THERMODYNAMICS OF BRANE-WORLDS

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We discuss the gross thermodynamical properties of brane-worlds with a focus on the Randall-Sundrum scenarios. We argue that the Gibbons-Hawking effect and the Hawking-Page phase transitions are both realized in these five dimensional brane world models, and hence they may both play a role in early universe cosmology. This could be particularly interesting during an de-Sitter phase associated with cosmic inflation, where thermal effects are responsible for the perturbations created.

1 Introduction

Current trends in high-energy physics suggest that it might be sensible to consider the possibility that the universe is in fact a hypersurface situated in a background spacetime of higher dimension. The main motivation for this comes from string theory where it has come to be accepted that the unification of gravity with the other fundamental forces of nature requires eleven dimensions. These extra dimensions must, however, be compactified so that they are not visible to observers in a low-energy world such as our own.

For many years it was thought that this should be done by use of a smooth manifold $\mathcal{M}$ of dimension $n-4$ with radius $R_{\rm c}\sim M_{\rm pl}^{-1}$, where n is the total number of spacetime dimensions and $M_{\rm pl}$ is the Planck mass. However, a smooth manifold cannot lead to a low-energy phenomenology with is chiral[2], and hence it has become popular to use orbifolds - manifolds which are smooth except for at a finite number of points which are fixed - for compactification. The Horava-Witten corner[3] of the M-theory moduli space is perhaps the most celebrated example of an orbifold compactification. In this model two 3-branes are at two orbifold fixed points, separated by a single extra-dimension, the other six-dimensions being compactified in the standard way via a Calabi-Yau manifold, for example.

This basic concept has spawned a number of phenomenological variants which have attempted to understand the general principles involved. All the models assume that the fields of the standard model only have support on our 3-brane, while gravity, and possibly other fields such as a dilaton, propagate in the whole spacetime. In particular, if the metric of the higher dimensional space is a direct product of the 3-brane universe and the compactified dimensions[1], it is possible for this kind of model to solve the hierarchy problem: if the fundamental scale is M then $M_{\rm pl}^2 = VM^{n+2}$ where $V \sim r_{\rm c}^{n-4}$ is the volume of the internal space, and, for example, $M \sim 1{\rm TeV}$ if the number of extra dimensions is two and the size of the extra dimension is about a millimetre. The case of a single extra dimension cannot solve the hierarchy problem in this scheme since it requires an extra dimension of size $R \sim 10^{13}{\rm cm}$ and Newton's gravitational force law has been probed by experiments down to about a millimetre.

This does not, however, exclude the possibility of a single extra dimension as in the Horava-Witten theory[a]. This phenomenologically well motivated theory is extremely complicated involving many fields, but there are simple models which are thought to have some interesting similarities, the best known of these being the Randall-Sundrum scenario[4,5]. The action for this model is

$$S = \int_M d^5x\sqrt{g}\left(\frac{R}{2\kappa_5^2} - \Lambda\right) + \frac{1}{\kappa_5^2}\int_{\partial M} d^4x\sqrt{h}K + \sigma\left(\int_{B_1} d^4x\sqrt{h} - \int_{B_2} d^4x\sqrt{h}\right), \qquad (1)$$

where $\Lambda(<0)$ is the bulk cosmological constant, σ is the tension of the branes and $\kappa_5^2 = 8\pi G_5$ is the five dimensional gravitational coupling constant. The metric of the five dimensional spacetime M (∂M is the boundary of M) is $g_{\mu\nu}$ and that on the four dimensional branes B_1 and B_2 is $h_{\mu\nu}$. Two solutions have been considered in the context of this model: the first (RS1) has the two branes and can solve the hierarchy problem, by using an exponential warp factor to generate a hierarchy between the two scales. The second (RS2), which is the subject of this talk, takes the negative tension brane off to infinity, creating an effectively infinite extra-dimension. The simple closed form solution is

$$ds^2 = \exp\left[-\kappa_5^2\sigma z/3\right]\left(-dt^2 + d\Omega_3^2\right) + dz^2, \qquad (2)$$

where $d\Omega_3^2$ is the metric on a three-sphere and the existence of this solution requires that $\kappa_5^2\sigma^2 = -6\Lambda$. This is a particularly interesting model since it has been shown that four dimensional gravity is recoverable on the 3-brane[5].

2 The Gibbons-Hawking effect

In order to study the RS2 model in a cosmological setting, we generalize the metric (2) to[6]

$$ds^2 = \left(\cosh\left[\frac{\kappa_5^2\sigma z}{6}\right] - \frac{\kappa_5\sigma}{\sqrt{6|\Lambda|}}\sinh\left[\frac{\kappa_5^2\sigma z}{6}\right]\right)^2\left(-dt^2 + \exp[2Ht]d\Omega_3^2\right) + dz^2, \qquad (3)$$

where $H = (\kappa_5^4\sigma^2 - 6\kappa_5^2|\Lambda|)/36$ is the Hubble parameter of the four-dimensional de-Sitter (dS) slice at $z = 0$ in the five dimensional background anti-de-Sitter space (AdS_5). This metric has an acceleration horizon in the bulk at

$$z_{\rm h} = \sqrt{\frac{3}{2{\kappa_5}^2|\Lambda|}}\log\left(\frac{\kappa_5\sigma + \sqrt{6|\Lambda|}}{\kappa_5\sigma - \sqrt{6|\Lambda|}}\right), \qquad (4)$$

which will give rise to thermal effects in the bulk via the Gibbons-Hawking effect[7].

[a]Typically, the size of the extra dimension is about 10^{-24} cm in the Horava-Witten model.

