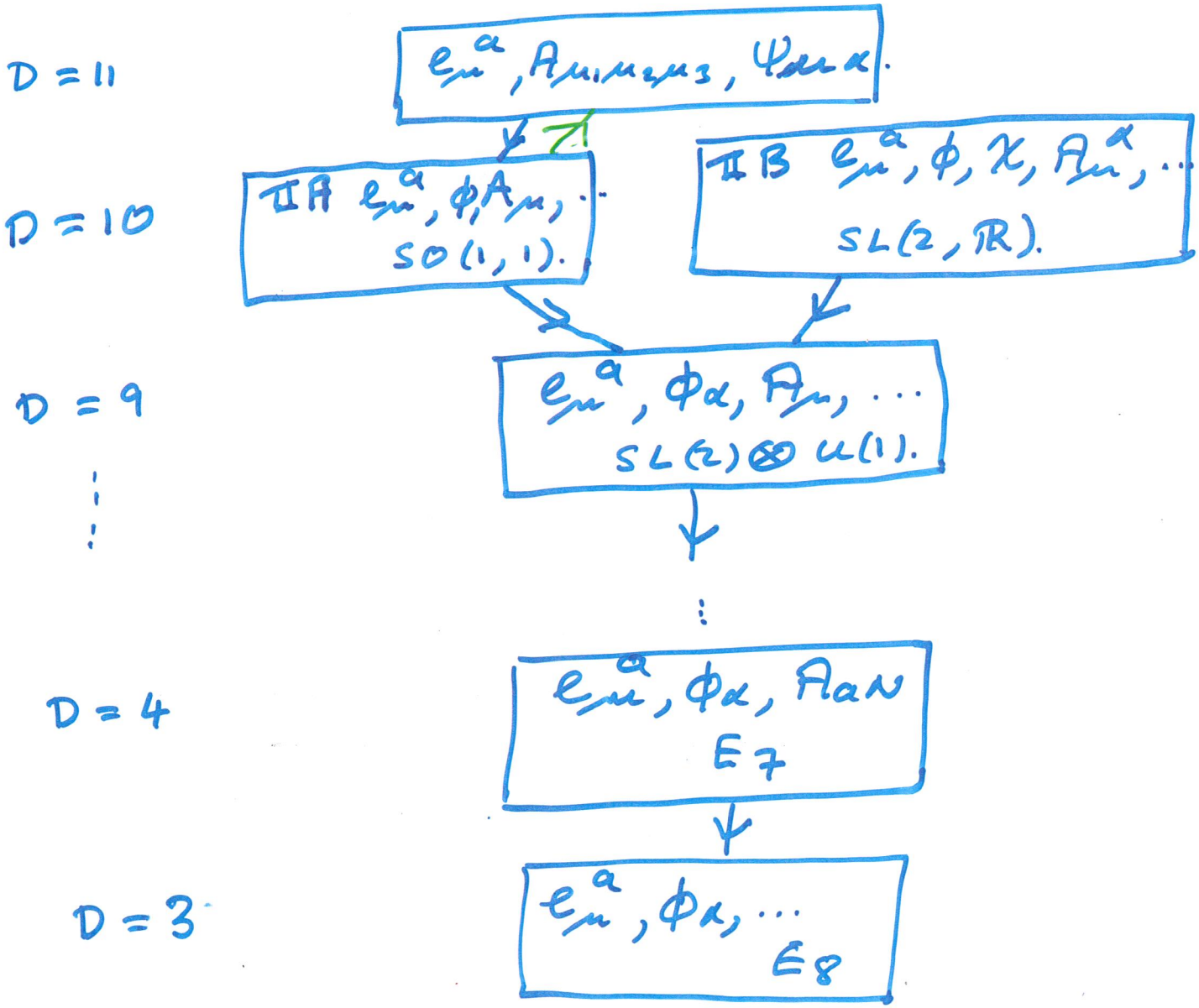
E theory

Supersymmetry and string theory lead to a consistent perturbative theory of quantum gravity However non-perturbative effects are difficult to compute and we must include branes about which we know very little.

Almost nothing is known about the underlying theory of strings and branes but many effects at low energy are contained in the maximal supergravity theories. However there are many such theories, some of which are connected (M theory).

In 2001 I proposed that the low energy effective theory of strings and branes had the Kac-Moody symmetry E_{11}. It is the non-linear realization of the semi-direct product of E_{11} and its vector representation.

Maximal Supergravities and M theory



- Maximal supergravities exist in dimensions $D \leq 11$. They have E_{11-D} (Cremmer-Julia) symmetries with $SL(2,\mathbb{R})$ in IIB

- The relations between these theories are known as M theory.

