Inflation models, spectral index and observational constraints.

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We have evaluated the observational constraints on the spectral index n, in the context of a ΛCDM model. For n scale-independent, as predicted by most models of inflation, present data require $n \simeq 1.0 \pm 0.1$ at the 2-σ level. We have also studied the two-parameter scale-dependent spectral index, predicted by running-mass inflation models. Present data allow significant variation of n in this case, within the theoretically preferred region of parameter space.

1 Introduction

It is generally supposed that structure in the Universe originates from a primordial gaussian curvature perturbation, generated by the quantum fluctuations of the inflaton field during slow-roll inflation. Then the spectrum of the curvature perturbation $\delta_H(k)$ is determined by the inflaton potential $V(\phi)$. In this paper we will consider the scale–dependence of the primordial spectrum, defined by the spectral index n:

$$n(k) - 1 \equiv 2\frac{\partial \ln \delta_H}{\partial \ln k} = 2M_{Pl}^2(V''/V) - 3M_{Pl}^2(V'/V)^2 \,, \qquad (1)$$

where[a] the potential and its derivatives are evaluated at the epoch of horizon exit $k = aH$. The value of ϕ at this epoch is given by $\ln(k_{end}/k) = N(\phi) = M_{Pl}^{-2}\int_{\phi_{end}}^{\phi}(V/V')d\phi$, where k_{end} is the scale leaving the horizon at the end of slow roll inflation and $N(\phi)$ is the number of e-folds. In the majority of the inflation models, n is practically scale-independent so that $\delta_H^2 \propto k^{n-1}$, but we shall also discuss an interesting class of models giving significant scale dependence.

[a] $M_{Pl} = 2.4 \times 10^{18}$ GeV is the Planck mass, a is the scale factor and $H = \dot{a}/a$ is the Hubble parameter, and k/a is the wavenumber.

Table 1: Fit of the ΛCDM model to presently available data. The spectral index n is a parameter of the model, as are the next four quantities. Every quantity except n is a data point, with the value and uncertainty listed in the first two rows taken from the references in superscript. The result of the least-squares fit is in the lines three to five for $z_R = 20$. All uncertainties are at the nominal 1-σ level. The total χ^2 is 2.4 for 2 degrees of freedom.

	n	$\Omega_b h^2$	Ω_0	h	$10^5 \tilde{\delta}_H$	$\tilde{\Gamma}$	$\tilde{\sigma}_8$	$\sqrt{\tilde{C}_{\rm peak}}$
data	—	0.019[4]	0.35[1]	0.65[1]	1.94[5]	0.23[2,6]	0.56[7]	80 μK [8]
error	—	0.002	0.075	0.075	0.075	0.035	0.055	10 μK
fit	1.01	0.019	0.36	0.63	1.95	0.19	0.58	72 μK
error	0.05	0.002	0.06	0.06	0.075	—	—	—
χ^2	—	4×10^{-5}	1×10^{-2}	0.1	5×10^{-3}	1.3	0.2	0.8

2 The observational constraints on the ΛCDM model

In the interest of simplicity and due to present observations[1], we adopt the ΛCDM model, with $\Omega_{tot} = 1$ and cold non-baryonic dark matter with negligible interaction. We shall constrain this model, including the spectral index, by performing a least-squares fit to the key observational quantities.

The parameters of the ΛCDM model are the primordial spectrum $\delta_H(k)$, the Hubble constant h (in units of $100\,{\rm km\,s^{-1}\,Mpc^{-1}}$), the total matter density Ω_0, the baryon density Ω_b, and the reionization redshift z_R (we consider complete and sudden reionization). z_R can be estimated in terms of the other parameters because it can be related to the density perturbation and the the fraction of collapsed matter f at the epoch of reionization, so we exclude it from the least-squares fit. In the case of the constant n models we fix it at a reasonable value ($z_R = 20$), while in the case of the running mass models we compute it assuming that reionization occurs when a fixed fraction of the matter $f \simeq 1$ collapses. The spectrum is conveniently specified by its value at the COBE scale $k_{COBE} = 6.6 H_0$, and the spectral index $n(k)$. Excluding z_R, the ΛCDM model is therefore specified by five parameters in the case of a constant spectral index, or by six parameters in the case of running mass inflation models.

