Grand Unification & Six-dimensional SuperGravity

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UV Completion of the Standard Model

- Structure of Standard Model points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...)

- Strong theoretical arguments for supersymmetry at "high" energy scales (gravity, extra dimensions, string theory)

- Energy scale of grand unification: $\Lambda_{\rm GUT} \simeq 10^{15} \ldots 10^{16}$ GeV
  energy scale of supersymmetry breaking: $\Lambda_{\rm SB} \simeq$ **??**

- This talk: 6d supergravity GUTs, $\Lambda_{\rm SB} \sim R_{\rm c}^{-1} \sim \Lambda_{\rm GUT}$

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SUPERSYMMETRIC GRAND UNIFICATION IN A SIX-DIMENSIONAL SPACETIME

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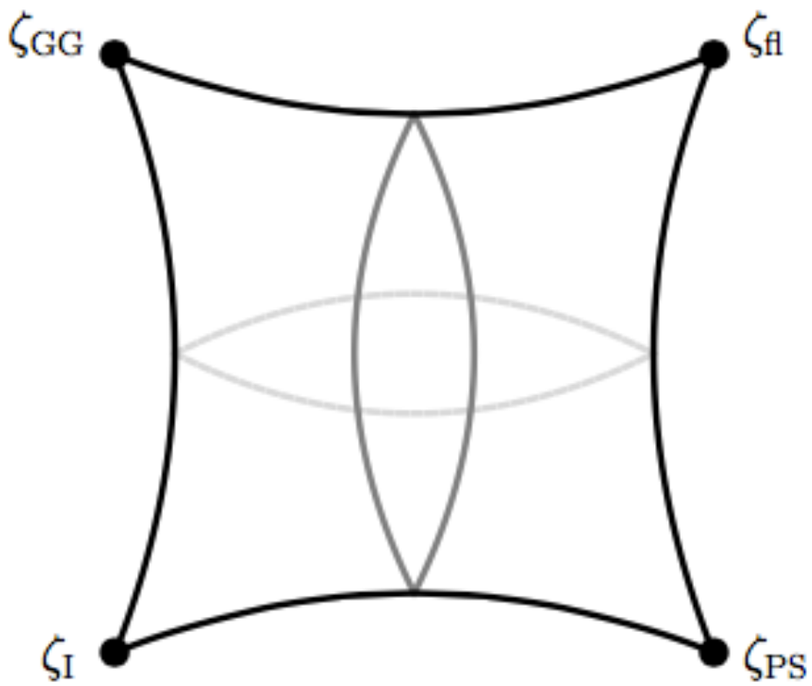
We formulate $N = 2$ supersymmetric GUTs in a six-dimensional spacetime, studying in particular the spontaneous breaking of gauge invariance and the construction of massive gauge multiplets. In six dimensions the photon and gluons are associated with a Weyl photino and Weyl gluinos; the massive $W^{\pm}$, with two Dirac Winos and three spin-0 Higgs bosons; the massive Z, with one Dirac Zino and three neutral Higgs bosons. The $X^{+4/3}$ is massless, and the $Y^{+1/3}$ has the same mass as the $W^{\pm}$. The grand-unification mass, generated by the breaking of the 6D Poincaré invariance, is associated with the large momenta carried by the grand-unification gauge bosons along the new compact dimensions.

Since then: 6d supergravity, orbifold GUTs (symmetry breaking, doublet-triplet splitting), 6d supergravity as intermediate step toward string theory, flux compactifications, connection with supersymmetry breaking ...

Split symmetries

WB, Dierigl, Ruehle, Schweizer '15, '16

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with Standard Model group as intersection; bulk fields 45, 16, 16*, 10's [Asaka, WB, Covi '02,'03]; full 6d gauge symmetry:



$$SO(10) \times U(1)_A$$

N 16's from charged bulk 16-plet and N flux quanta:

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}}\ [G_{\rm SM}]$$

Higgs fields from uncharged bulk 10-plets, form split multiplets:

$$H_1 \supset H_u\,, \quad H_2 \supset H_d\,, \quad \Psi \supset D^c, N^c\,, \quad \Psi^c \supset D, N$$

Flux **breaks supersymmetry** [Bachas '95], soft SUSY breaking only for quark-lepton families:

$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15}\ \mathrm{GeV})^2$$

$$m_{3/2} \sim 10^{14}\ \mathrm{GeV}\,, \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (reminiscent of "split/spread SUSY" [Arkani-Hamed, Dimopoulos; Giudice, Romanino '04; Hall, Nomura '11]):

