Cosmic Data Fusion

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We compare and combine cosmological parameter constraints from galaxy peculiar velocities, the CMB and supernovae in the parameter space $\Omega_{\rm m}$, h and σ_8. We also outline a method for combining data sets using weightings, or 'Hyper-Parameters', and we compare the results of this approach with those of a conventional approach by applying it to subsets of a CMB data compilation.

1 Introduction

Single cosmological data sets can only yield information on degenerate combinations of cosmological parameters. These combinations are often different for different cosmological data sets therefore we may estimate single cosmological parameters by combining the information from different data sets. In Section 2 we compare and combine galaxy peculiar velocities, CMB and supernova constraints. In Section 3 we consider the general problem of combining data sets and propose a variable weighting scheme for each data set. We illustrate this Hyper-Parameters approach in Section 4 by applying it to various subsets of our CMB data compilation.

2 Velocities, CMB and Supernovae

We use constraints on $\Omega_{\rm m}$, h and σ_8 from the SFI galaxy peculiar velocity (PV) catalogue (Haynes et al. 1999 AJ, 117, 1668 & AJ, 117, 2039), which consists of ~ 1300 spiral galaxies within $\sim 70 h^{-1}$ Mpc ($h \equiv H_0/(100 \ {\rm kms^{-1} Mpc^{-1}})$). The analysis follows in general the maximum-likelihood method of Zaroubi et al. (1997 ApJ, 486, 21) and Freudling et al. (1999 ApJ, 523, 1). We use a zero-lag velocity dispersion parameter set to its best fit value, $\sigma_{\rm v} = 200$. We use the same compilation of CMB anisotropy measurements as in Bridle et al. (1999 MNRAS, 310, 565), supplemented by the new TOCO points (Miller et al. 1999 astro-ph/9906421) and the

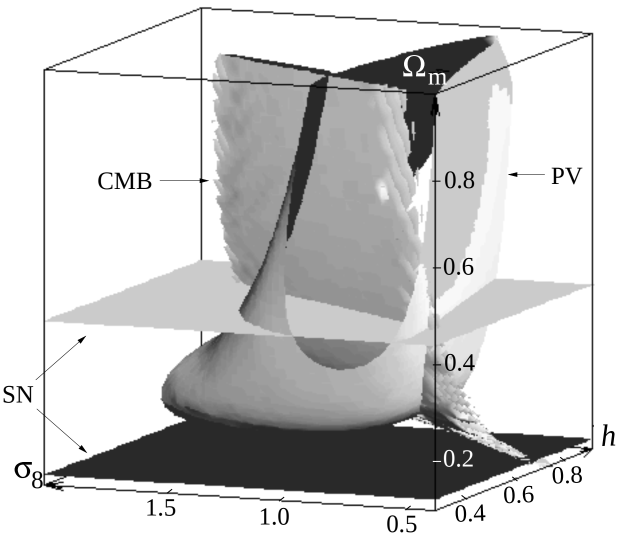

Figure 1: PV, CMB and SN 2σ iso-probability surfaces. For PV and CMB the surfaces are at Δlog(Likelihood)=4.01, and for the SN the surfaces are at Δlog(Likelihood)=2.00, corresponding to the 95 per cent limits for 3 and 1 dimensional Gaussian distributions respectively. The SN surfaces are two horizontal planes.

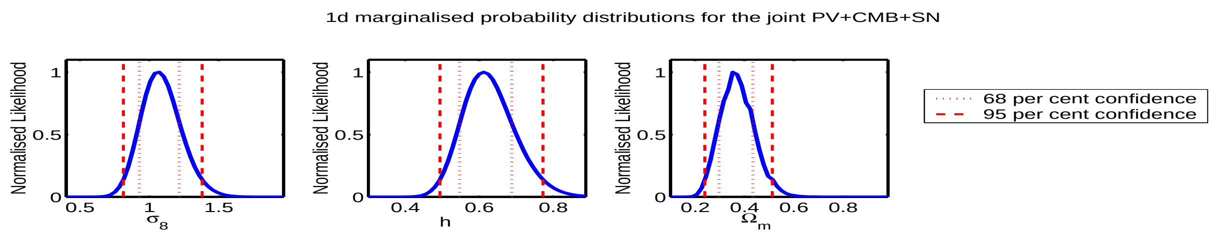

Figure 2: The 1-dimensional marginalised likelihood distributions from the joint PV, CMB and SN likelihood function.

