COSMOLOGY ON A 3-BRANE

C. DEFFAYET
Laboratoire de Physique Théorique, Bâtiment 210,
Universit Paris XI, 91405 Orsay Cedex, France

I consider the cosmology of a "3-brane universe" in a five dimensional (bulk) space-time with a cosmological constant. After deriving, in a nutshell, the analogous of the Friedmann equations for this system. I discuss various consequences, in particular the cosmology of the Randall-Sundrum scenario.

1 Introduction

A lot of interest has recently been raised by the possibility that our three-dimensional world could be a surface (3-brane) embedded in a higher dimensional space.[1] In contrast with the old Kaluza-Klein approach, these recent developpements are based on the idea that standard model fields could be confined to a three-dimensional world, corresponding to our apparent Universe, while gravity could live in a higher dimensional space. This relaxes the usual constraints on Kaluza-Klein models[1] and extra dimensions up to a *mm* size are conceivable. In the same spirit, Randall and Sundrum[2] reviving an older idea,[3] have recently pointed out that the extra dimension (single in their case) need not even be compact. In their scenario, our world is viewed as embedded in a particular slice of five dimensional anti-de Sitter space-time.

Several groups have begun to work on possible experimental signatures of these kinds of models. Although less directly, one would expect that cosmology could provide a lot of interesting constraints, as well as a way to better understand the gravitational dynamics in these scenarios. A first crucial question is the viability of such models with respect to the cosmological evolution of our Universe. As shown in our work,[4] the equations governing the cosmological evolution of the brane (that I will call here *brane Friedmann equations*) will, under minimal assumptions, be different from the analogous Friedmann equations of standard cosmology (see also[5]). Essentially, the difference lies in the fact that the energy density of the brane appears quadratically in the brane Friedmann equations in contrast with the linear behaviour of the usual equations. In the simplest model where the bulk is supposed to be empty, we found explicit solutions which showed that the corresponding cosmological scenario would be incompatible with nucleosynthesis constraints. In the first part of this talk I will very briefly review the derivation of the brane Friedmann equations in the particular case of a bulk spacetime with a cosmological constant.[6] Different ways were proposed to reconcile brane cosmology with the required standard cosmological scenario (at least since nucleosynthesis), introducing in particular stabilization mechanisms.[7,8] I will not discuss here these interesting proposals, but concentrate on the Randall Sundrum scenario[2] where no additional feature seems to be necessary, as noticed by two groups.[9] Our recent work[6] enables indeed to solve explicitely for the cosmological evolution of the Randall Sundrum scenario as will be shown in section 3. I will also discuss briefly some other phenomenological consequences of the equations obtained in section 2. I refer the interested reader to the original work[4,6] for more details. In particular we discuss there solutions over the whole space time, which are not discussed in this talk.

2 Brane Friedmann equations

We consider five-dimensional spacetime metrics of the form

$$ds^2 = -n^2(\tau,y)d\tau^2 + a^2(\tau,y)\gamma_{ij}dx^i dx^j + b^2(\tau,y)dy^2, \tag{1}$$

where y is the coordinate of the fifth dimension and τ is the time coordinate, γ_{ij} is a maximally symmetric 3-metric whose curvature is parametrized by an integer k ($k = -1, 0, 1$). We will focus our attention on the hypersurface defined by $y = 0$, which we identify with the world volume of the brane that forms our universe. The five-dimensional Einstein equations take the usual form $\tilde{G}_{AB} = \kappa^2 \tilde{T}_{AB}$ where $\tilde{G}_{AB}$ is the five-dimensional Einstein tensor and the constant κ is related to the the five-dimensional reduced Planck mass $M_{(5)}$, by the relation $\kappa^2 = M_{(5)}^{-3}$. We decompose the stress-energy-momentum tensor into two parts, $\tilde{T}^A{}_B = \check{T}^A{}_B|_{\rm bulk} + T^A{}_B|_{\rm brane}$. Where $\check{T}^A{}_B|_{\rm bulk}$ is the energy momentum tensor of the bulk "matter", which will be assumed to be a cosmological constant. And the second term $T^A{}_B|_{\rm brane}$ corresponds to the matter content in the brane $(y = 0)$. Since we consider here only strictly homogeneous and isotropic geometries inside the brane, the latter can be expressed quite generally in the form

$$T^A{}_B|_{\rm brane} = \frac{\delta(y)}{b}\,\mathrm{diag}\,(-\rho_b, p_b, p_b, p_b, 0)\,, \tag{2}$$

where the energy density ρ_b and pressure p_b are independent of the position inside the brane, i.e. are functions only of time.

The assumption that $\tilde{T}_{\tau y} = 0$, which physically means that there is no flow of matter along the fifth dimension, implies that $\tilde{G}_{\tau y}$ vanishes. It then turns out, remarkably, that the component (τ,τ) and (y,y) of Einstein's equations in the bulk, can be integrated to give

$$\left(\frac{\dot a}{na}\right)^2 = \frac{1}{6}\kappa^2\rho_B + \left(\frac{a'}{ba}\right)^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}, \tag{3}$$

where $\mathcal{C}$ is an integration constant, a dot means ∂_τ, a prime ∂_y and ρ_B is the energy density in the bulk. One can then show[a] that any set of functions a, n, and b satisfying (3) together with $\tilde{G}_{\tau y} = 0$, will be solution of all Einstein's equations, locally in the bulk[b].

