

CURRENT STATUS OF COSMOLOGICAL MDM MODELS

E.V. MIKHEEVA, V.N. LUKASH, N.A. ARKHIPOVA, A.M. MALINOVSKY
*Astro Space Center of Lebedev Physical Institute,
84/32 Profsoyuznaya, Moscow 117810, Russia*



An analysis of cosmological models in spatially flat Friedmann Universe with cosmic gravitational wave background and zero Λ -term is presented. The number of parameters is equal to 5, they are (1) σ_8 , the dispersion of the mass fluctuations in the sphere with radius $8h^{-1} Mpc$, (2) n , the slope of the density perturbation spectrum, (3) Ω_ν , the density of hot dark matter, (4) Ω_b , the density of baryons, and (5) h , the Hubble constant $H_0 = 100h \text{ km s}^{-1} Mpc^{-1}$. The cold dark matter density parameter is equal to $\Omega_{cm} = 1 - \Omega_\nu - \Omega_b$. Our analysis is based on the confrontation with the number density and mass function of clusters of galaxies and CMBR anisotropy. The implication of Press-Schechter formalism allowed us to constrain $\sigma_8 = 0.52 \pm 0.04$. This normalization of the spectrum of density perturbations has been used to calculate numerically the value of the large scale CMB anisotropy ($\ell \simeq 10$) and the relative contribution of cosmological gravitational waves T/S. Increasing Ω_ν weaken the requirements to the value of T/S, however even for $\Omega_\nu \leq 0.4$ the models with $h + n \geq 1.5$ suggest considerable abundance of gravitational waves: $T/S \gtrsim 0.3$. In models with $\Omega_\nu \leq 0.4$ and scale-invariant spectrum of density perturbations ($n = 1$): $T/S \gtrsim 10(h - 0.47)$. Minimization of the value T/S is possible only in the range of the red spectra ($n < 1$) and small h (< 0.6). However, the parameter Ω_ν is strongly constrained by $\Delta T/T$ data on the first acoustic peak of Sacharov oscillations ($\ell \simeq 200$). Assuming that $T/S \in [0, 3]$ and taking into account observational data on the primordial nucleosynthesis and the amplitude of the acoustic peak we constrain the model parameters Ω_ν and n : it is shown that the models considered admit both moderate red and blue spectra of density perturbations, $n \in [0.9, 1.2]$, with rather high abundance hot dark matter, $\Omega_\nu \in [0.2, 0.4]$. Any condition, $n < 0.9$ or $\Omega_\nu < 0.2$, decreases the relative amplitude of the first acoustic peak for more than 30% in comparison with its height in the standard CDM normalized by COBE data.

1 Model description

We considered a family of models with the following free parameters:

- $\sigma_8 \in [0.47, 0.61]$, (15 models with step 0.01);

- $n \in [0.8, 1.4]$, (7 models with step 0.1);
- $\Omega_\nu \in [0, 0.4]$, (5 models with step 0.1);
- $\Omega_b \in [0.01, 0.11]$, (6 models with step 0.02);
- $h \in [0.45, 0.7]$, (6 models with step 0.05).

We use the analytic approximation of the transfer function by Novosyadlyj et al (1998). Altogether we have 18900 variants of the model. The derived parameters are abundance of the cold matter, $\Omega_{cm} = 1 - \Omega_\nu - \Omega_b$, and the contribution of tensor mode to large-scale CMBR anisotropy, T/S. Our goal is to constrain the model parameters by data of the mass function of galaxy clusters and $\Delta T/T$ anisotropy in both large ($\ell \sim 10$) and small ($\ell \sim 200$) angular scales.

2 Mass function of galaxy clusters

The number of massive halos with the mass larger then M is calculated with help of Press-Schechter formalism². Observational data are taken from Bahcall & Cen paper³.

The χ^2 analysis allows us to delimit the amplitude of the power spectrum at cluster scale with high accuracy, $\sigma_8 = 0.52 \pm 0.01$; taking into account the uncertainties of the Press-Schechter approximation and experimental systematics enhances the total errorbar by a value of $0.04^4, ^5$. Other parameters ($n, \Omega_\nu, \Omega_b, h$) are not constrained within their ranges by the cluster data.