BOOMERANG North American test flight results (Mauskopf et al. 1999 astro-ph/9911444)). We use the constraints obtained by Perlmutter et al. (1999 ApJ, 517, 565) whose sample consists of 42 high-redshift supernovae (SN) ($0.18 \le z \le 0.83$), supplemented by 18 low-redshift SN ($z < 0.1$). We assume a spatially flat universe ($\Omega_m = 1 - \Omega_\Lambda$) which begain from adiabatic scale-invariant initial conditions with negligible tensor contributions, as predicted by most inflation models. For the baryonic content we adopt the value favored by Deuterium abundance in the context of Big-Bang nucleosynthesis $\Omega_b h^2 = 0.019$ (Burles et al. 1999 PRL, 82, 4176).

In order to examine how well the constraints from PVs, CMB and SN agree with each other we plot in Fig. 1 the three corresponding iso-likelihood surfaces, at the 2-sigma level. The three data sets are nicely complementary and, despite our large number of assumptions, agree well at the 2σ level. Therefore we may combine the cosmological parameter constraints, and we marginalise over the other parameters to produce one-dimensional marginalised distributions shown in Fig. 2. The values obtained from the joint analysis for h and Ω_m, are in general agreement with other estimates but this analysis tends to favor a slightly higher value for σ_8. In particular, the result for σ_8 is higher than the Bridle et al. (1999 MNRAS 310, 565) constraint, $\sigma_8 = 0.74 \pm 0.1$ (95% confidence) obtained by combining the CMB with cluster abundance and IRAS and allowing for linear biasing. This may reflect the preference of the peculiar velocities for a slightly higher value of $\sigma_8 \Omega_m^{0.6}$ than favored by the cluster abundance analysis. This partially motivates the work of the next sections.

3 Hyper-Parameters

If data sets disagree, for example their 2-σ limits do not overlap, how should we deal with this? Ideally we would find the root of the disagreement and resolve it, but failing that two popular methods are (i) to carry on regardless and combine the constraints anyway or (ii) chose a subset of data sets that do agree with each other and ignore others. Here we propose a middle road option in which each data set is given a weighting parameter, which is treated as a nuisance parameter and marginalised out. For the case where likelihoods would conventionally be calculated by adding χ^2 values:

$$\chi^2_{\rm joint} = \chi^2_A + \chi^2_B \tag{1}$$

we generalise this to

$$\chi^2_{\rm joint} = \alpha\chi^2_A + \beta\, \chi^2_B \tag{2}$$

where α and β the weights or 'Hyper-Parameters' (HPs).

It may be shown (Lahav et al. 2000) that the new probability is given by

$$-2\, \ln P(\mathbf{w}|D_A, D_B) = N_A \ln(\chi^2_A) + N_B \ln(\chi^2_B) \tag{3}$$

where the HPs have been marginalised out.

4 Application to CMB data

We illustrate the effect of using HPs by application to measurements of the angular power spectrum of the cosmic microwave background (CMB). There are clearly a large number of possible combinations of CMB data sets that could be investigated. For the purpose of illustration we divide a selection of the current CMB power spectrum estimates into six subsets: Saskatoon, Python V, MSAM1, TOCO, BOOMERANG / NA and an early compilation which we refer to as 'Other'. See Lahav et al. (2000) for more details.