The brane can then be taken into account by using the junction conditions,[10] which simply relate the jumps of the derivative of the metric across the brane to the stress-energy tensor

[a] using the Bianchi identity,

[b] one need also to assume that the $\dot a \neq 0$.

inside the brane. Assuming the symmetry $y \leftrightarrow -y$ for simplicity, the junction condition can be used to compute a' on the two sides of the brane, and by continuity when $y \to 0$, (3) will yield the generalized brane (first) Friedmann equation:

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^2}{6}\rho_B + \frac{\kappa^4}{36}\rho_b^2 + \frac{\mathcal{C}}{a_0^4} - \frac{k}{a_0^2}, \tag{4}$$

where a subscript 0 means that the corresponding function is taken at $y = 0$ (and we have set $n_0 = 1$ by a suitable redefinition of time). This equation together with the equation of conservation of the usual form[c] $\dot{\rho}_b + 3(p_b + \rho_b)\dot{a}_0/a_0 = 0$, are the starting point for studying the cosmological evolution in the brane, independently of the value of the metric in the bulk and in particular of the time evolution of b.

3 Cosmological scenarios

The salient features of equation (4) are: (i) the bulk energy density enters linearly, (ii) the brane energy density enters quadratically, (iii) the coefficient multiplying the brane energy density does not depend on the radius of the extra dimension, and (iv) the cosmological evolution depends on a free parameter $\mathcal{C}$ (related to the choice of initial conditions in the whole space-time; see also[11]) whose influence corresponds to an effective radiation term.

Let us now assume that the energy density in the brane can be decomposed into two parts $\rho_b = \rho_\Lambda + \rho$, where ρ_Λ is a constant that represents an intrinsic tension of the brane and ρ stands for the ordinary energy density in cosmology. Substituting in (4) one gets

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^2}{6}\rho_B + \frac{\kappa^4}{36}{\rho_\Lambda}^2 + \frac{\kappa^4}{18}\rho_\Lambda\rho + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a_0^4} - \frac{k}{a_0^2}. \tag{5}$$

If we follow Randall and Sundrum[2] by choosing ρ_Λ such that $\kappa^2\rho_B/6 + \kappa^4{\rho_\Lambda}^2/36 = 0$, then one sees that standard cosmology is recovered with the identification[9] $8\pi G_{Newton} \simeq \kappa^4\rho_\Lambda/6$, when $\rho \ll \rho_\Lambda$. Let us however keep ρ_Λ unspecified at this stage.

To get analytic solutions we will now assume that ordinary matter is described by an equation of state of the form $p = w\rho$, with w constant. Then, using the conservation equation for ρ and p, one can write $\rho = \rho_*(a_0/a_*)^{-q}$ with $q = 3(1+w)$ and where ρ_* and a_* are constants. Inserting this expression in (5), it is possible to integrate explicitly the resulting equation in the case where $\mathcal{C} = 0$ and $k = 0$. Assuming the first term on the right hand side of (5) to be positive and defining

$$\lambda = \sqrt{\frac{\rho_B}{6\kappa^2} + \frac{{\rho_\Lambda}^2}{36}}, \tag{6}$$

one finds, for $\lambda > 0$,

$$a_0 = a_*\rho_*^{1/q}\left\{\frac{\rho_\Lambda}{36\lambda^2}\left[\cosh\left(q\kappa^2\lambda t\right) - 1\right] + \frac{1}{6\lambda}\sinh\left(q\kappa^2\lambda t\right)\right\}^{1/q}, \tag{7}$$

and, for $\lambda = 0$,

$$a_0(t) = a_*\left(\kappa^2\rho_*\right)^{1/q}\left(\frac{q^2}{72}\kappa^2\rho_\Lambda t^2 + \frac{q}{6}t\right)^{1/q} \tag{8}$$

(the origin of time being chosen so that $a_0(0) = 0$). It is clear, from the latter expression, in the case $\lambda = 0$, how one passes from a very early universe, characterized by a non-conventional evolution $a(t) \sim t^{1/q}$, to a late time phase described by standard cosmology, $a(t) \sim t^{2/q}$. In the

[c] which can be obtained through the (τ, y) component of the Einstein's equations.

case $\lambda > 0$, with λ sufficiently small, one obtains three successive phases, a non-conventional phase dominated by ρ^2, a conventional phase dominated by ρ and, finally, an exponential phase, where λ plays the rôle of an effective cosmological constant in our Universe.

Let us finally examine the case of a universe filled with radiation ($w = 1/3$). It is then possible, still with $k = 0$, to integrate explicitly the case $\mathcal{C} \neq 0$, because the $\mathcal{C}$-term has the same dependence on a_0 as the term proportional to ρ in (5). Moreover, the free parameter $\mathcal{C}$ can be constrained by nucleosynthesis (see also R. Battye's talk in this volume). Indeed, at the time of nucleosynthesis, the universe is dominated by the radiation energy density, which can be written

$$\rho_{rad}(t_N) = g_* \frac{\pi^2}{30} T_N^4, \tag{9}$$

where g_* is the effective number of relativistic degrees of freedom at that time. In the standard model, $g_*(standard) = 10.75$, and any deviation Δg_* is strongly constrained by the observed abundances of light elements, typically $\Delta g_* < 2$. In our model, since the additional $\mathcal{C}$-term evolves like radiation, it can also be seen effectively as additional relativistic degrees of freedom, subject to the usual constraint, so that at nucleosynthesis

$$\rho_{\mathcal{C}}(t_N) \equiv \frac{3\mathcal{C}}{8\pi G a^4(t_N)} \leq \frac{\pi^2}{15} T_N^4. \tag{10}$$

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