Taking as our starting point a study performed three years ago[2], we consider seven observational quantities: the cosmological quantities h, Ω_0, Ω_B, which we are also taking as free parameters, and moreover the shape parameter Γ, σ_8, the COBE normalization and the first peak height in the cmb anisotropy. The adopted values and errors are given in the second and third line of Table 1. For a discussion of the data, see[3]. In common with earlier investigations, we assume the errors to be uncorrelated and random errors to dominate over systematic ones.

3 Results

We perform the least–squares fit with the CERN Minuit package; the quoted error bars use the parabolic approximation, while the exact errors computed by Minuit agree with the approximated ones to better than 10%.

In order to simplify the numerical procedure, we follow[9] and parameterize the predicted value of $\sqrt{\tilde{C}_{\rm peak}}$ with the analytical formula $\sqrt{\tilde{C}_{\rm peak}} = 77.5\,\mu{\rm K} \left(\frac{\delta_H(k_{COBE})}{1.94 \times 10^{-5}}\right) \left(\frac{220}{10}\right)^{\nu/2}$ where

$$\nu \equiv 0.88(n_{COBE} - 1) - 0.37 \ln(h/0.65) - 0.16 \ln(\Omega_0/0.35) + 5.4 h^2 (\Omega_b - 0.019) - 0.65 g(\tau)\tau \quad (2)$$

and $\tau = 0.035 \frac{\Omega_b}{\Omega_0} h \left(\sqrt{\Omega_0 (1 + z_R)^3 + 1 - \Omega_0} - 1\right)$. The formula is fitted to the CMBfast[10] results and agrees within 10% for a 1-σ variation of the cosmological parameters, h, Ω_0 and Ω_b, and

$n = 1.0 \pm 0.05$. With the function $g(\tau)$ set equal to 1, the formula contains the usual factor $\exp(-\tau)$. By fitting the output of CMBfast, we introduce also $g(\tau) = 1 - 0.165\tau/(0.4 + \tau)$, so that our formula is accurate to a few percent over the interesting range of τ.

Constant spectral index. For the case of a constant spectral index our result is given in Table 1 for $z_R = 20$. In the case of no reionization ($z_R = 0$) we obtain a slightly smaller spectral index, $n = 0.98 \pm .05$, and cosmological parameters within the observational error bar, in agreement with previous analysis[11]. This result is not enough yet to exclude completely proposed inflationary models, but a better determination of the peak height could strengthen the bound sufficiently to discriminate between them, especially in the case of new inflation models, which give low values of n[3].

Running mass models. We have also considered the scale-dependent spectral index, predicted in inflation models with a running inflaton mass[12]. In these models, one–loop corrections to the potential are taken into account by evaluating the scale dependent inflaton mass $m^2(Q)$ at $Q \simeq \phi$. Then the spectral index can be parameterized by just two quantities:

$$\frac{n(k) - 1}{2} = \sigma e^{-cN(\phi)} - c \,, \tag{3}$$

where σ is an integration constant and c is related to the inflaton coupling responsible of the mass running. The different signs of σ and c give raise to four different models of inflation. In general, without fine tuning, we expect

$$|c| \lesssim |\sigma| \lesssim 1 \qquad |c| \simeq g^2 \frac{\tilde{m}^2 M_{Pl}}{V_0} \tag{4}$$

with g denoting the gauge or Yukawa coupling of the inflaton, $\tilde{m}^2$ the soft supersymmetry breaking mass of the particles in the loop and V_0 the value of the potential energy during inflation. With gravity-mediated susy breaking, typical values of the masses are $\tilde{m}^2 \sim V_0/M_{Pl}^2$, which makes c of order of the coupling strength. For a gauge coupling, or an unsuppressed Yukawa coupling, we expect $|c| \sim 10^{-1}$ to 10^{-2}.

Extremizing with respect to all other parameters, we have computed the χ^2 in the σ vs. c plane and obtained contour levels for χ^2 corresponding to the 70% and 95% confidence level in two variables. The results are shown in Figure 1.