There are a large number of possible groupings of the data subsets. We show here the results from just five groupings, which are a fair sample and also highlight some of the properties of HPs. Firstly we consider the case of two relatively discrepant data sets, Saskatoon and BOOMERANG / NA; the h values that they prefer do not overlap significantly. Combining their χ^2 values for each h in the conventional manner (Eq. 1) yields the likelihood function plotted with the dotted line in Fig. 3 (Top). An intermediate value of h is preferred, and in fact the best fitting h values for each data set alone are essentially ruled out. In contrast, when HPs are used, i.e. the χ^2 values are combined using Eq. 3, the dotted line in Fig. 3 (Bottom) is obtained. There are two peaks in the probability distribution corresponding to the two different values of h preferred by each data set alone. This is perhaps closer to what we would actually believe given just these two data subsets.

Next we consider the effect of adding in a data subset that agrees strongly with one of the above two data subsets. That is, we consider TOCO with Saskatoon and BOOMERANG / NA. The probability distribution calculated using HPs now loses its second peak, retaining the one that agrees with TOCO and Saskatoon. On combining two data sets that do agree well, BOOMERANG / NA and 'Other', there is little difference between the conventional and HP analyses, although the error bar on h is slightly decreased when using HPs. Adding in a data subset that has a poor χ^2 (given the range of models considered), PythonV, makes a large difference to the conventional analysis but only a very small difference to the HP analysis. Finally we use all of the data subsets, obtaining the solid lines in Fig. 3. It turns out that the best fitting value of h is similar in both the conventional and HP analyses, but the error bars are significantly wider in the HP analysis, which corresponds better to what we would naturally believe.

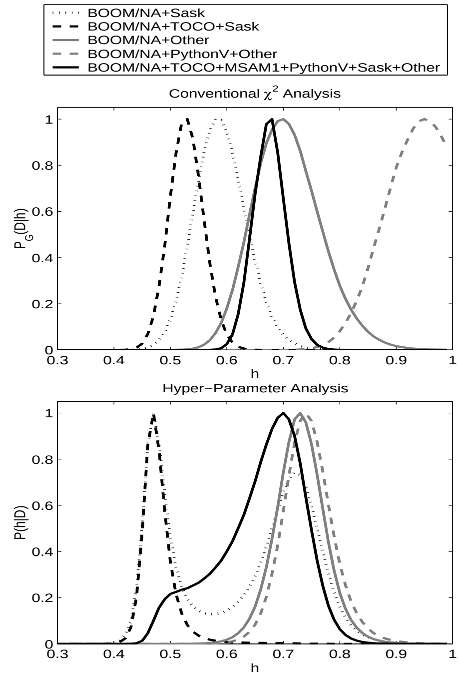

Figure 3: The probability of the Hubble constant h as a function of h from different subsets of CMB data (as indicated in the legend) resulting (Top figure) from a conventional χ^2 analysis and (Bottom figure) the Hyper-Parameter analysis, in which $P(h|D_A, D_B, \ldots) = \exp[-\frac{1}{2}\sum N_j \ln(\chi_j^2)]$.

5 Conclusions

Given a number of popular assumptions we find that constraints on $\Omega_{\rm m}$, h and σ_8 from galaxy velocities, CMB and supernovae agree at the 2-σ level. We find cosmological parameter constraints $0.24 < \Omega_{\rm m} < 0.51$, $0.49 < h < 0.77$ and $0.81 < \sigma_8 < 1.38$ at the 95 per cent confidence level. For details of a more thorough investigation see Bridle et al. (2000).

We propose a new method for combining data sets in which each data set is given a weighting, which we call a Hyper-Parameter. The result of marginalising over these weighting parameters is simple and easy to implement. We illustrate the results of this Hyper-Parameters approach by applying it to h estimation from various subsets of our CMB data compilation. The constraints are more conservative, but downweight badly fitting outliers. See Lahav et al. (2000) for more information.

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References

1. Bridle, S.L., Zehavi, I., Dekel, A., Lahav, O., Lasenby, A.N., Hobson, M.P. 2000 MNRAS, submitted (astro-ph / 0006170)
2. Lahav, O., Bridle, S.L, Hobson, M.P., Lasenby, A.N., & Sodre Jr., L. 2000 MNRAS, 315, 45 (astro-ph / 9912105)