- supersymmetry breaking is large for scalar quarks and leptons *because* they form *complete* GUT multiplets

- supersymmetry breaking can be small for gauge and Higgs fields *because* they form *incomplete* GUT multiplets

Quantum Corrections

WB, Dierigl, Dudas, Schweizer '16

Simplest example: 6d SUSY QED, compactified on torus:

$$S_6 = \int d^6x \Big\{ \tfrac{1}{4} \int d^2\theta\, W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left(\partial V \overline{\partial} V + \phi\overline{\phi} + \sqrt{2} V \left(\overline{\partial}\phi + \partial\overline{\phi} \right) \right)$$
$$+ \int d^2\theta\, \tilde{Q}(\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left(\overline{Q} e^{2gqV} Q + \overline{\tilde{Q}} e^{-2gqV} \tilde{Q} \right) \Big\} ,$$

$$\partial = \partial_5 - i\partial_6 \,, \quad \phi|_{\theta=\bar\theta=0} = \tfrac{1}{\sqrt{2}}(A_6 + iA_5)$$

Wilson lines and flux, mode expansion of superfields:

$$\phi_0|_{\theta=\bar\theta=0} = \frac{f}{2\sqrt{2}} (x_5 - ix_6) + \varphi \,, \quad \varphi = \tfrac{1}{\sqrt{2}}(a_6 + ia_5) \,,$$

$$Q(x_M, \theta, \bar\theta) = \sum_{n,j} Q_{n,j}(x_\mu, \theta, \bar\theta)\, \psi_{n,j}(x_m) \,, \quad \tilde{Q}(x_M, \theta, \bar\theta) = \ldots$$

Bachas '95: Landau levels

→ effective 4d action, compute Wilson line potential

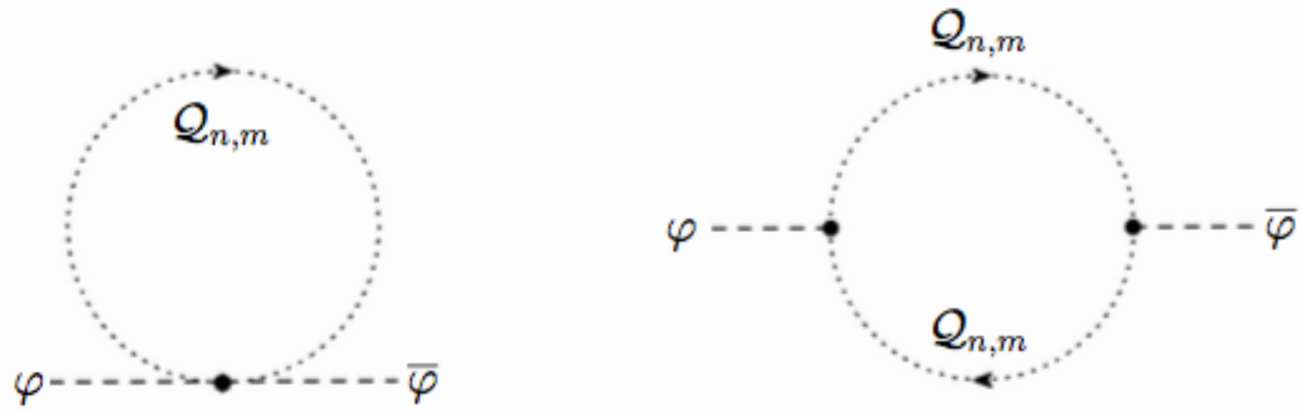


Figure 1: Bosonic contributions to the Wilson line mass without flux.

$$\delta m_b^2 = 2g^2q^2 \sum_{n,m} \int_0^\infty dt\, t\, e^{-|M_{n,m}|^2 t} \int \frac{d^4k}{(2\pi)^4} k^2 e^{-k^2 t}$$

$$= \frac{g^2q^2L^2}{16\pi^3} \int_0^\infty \frac{dt}{t^3}\, \Theta_3\left(0; \frac{iL^2}{4\pi t}\right)^2$$

$$= \frac{g^2q^2L^2}{16\pi^3} \int_0^\infty du\, u\, \Theta_3\left(0; \frac{iL^2 u}{4\pi}\right)^2$$

$$= \frac{g^2q^2}{\pi^3 L^2} \sum_{r,s} \frac{1}{(r^2+s^2)^2} \sim \frac{g^2q^2}{\pi^3 L^2}\left(\frac{1}{0^4}+1\right)$$

Antoniadis, Benakli, Quiros '01

mass correction finite, **after** subtraction of divergent constant (remnant of 6d gauge symmetry, invariance w.r.t. large gauge transformations)