3 CMBR Anisotropy

The contribution of cosmic gravitational waves into the large-scale CMBR anisotropy is estimated by the T/S parameter:

$$\left\langle \left(\frac{\Delta T}{T} \right)^2 \right\rangle_{10^\circ} = S + T = S \left(1 + \frac{T}{S} \right) \simeq 1.1 \times 10^{-10},$$

where S is the contribution of the perturbations of matter density normalized by $\sigma_8 = 0.52$:

$$S = \sum_{\ell=2}^{\infty} S_\ell W_\ell, \quad S_\ell = \frac{2\ell+1}{64\pi} A H_0^{n+3} \frac{\Gamma(3-n)\Gamma(\ell+(n-1)/2)}{\Gamma^2(2-n/2)\Gamma(\ell+(5-n)/2)}, \quad W_\ell = \exp \left[- \left(\frac{2\ell+1}{27} \right)^2 \right],$$

A and W_ℓ are the normalization constant and DMR window function, respectively. The accuracy of this approximation is better than 3%, the harmonics with $\lesssim 10$ ensure the dominant contribution. The result of calculation of $T/S \in [0, 3]$ is presented in Fig.1.

The value T/S increases linearly with h and decreases with Ω_ν growing, therefore the curves T/S with the maximum parameter $\Omega_\nu = 0.4$ can be used to put the lower limit on T/S (see Conclusions).

Taking a moderate $T/S < 0.5$ and nearly flat power spectrum ($0.92 \leq n \leq 1.02$), we put an upper limit on the Hubble constant, $h < 0.6$, and lower limit on the hot dark matter abundance, $\Omega_\nu > 0.1$. However, the hardest constraint for the parameter Ω_ν can be got when we confront the amplitude of the first acoustic peak in $\Delta T/T$ ($\ell \simeq 200$) with the observational data.

We compare the hight of the acoustic peak generated in our models with its value in the standard CDM (without gravitational waves) normalized by the COBE data. The parameter for such a comparison is the relative amplitude of the peak, $\mathfrak{R} \equiv \mathfrak{R}_{\ell=200}/1.1 \times 10^{-10}$, where $\mathfrak{R}_\ell \equiv \ell(\ell+1)S_\ell/(\ell+0.5)$; $\mathfrak{R} = 5.1$ for sCDM.

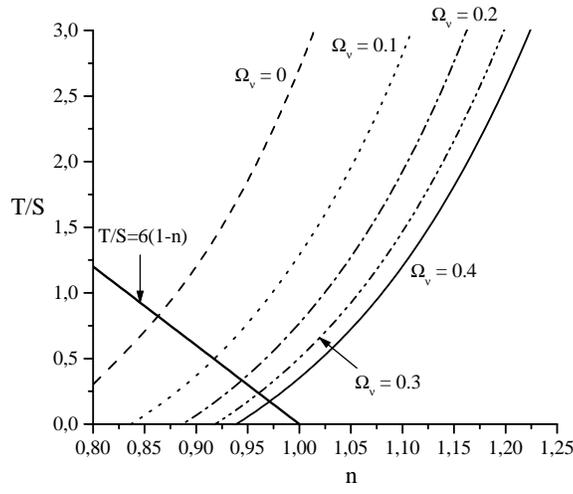


Figure 1: T/S as a function of n and Ω_ν ($\sigma_8 = 0.52$, $\Omega_b = 0.05$, $h = 0.5$).

Evidently, \mathfrak{R} decreases with T/S growing (and other parameters fixed). E.g. for CDM models ($\Omega_\nu = 0$ and 'standard' values for n , Ω_b and h) the relative amplitude of the acoustic peak decays by a factor $T/S+1 \simeq 4$.

The more efficient ways to enhance the acoustic peak is a transition to the 'red' power spectra and/or high abundance of the hot matter. The role of the 'blue' spectra becomes important when the parameter Ω_ν rises up (since the 'red' spectra will violate the condition $T/S \geq 0$). The results of the derivation are presented in Fig.2. The condition for a 'considerable' acoustic peak ($\mathfrak{R} \geq 4$) with the standard BBN constraint for the baryonic density, leaves us with a broad set of the power spectra ($n \in [0.9, 1.2]$) but requires high fraction of the hot matter ($\Omega_\nu \in [0.2, 0.4]$) in the class of the models considered.

