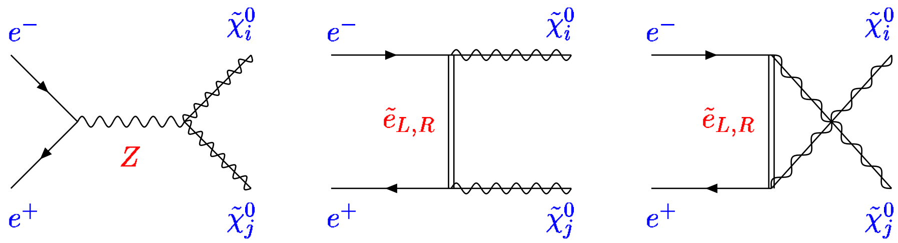
$$\mathsf{R}_{12} = \begin{pmatrix} c_{12} & s_{12}^* & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \begin{array}{l} c_{jk} \equiv \cos\theta_{jk} \\ s_{jk} \equiv \sin\theta_{jk}\, e^{i\delta_{jk}} \end{array}$$

- if $\delta_{ij} = 0 \mod \pi$, and $\alpha_i = 0 \mod \pi/2$, CP is conserved
- unitarity constraints $\longrightarrow$ quadrangles built up by
  - $\oplus$ the links $N_{ik}N_{jk}^*$ connecting two rows i and j
  - $\oplus$ the links $N_{ki}N_{kj}^*$ connecting two columns i and j
- unlike in the CKM or MNS cases of quark and lepton mixing, the orientation of all quadrangles is physical, and determined by the phases

Chargino and neutralino production at e^+e^- LC



The diagrams contributing to $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+$



The diagrams contributing to $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$

The production amplitude

$$T\left(e^+e^- \to \tilde{\chi}_i \tilde{\chi}_j\right) = \frac{e^2}{s} Q_{\alpha\beta} \left[\bar{v}(e^+)\gamma_\mu P_\alpha u(e^-)\right] \left[\bar{u}(\tilde{\chi}_i)\gamma^\mu P_\beta v(\tilde{\chi}_j)\right]$$

is expressed in terms of four generalized bilinear charges $Q_{\alpha\beta}$

- for example, for chargino diagonal pairs $\{11\}/\{22\}$:

$$Q_{LL} = D_L \mp F_L \cos 2\phi_L \qquad Q_{RL} = D_R \mp F_R \cos 2\phi_L$$
$$Q_{LR} = D'_L \mp F'_L \cos 2\phi_R \qquad Q_{RR} = D_R \mp F_R \cos 2\phi_R$$

- where

$$D_L = 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2})(s_W^2 - \frac{3}{4}) \qquad F_L = \frac{D_Z}{4 s_W^2 c_W^2}(s_W^2 - \frac{1}{2})$$
$$D_R = 1 + \frac{D_Z}{c_W^2}(s_W^2 - \frac{3}{4}) \qquad F_R = \frac{D_Z}{4 c_W^2}$$
$$D'_L = D_L + \left(\frac{g_{\tilde{W}}}{g_W}\right)^2 \frac{D_{\tilde{\nu}}}{4 s_W^2} \qquad F'_L = F_L - \left(\frac{g_{\tilde{W}}}{g_W}\right)^2 \frac{D_{\tilde{\nu}}}{4 s_W^2}$$

- D_Z and $D_{\tilde{\nu}}$ are normalized Z and $\tilde{\nu}_e$ propagators
- amplitudes linear in $c_{2L} \equiv \cos 2\phi_L$, c_{2R}, s_{2L}, s_{2R}
- sneutrino only in LR amplitude, breaks $c_{2L} \longleftrightarrow c_{2R}$ symmetry

Extracting SUSY parameters M_1, M_2, μ and $\tan\beta$

- numerical analyses for one CP–invariant (one of the Snowmass points) and one related CP–noninvariant case

		$\tan\beta$	$\lvert M_1\rvert$	M_2	$\lvert\mu\rvert$	Φ_1	Φ_μ
RP1	:=	10	100.5 GeV	190.8 GeV	365.1 GeV	0	0
RP1′	:=	10	100.5 GeV	190.8 GeV	365.1 GeV	$\frac{\pi}{3}$	$\frac{\pi}{8}$