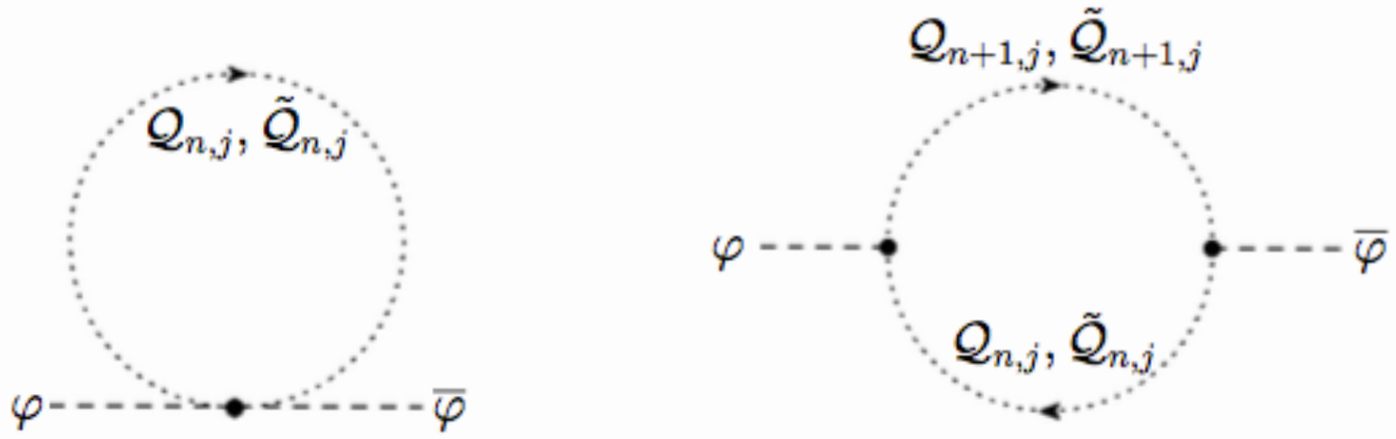


Figure 3: Bosonic contributions to the Wilson line mass with flux.

$$
\begin{aligned}
\delta m_b^2 &= -4q^2g^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\left(\frac{n}{k^2+\alpha(n+\frac{1}{2})} - \frac{n+1}{k^2+\alpha(n+\frac{3}{2})}\right) \\
&= -\frac{q^2g^2}{4\pi^2}|N|\sum_n \int_0^\infty dt\,\frac{1}{t^2}\left(ne^{-\alpha(n+\frac{1}{2})t} - (n+1)e^{-\alpha(n+\frac{3}{2})t}\right) \\
&= -\frac{q^2g^2}{4\pi^2}|N|\int_0^\infty dt\,\frac{1}{t^2}\left(\frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} - \frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2}\right) \;=\; 0
\end{aligned}
$$