In the case of Models (ii) and (iv), the allowed region corresponds to $|c|$ and $|\sigma|$ both small, giving a practically scale-independent spectral index, with a red and blue spectrum respectively.

In contrast, the allowed region for Models (i) and (iii) allows strong scale-dependence. In Model (i), a large departure from a constant spectral index is allowed for large σ; for the theoretically favored value $\sigma \sim 1$ the variation between k_{COBE} and $8^{-1}h\text{Mpc}^{-1}$ can be as large as 0.05, while the maximal change allowed by the data is 0.2. For Model (iii), a much larger departure from a constant spectral index is allowed, but in the theoretically favored regime $|\sigma| \geq c$ one again finds a variation of at most 0.05.

4 Conclusion

We have evaluated the observational constraints on the spectral index n, using a range of data, and we find, for constant n at 2-σ level, $0.88 \leq n \leq 1.11$ for $0 \leq z_R \leq 20$.

We have also investigated the running mass models, parameterized by c and σ. For c and σ with the same sign, we have found that indeed n can vary by about 0.05 between the COBE scale and $8h^{-1}$Mpc. Moreover, if c is positive as it would be for a gauge coupling, $n - 1$ *can change sign between the COBE and* $8h^{-1}$ *Mpc scales.* It will be very interesting to see how the present situation changes with the advent of better data.

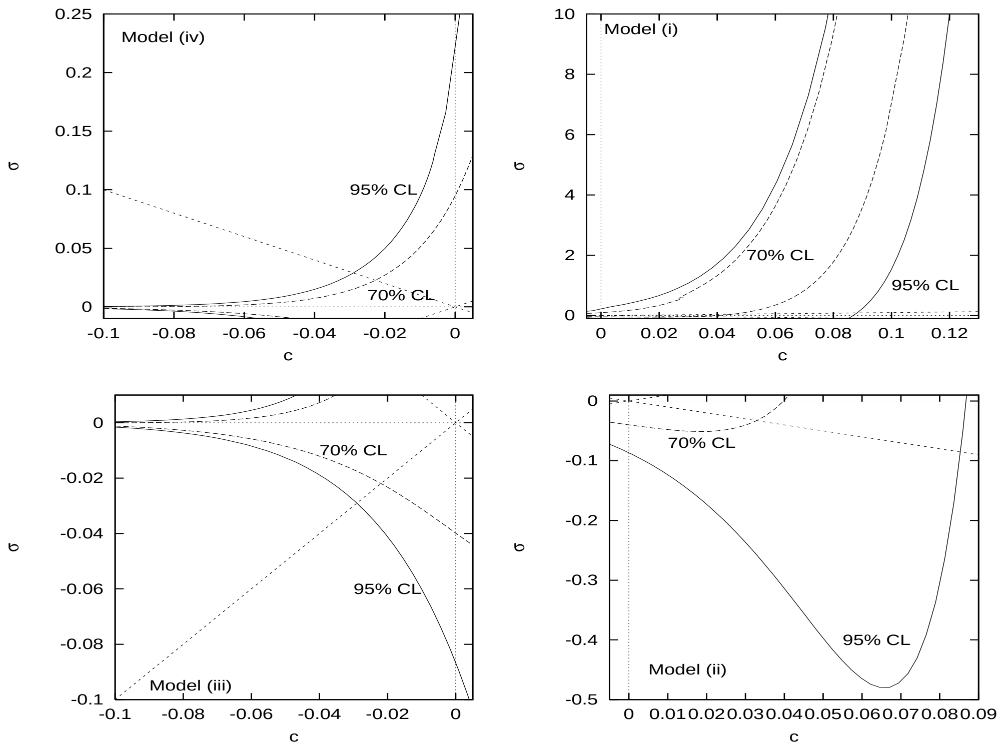

Figure 1: Allowed region in the σ vs c plane for $N_{COBE} = 50$; the solid line is the 95% CL contour, while the dashed line the 70% CL. The theoretically favoured region is above (below) the dotted line $|\sigma| = |c|$ in the upper (lower) half plane. For positive c and σ the contours close at $\sigma \sim 200, c \simeq 0.15$ and $\sigma \sim 1250, c \simeq 0.19$, for the 70% and 95% CL respectively.

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