Given any event horizon in a spherically symmetric and stationary spacetime, one can recover thermodynamic information by calculating the surface gravity, $\kappa_{\rm H}$, of the horizon, which measures the frequency decrease (or redshift) of an outgoing light signal propagating along the horizon, that is, the redshift measured relative to Killing or stationary time. A straightforward calculation yields $\kappa_{\rm H} = (\kappa_5/6)\sqrt{\kappa_5{}^2\sigma^2 - 6|\Lambda|}$ from which we recover the temperature $2\pi T = \kappa_{\rm H} = H$. In order to interpret this result one has to realise that the 3-brane can be thought of as a uniformly accelerating observer in the background AdS space, and that this is the temperature seen by the observers on the 3-brane, not those in the bulk. In a similar calculation to the one presented here[8], it was shown that the temperature of experienced by a uniformly accelerating observer in AdS is given by $2\pi T = \sqrt{a^2 - \kappa_5^2|\Lambda|/6}$, where a is the acceleration of the observer. Clearly, the acceleration of the 3-brane in our model is given by $a = \kappa_5^2\sigma/6$, and hence the two calculations agree.

The most obvious cosmological consequence of this temperature is that the amplitude of fluctuations of gravitons and the inflaton created during an inflationary phase would be given by the usual formula, for example, $\delta\phi = H/2\pi$ for an inflaton ϕ, but now the H is the modified Hubble parameter[9]. There is, however, another more subtle consequence of this which is also of interest in cosmology. In four dimensions the gravitons associated with the Gibbons-Hawking temperature are often described as being thermal. But the gravitons cannot be truly thermal since in pure de-Sitter space there is prefered direction by which one can define a thermal temperature, and hence if one computes the stress-energy tensor of the gravitons it is observer dependent. In the five dimensional models under consideration here, there is still an observer dependent effect when one considers the whole spacetime, but from the point of view of the de-Sitter brane, which is accelerating in the background space time, and hence has a preferred direction, the stress-energy tensor on the brane will be finite. Moreover, the energy density of the gravitons will contribute to the cosmological expansion of the de-Sitter brane as a isotropic radiation fluid.

This is explicit in the (4+1) Gauss-Codacci formalism which can be used to compute the four dimensional Einstein equations[10]. It was shown that there is an extra contribution to the Einstein equations which is the projection of the five dimensional Weyl tensor on to the brane $E_{\mu\nu} = n^\alpha n^\gamma h_\mu{}^\beta h_\nu{}^\delta C_{\alpha\beta\gamma\delta}$ where $C_{\alpha\beta\gamma\delta}$ is the Weyl curvature of the background spacetime, n_μ is the unit normal to the brane-world and $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ is the fundamental tensor (or metric) of the brane. The Weyl tensor corresponds to the tidal degrees of freedom of the background spacetime, for example, gravitational waves, and therefore $E_{\mu\nu}$ can be thought of as being equivalent to the stress-energy of the projected bulk gravitons. Even though the averaged Weyl tensor is likely to be zero for the whole spacetime, the average of this projection will not and should contribute in an obvious way to the radiation density on the 3-brane. It is, therefore, constrained by nucleosynthesis as suggested in ref.[9], where a similar term was derived as an integration constant.

3 The Hawking-Page phase transition

The simplest way to generate Weyl curvature in the bulk is to place a black hole there, since such a black hole would introduce a temperature into the system. The black hole would contribute to the Friedmann equation of the brane world as an effective radiation fluid in an analogous way to the gravitational waves described above[11], and the temperature of the black hole would be the effective thermal temperature of radiation fluid experienced by observers on the brane. This will related to the mass of the bulk black hole and hence nucleosynthesis will impose a constraint on the black holes mass. However, the stability of such a system would require the bulk black hole exist in thermal equilibrium with the brane-world. In other words, is there a generalization of the Hawking-Page phase transition to the RS2 model?

Hawking and Page showed[12] that there is a critical temperature, T_c, past which thermal radiation in pure AdS is unstable to the formation of a Schwarzschild-AdS black hole. In fact, they found that for $T > T_c$ there are two values of the black hole mass at which the Hawking radiation can be in equilibrium with the thermal radiation of the background. The lesser of these two masses is a point of unstable equilibrium, since it has negative specific heat, whereas the greater mass is stable. In order to prove that this phase transition occurs, they calculated the action (at a given temperature) of the Euclidean section of Schwarzschild-AdS, and compared it to the equivalent action of the Euclidean section for pure AdS[b]. At $T = T_c$ the values are equal and for $T > T_c$, Schwarzschild-AdS is prefered.

It should be possible calculate the action (at a fixed temperature) of the Euclidean section of a pure de Sitter brane-world moving in AdS, and compare it to the action of the Euclidean section of a brane-world moving in Schwarzschild-AdS with the same temperature. There are now bulk regions, on each side of the domain wall and furthermore, there will be a non-trivial matching condition for the domain wall on the Schwarzschild-AdS 'cigar' instanton[13], which make the calculation technically more challenging. There is, however, a conceptual issue to be understood: the size of the region of AdS which is bounded by the brane-world. Ordinarily, the black holes which are stable after the Hawking-Page transition has occured, are those which are large relative to the length scale of AdS. But the region of AdS bounded by the de Sitter brane-world can be arbitrarily small relative to the length scale of AdS. Presumably, if the effective cosmological constant on a de Sitter brane-world in RS2 is sufficiently large relative to the AdS length scale in the bulk, then it is *impossible* for a bulk black hole to be in thermal equilibrium with the brane-world. Research on this and related issues is currently underway.

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[b] Both of these actions are a priori infinite, so one has to take care to match the two instantons in the asymptotic region to get a finite answer for the difference.