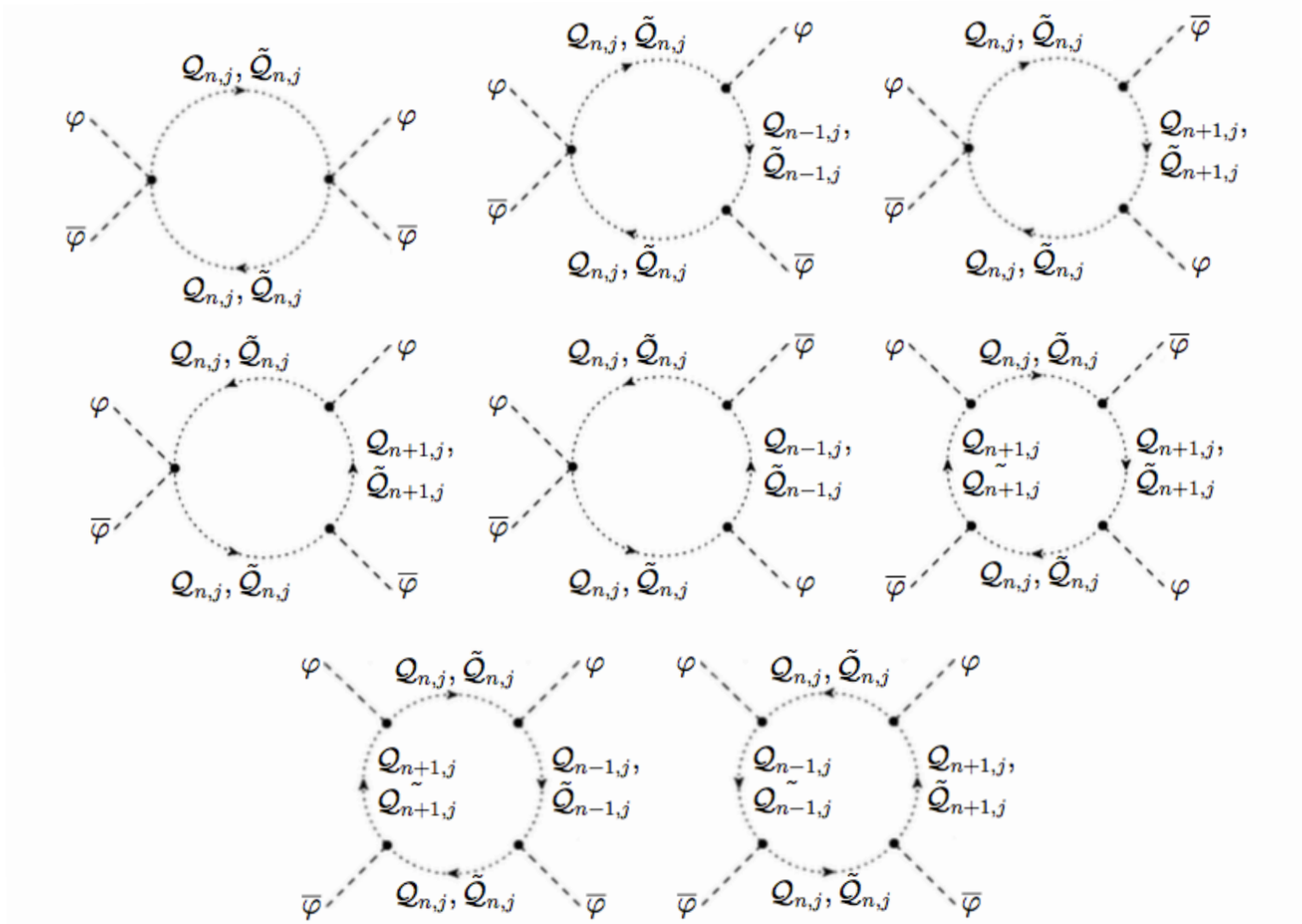


Figure 6: Bosonic contributions to the Wilson line quartic coupling with flux.

$$\delta\lambda_f = -\frac{q^4 g^4}{2\pi^2}|N|\int_0^\infty dt \sum_n \Big[\frac{1}{t}\left(-n^2 e^{-\alpha n t} + (n+1)^2 e^{-\alpha(n+1)t}\right) + \frac{1}{\alpha t^2}\left(n(n+1)e^{-\alpha n t} - (n+1)(n+2)e^{-\alpha(n+1)t} - n(n+1)e^{-\alpha(n+1)t} + (n+1)(n+2)e^{-\alpha(n+2)t}\right)\Big] = 0$$

Wilson lines as Goldstone bosons

Action of charged matter field invariant w.r.t. translations,

$$S_6 = \int d^6x \, \left(-D_M \bar{Q} D^M Q\right) \,, \quad D_M Q = (\partial_M + iqgA_M)Q$$

$$\delta Q = \epsilon^m \partial_m Q \,, \quad \delta A_n = \epsilon^m \partial_m A_n$$

Symmetries for constant Wilson line background $\varphi = \frac{1}{\sqrt{2}}\left(a_6 + ia_5\right)$,

$$\delta Q = \epsilon^m \partial_m Q \,, \quad \delta a_n = 0$$

Flux background breaks translational symmetries spontaneously,

$$D_m Q = \left(\partial_m + iqg\left(a_m + \tfrac{f}{2}\epsilon_{mn}x_n\right)\right)Q \,, \quad \langle A_m \rangle = \tfrac{f}{2}\epsilon_{mn}x_n$$

Translational symmetries now nonlinearly realized with Wilson lines as Goldstone bosons,

$$\delta Q = \epsilon^m \partial_m Q \,, \quad \delta a_n = \epsilon^m \tfrac{f}{2}\epsilon_{nm}$$

Conclusions

- Supersymmetric extensions of Standard Model strongly motivated, but what is the scale of SUSY breaking??

- Higher-dimensional GUT models with flux suggests GUT scale for SUSY breaking; emerging low energy spectrum reminiscent of `spread' SUSY (2HDM + higgsino + ...)

- Hierarchy problem of electroweak scale? Flux has important effect on quantum corrections for scalar masses (protection from quadratic divergencies?) Possible applications: gauge-Higgs unification, orbifold GUTs, ...