- We can add a cosmological constant ($\int d^D x \det e_\mu^a$) and preserve the supersymmetry in many different ways.

Kac-Moody Algebras 3

Lie algebras $\rightsquigarrow$ Cartan subalgebra H_i

$$[H_i, E_\alpha] = \alpha_i E_\alpha .$$

$\rightsquigarrow$ simple roots α_a with Cartan matrix

$$A_{ab} = 2\frac{(\alpha_a, \alpha_b)}{(\alpha_a, \alpha_a)} \rightsquigarrow \text{Dynkin diagram}$$

Cartan - Killing found Lie algebras with A_{ab}'s such that

$$A_{ab} \text{ -ve integer if } a \neq b \quad -(1)$$

$$\sum_{a,b} v^a A_{ab} v^b > 0 \quad \forall\, v^a \quad -(2).$$

Serre found a way to go back from the Cartan matrix (Dynkin diagram) to the Lie algebra by imposing certain relations.

Kac-Moody just dropped (2).

when a rigid symmetry G is spontaneously broken to H there are dim G − dim H massless (Goldstone) particles whose low energy dynamics is "the non-linear realization of G with local subgroup H."

π dynamics	supergravity
$SU(2) \otimes SU(2)$	E_{11}
quarks	?

The E_n symmetries in the maximal supergravity theories were thought to be a quirk of dimensional reduction.

In $D = 4$ the scalars belong to the non-linear realization of E_7 with local subgroup $SU(8)$.

$$\phi_\alpha \longrightarrow R^\alpha \in E_7.$$

Can we generalize this to all fields. In $D = 11$ this means

$$h_a{}^b \longleftrightarrow K^a{}_b$$

$$A_{a_1 a_2 a_3} \longleftrightarrow R^{a_1 a_2 a_3} \Big\} E_{11}$$

$$\vdots$$

we need an infinite number of fields.

E_{11} and its vector representation



E_{11} has a A_{10}, or $SL(11)$, subalgebra. and when decomposed into this algebra has the generators

$$R^{\underline{\alpha}} = \{ \ldots, K^a{}_b, R^{a_1 a_2 a_3}, R^{a_1 \ldots a_6}, R^{a_1 \ldots a_8, b}, \ldots \}$$

The commutators known up to level four.

The fundamental representation for node 1, that is, the vector representation has the objects

$$\ell_A = \{ P_a, Z^{a_1 a_2}, Z^{a_1 \ldots a_5}, Z^{a_1 \ldots a_7, b}, \ldots \}$$

The semi-direct product of $E_{11} \otimes_s \ell$

is the algebra

$$[R^{\underline{\alpha}}, R^{\underline{\beta}}] = f^{\underline{\alpha}\,\underline{\beta}}{}_{\underline{\gamma}}\, R^{\underline{\gamma}}$$

ie the E_{11} algebra and

$$[R^{\underline{\alpha}}, \ell_A] = -(D^{\underline{\alpha}})_A{}^B \ell_B$$

Note Poincaré = $\text{dvect}_3 \otimes_s T^4$

$$= \{ J_{a_1 a_2} \} \otimes_s \{ P_a \}$$

Non-linear Realization of $E_{11} \otimes_s l$

with local subgroup $I_c(E_{11})$ is constructed from $g \in E_{11} \otimes_s l$ subject to

$$g \to g_0 \, g \quad \text{and} \quad g \to g \, h$$

$g_0 \in E_{11}$ (rigid), $h \in I_c(E_{11})$.

The group element g has the form

$$g = g_c \, g_E = e^{x^a P_a} \, e^{x^{a_1 a_2} Z_{a_1 a_2}} \ldots$$
$$e^{h_a{}^b K^a{}_b} \, e^{A_{a_1 a_2 a_3} R^{a_1 a_2 a_3}} \, e^{A_{a_1 \ldots a_6} R^{a_1 \ldots a_6}} \ldots$$

It is a theory with fields

$$h_a{}^b, \; A_{a_1 a_2 a_3}, \; A_{a_1 \ldots a_6}, \ldots$$

which depend on the coordinates

$$x^a, \; x_{a_1 a_2}, \; x_{a_1 \ldots a_5}, \ldots$$

However if we keep only the low level fields and the usual spacetime x^a then the equations of motion are unique and are precisely those of eleven dimensional supergravity.

E_{11} as a unified theory

— By taking different decompositions one finds the maximal supergravities in D dimensions



— It contains all the maximal supergravities with a cosmological constant (gauged supergravities).

— It is essentially unique given the E_{11} Dynkin diagram and the choice of vector representation

— It includes all we know about strings and branes at low energy but it contains many new effects. It must be the low energy effective action for strings and branes.