- The induced chargino and neutralino masses

$$m_{\tilde\chi_1^\pm} = 175.6/176.0\,\text{GeV} \qquad m_{\tilde\chi^\pm} = 389.0/389.3\,\text{GeV}$$

$$m_{\tilde\chi_1^0} = 97.6/98.7\,\text{GeV} \qquad m_{\tilde\chi_2^0} = 176.2/176.3\,\text{GeV}$$

$$m_{\tilde\chi_3^0} = 371.4/371.8\,\text{GeV} \qquad m_{\tilde\chi_4^0} = 388.9/388.2\,\text{GeV}$$

for the two points RP1/1′, respectively

- the sneutrino and selectron masses are

$$m_{\tilde\nu} = 192.8\,\text{GeV} \quad m_{\tilde e_L} = 208.7\,\text{GeV} \quad m_{\tilde e_R} = 144.1\,\text{GeV}$$

for both points

consider two cases:

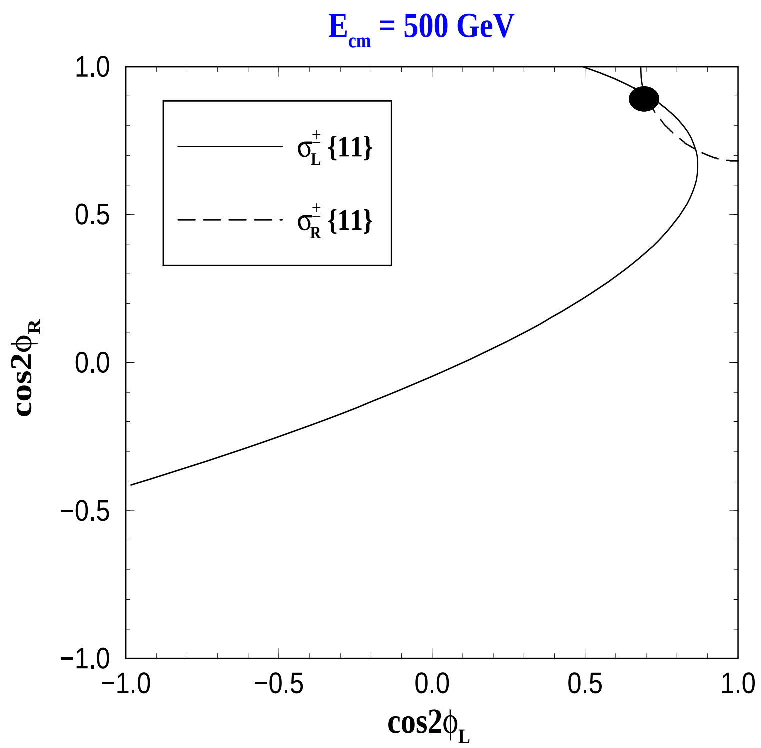
(a) early stage of the LC: only $\tilde\chi_1^\pm\tilde\chi_1^\mp$ and $\tilde\chi_1^0\tilde\chi_2^0$

(b) all charginos and neutralinos produced

Early stage of the LC

CP conserving case:

- from chargino sector: mass of $\tilde{\chi}_1^\pm$, and $\sigma_L\{11\}$ and $\sigma_R\{11\}$ known from experiment
- polarized cross sections are quadratic functions of $\cos 2\phi_L$ and $\cos 2\phi_R$
- in general contour lines for $\sigma_L\{11\}$ and $\sigma_R\{11\}$ in the $\{\cos 2\phi_L, \cos 2\phi_R\}$ plane may cross at most in four points ($m_{\tilde{\nu}}$ and $g_{\tilde{W}}$ assumed to be known)



$\sigma_L\{11\}$ and $\sigma_R\{11\}$ for the set $RP1$ at the e^+e^- c.m. energy of 500 GeV.