4 Conclusions

- The data on the galaxy cluster abundance determine the value σ_8 with a high accuracy, the other parameters (n , Ω_ν , Ω_b , h) remain free within their ranges.
- None of the MDM models with $n = 1$ and $T/S = 0$ satisfies both normalizations, on the galaxy cluster abundance and large-scale $\Delta T/T$ anisotropy, which leads either to rejection from the flat spectrum or to the introduction of a non-zero T/S (or both).
- Small values of T/S are realised for the red spectra ($n < 1$) and moderate $h (< 0.6)$, the violation of these conditions leads to a high $T/S (\gtrsim 1)$.
- Increasing Ω_ν weakens the requirement to the value of T/S , however even for $\Omega_\nu \leq 0.4$ the models with $h + n \geq 1.5$ suggest considerable abundance of gravitational waves: $T/S \gtrsim 0.3$.
- In models with $\Omega_\nu \leq 0.4$ and scale-invariant spectrum of density perturbations ($n = 1$): $T/S \gtrsim 10(h - 0.47)$.
- In models with $\Omega_b = 0.05$ and $h = 0.5$ we have the following approximation for the primordial gravitational waves (the accuracy is better than 11 % for $0.1 \leq T/S \leq 3$):

$$\frac{T}{S} = \frac{30(n - 0.7)^2}{10\Omega_\nu + 1} + 10\Omega_\nu (n^{3/2} - 1.06).$$

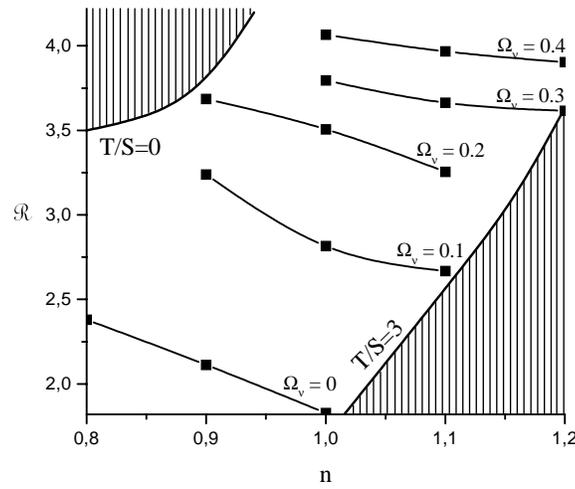


Figure 2: \mathfrak{R} as a function n for different values of Ω_ν ($\Omega_b = 0.05$, $h = 0.5$, $\sigma_8 = 0.52$). Models in the non-shaded region have $T/S \in [0, 3]$.

- In double-normalized models with $T/S > 0$ the height of the acoustic peak is less than its 'standard' value ($\mathfrak{R} = 5.1$). The decrease of the parameter \mathfrak{R} does not exceed 30% in models with large $\Omega_\nu \in [0.2, 0.4]$ and any spectrum slope, $n \in [0.9, 1.2]$. Any condition, $n < 0.9$ or $\Omega_\nu < 0.2$, decreases the relative amplitude of the first acoustic peak for more than 30% (i.e. $\mathfrak{R} < 3.5$ in models with $\Omega_b = 0.05$, $h = 0.5$). The acoustic peak practically disappears in CDM models.
- When increasing the baryonic abundance the difference between \mathfrak{R} in our models and that in Λ CDM decreases. The amplitude of the acoustic peak coincides with its 'standard' value ($\mathfrak{R} \geq 4.5$) in models with $\Omega_b = 0.1$ and either 'blue' spectrum $n \in [1, 1.2]$ and $\Omega_\nu \geq 0.3$, or moderate 'red' spectrum $n \in [0.9, 1]$ and $\Omega_\nu \geq 0.2$.
- Thus, rising the parameter Ω_ν up to the values in the interval $[0.2, 0.4]$ solves the problem of the first acoustic peak in $\Delta T/T$, leaving the baryonic density within the primordial nucleosynthesis constraints.

Acknowledgments

The work was partly supported by Swiss National Foundation (SNSF 71P 050163.96/1), INTAS grant (97-1192), and Russian Foundation "Development and Support of Radioastronomy Scientific and Educational Center" (N 315). E.V.M., V.N.L. and N.A.A. are grateful to the Organizing Committee for the hospitality.

References

1. B. Novosyadlyj, R. Durrer, V.N. Lukash, *A&A* **347**, 799 (1999).
2. W.H. Press, P. Schechter, *ApJ* **187**, 425 (1974).
3. N.A. Bahcall, R. Cen, *ApJ* **407**, L49 (1993).
4. V.R. Eke, S. Cole, C.S. Frenk, *MNRAS* **282**, 263 (1996).
5. S. Borgani, M. Girardi, R.G. Carlberg, astro-ph/9907323.