- for $RP1$ the two ellipses cross at two points:

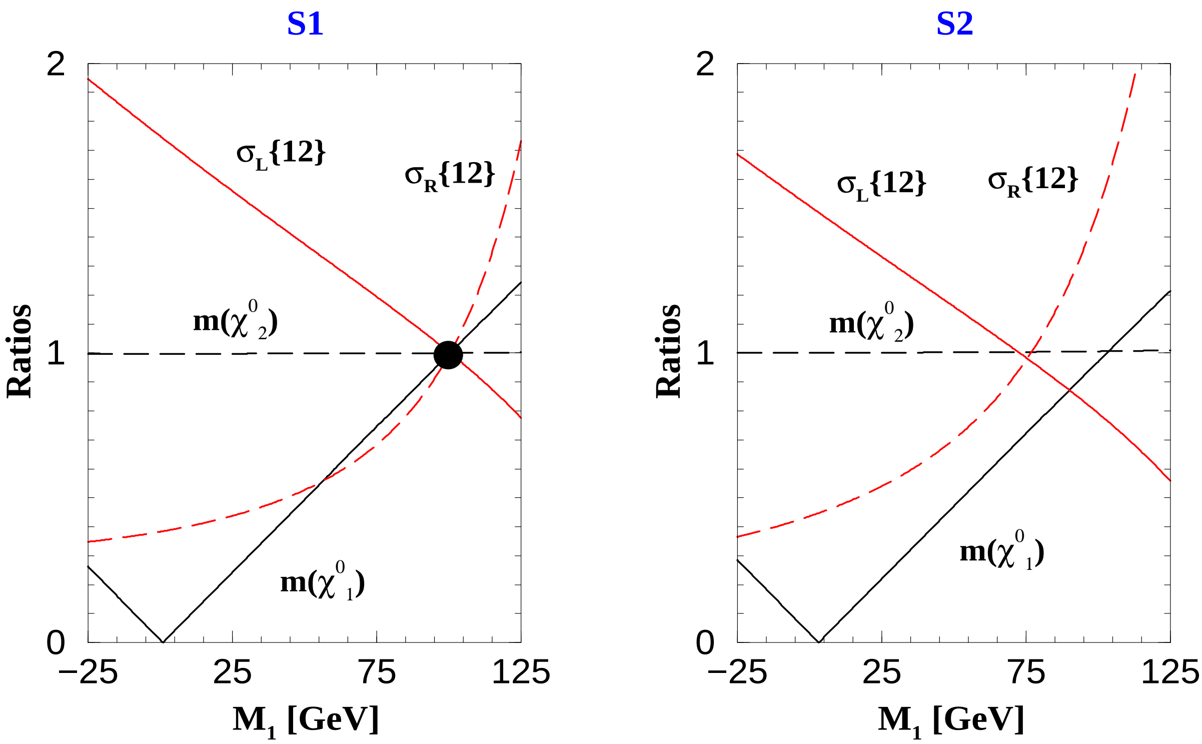
$$\{\cos 2\phi_L, \cos 2\phi_R\} = \{0.699, 0.906\} \ \text{ and } \ \{0.862, 0.720\}$$

- two possible solutions for $\{\tan\beta, M_2, \mu\}$:

$$\text{S1} : \{0.699, 0.906\} \Rightarrow \{\tan\beta = 10, M_2 = 191\,\text{GeV}, \mu = 365\,\text{GeV}\}$$
$$\text{S2} : \{0.862, 0.720\} \Rightarrow \{\tan\beta = 0.35, M_2 = 198\,\text{GeV}, \mu = 387\,\text{GeV}\}$$

to resolve ambiguity:

- add chargino cross section for transverse beam polarization, if available
- or exploit neutralino sector: compare predicted masses and cross section as functions of M_1 with measured values



Ratios of $m_{\tilde{\chi}^0_1}$, $m_{\tilde{\chi}^0_2}$, $\sigma_L\{12\}$ and $\sigma_R\{12\}$ with respect to their measured values. The left panel gives a unique value $M_1 = 100.5$ GeV.

- unique solution for couplings $\cos 2\phi_L$ and $\cos 2\phi_R$
- at the same time determine M_1, *i.e.* unique solution for M_1, M_2, μ and $\tan\beta$

CP-violating case:

- unique determination of chargino couplings $\cos 2\phi_L$ and $\cos 2\phi_R$ not enough to determine M_2, μ and $\tan\beta$.
- they depend on the unknown mass of heavy chargino

The strategy is to parametrize M_2, μ and $\tan\beta$ as functions of $m_{\tilde{\chi}^\pm}$and exploit neutralinos

- for each value of $m_{\tilde{\chi}^\pm}$calculate M_2, μ and $\tan\beta$
- the neutralino masses satisfy the characteristic equation

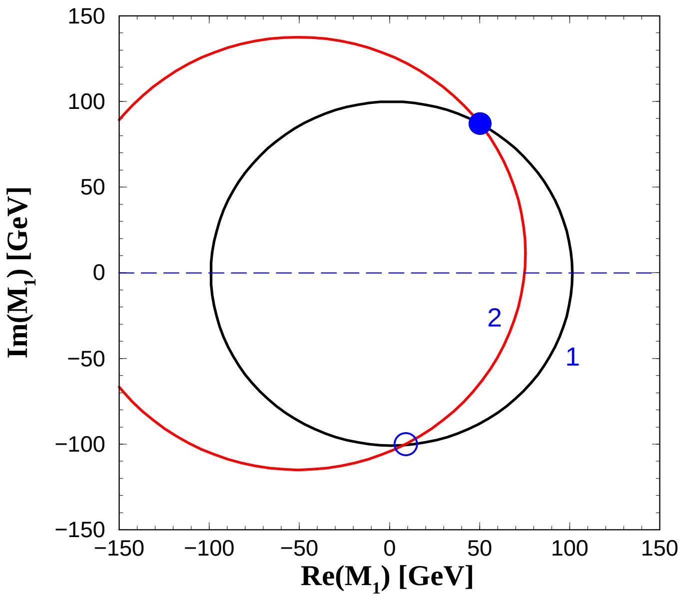
$$m^8_{\tilde{\chi}^0_i} - a\, m^6_{\tilde{\chi}^0_i} + b\, m^4_{\tilde{\chi}^0_i} - c\, m^2_{\tilde{\chi}^0_i} + d = 0 \quad \text{for} \quad i = 1, 2, 3, 4$$

  where each invariant a, b, c and d is a binomial of $\Re\mathrm{e} M_1 = |M_1| \cos\Phi_1$ and $\Im\mathrm{m} M_1 = |M_1| \sin\Phi_1$

- each characteristic equation for the neutralino mass squared $m^2_{\tilde{\chi}^0_i}$ has the form

$$(\Re\mathrm{e} M_1)^2 + (\Im\mathrm{m} M_1)^2 + u_i\, \Re\mathrm{e} M_1 + v_i\, \Im\mathrm{m} M_1 = w_i \quad \text{for} \quad i = 1, 2, 3, 4$$

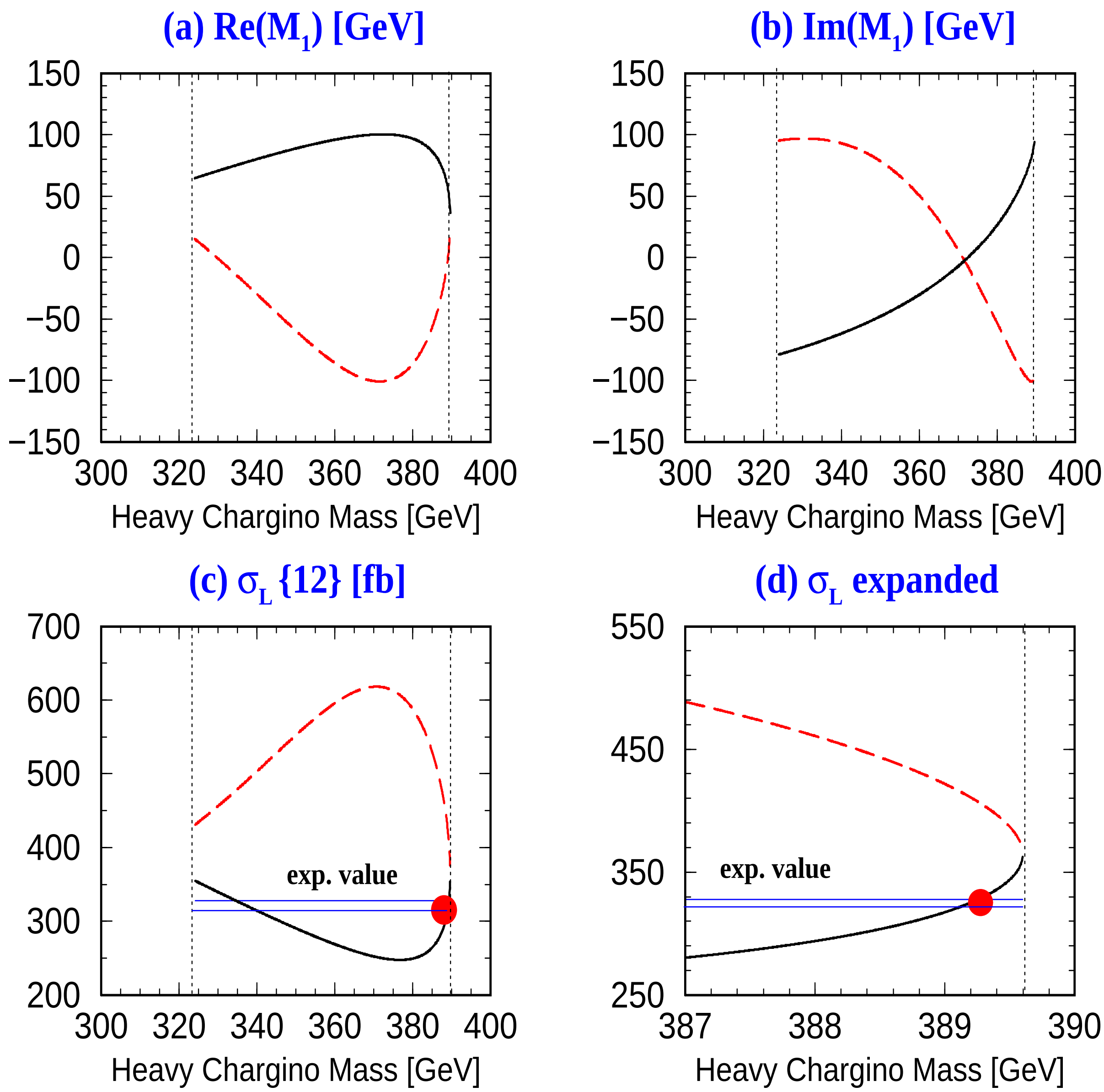
- v_i necessarily proportional to $\sin\Phi_\mu$ because masses are CP–even
- each neutralino mass defines a circle in the $\{\Re\mathrm{e} M_1, \Im\mathrm{m} M_1\}$ plane



- in the allowed mass range

$$\frac{1}{2}\sqrt{s} - m_{\tilde{\chi}_1^\pm} \leq m_{\tilde{\chi}^\pm} \leq \sqrt{m^2_{\tilde{\chi}_1^\pm} + 4m_W^2/|\cos 2\phi_L - \cos 2\phi_R|}$$

plot two possible solutions for $\{\Re e M_1, \Im m M_1\}$



- and use measured cross sections to select a unique solution for heavy chargino mass and M_1

Conclusions

- chargino and neutralino sectors with CP phases are solved analytically
- even at an early stage of e^+e^- LC, where only light charginos and neutralinos can be produced, the fundamental parameters M_1, M_2, μ and $\tan\beta$ can be determined including the CP-violating phases
- if $\Phi_\mu = 0$ however, a two–fold ambiguity for $\Im\mathrm{m} M_1$ remains: neutralino polarization needed to resolve it.
- the masses of the heavy chargino and neutralinos masses are predicted, and they can be searched for at the LHC
- with all charginos and neutralinos (not discussed in my talk)
  - ⊕ sum rules can be exploited to verify closure of the neutralino sctor
  - ⊕ threshold behaviour of nondiagonal neutralino pair production may signal nontrivial CP phases
  - ⊕ normal neutralino polarization - a unique probe of Majorana phases
  - ⊕ the basic structure of the chargino/neutralino complex can be reconstructed at e^+e^- colliders

- in the final analyses loop corrections will have to be included