DO WAVELETS REALLY DETECT NON-GAUSSIANITY IN THE 4-YEAR COBE DATA?

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We investigate the detection of non-Gaussianity in the 4-year COBE DMR-DSMB data reported by Pando, Valls-Gabaud & Fang (1998), using a technique based on the discrete wavelet transform. The analysis is performed on the two DMR faces centred on the North and South Galactic poles respectively, using the Daubechies 4 wavelet basis. We show that the results depend critically on the orientation of the data, and so should be treated with caution. For two distinct orientations, we calculate unbiased estimates of the skewness, kurtosis and scale-scale correlation of the corresponding wavelet coefficients in all of the available scale domains of the transform. We obtain several detections of non-Gaussianity at greater than the 99 per cent confidence level, but most of these occur on pixel-pixel scales and are therefore not cosmological in origin. Indeed, after removing all multipoles beyond $\ell = 40$ from the maps, only one robust detection remains. Moreover, using Monte-Carlo simulations, we find that the probability of obtaining such a detection by chance is 0.59. Thus, we conclude the wavelet technique does *not* yield strong evidence for non-Gaussianity of cosmological origin in the 4-year COBE DMR-DSMB data.

1 Introduction

Two apparently robust detections of non-Gaussianity in the 4-year COBE data have recently been reported [1,2]. The first applied a technique based on the normalised bi-spectrum to the 53+90 GHz inverse noise-variance weighted map, with an extended Galactic cut[3]. It was found that Gaussianity could be rejected at the 98 per cent confidence level, with the dominant non-Gaussian signal concentrated near the multipole $\ell = 16$. This non-Gaussian signal is certainly present in the COBE data, but has now been shown not to be cosmological in origin, but most likely the result of an observational artefact[4]. Nevertheless, an extended bi-spectrum analysis, has recently yielded a new non-Gaussian signal above the 97 per cent level, even after removing the observational artefact[5].

A second detection of non-Gaussianity was reported by Pando, Valls-Gabaud & Fang[2] (hereinafter PVF), who applied a technique based on the discrete wavelet transform (DWT) to Face 0 and Face 5 of the QuadCube pixelisation of the DMR-DSMB maps in Galactic coordinates. This map is constructed by first subtracting templates of synchrotron and dust emission and then removing free-free emission[6]. PVF computed the skewness, kurtosis and scale-scale correlation of the wavelet coefficients in certain domains of the wavelet transform, and compared these statistics with the corresponding probability distributions computed from 1000 realisations of simulated COBE observations of a Gaussian CMB sky. In particular, they found that the scale-scale correlation coefficient showed evidence for non-Gaussianity at the 99 per cent confidence level on scales of 11–22 degrees in Face 0 of the DMR-DSMB map.

In this paper, we also apply to the 4-year COBE data a non-Gaussianity test based on the skewness, kurtosis and scale-scale correlation of the wavelet coefficients. In the analysis presented here, however, we calculate the skewness and kurtosis statistics using *unbiased* estimators based on k-statistics[7], as opposed to the straightforward calculation of sample moments employed by PVF. For the scale-scale correlation, we adopt the same definition as that used by PVF. We also note that the analysis presented hre is slightly more general than that presented by PVF, since we calculate the statistics of the wavelet coefficients in *all* the available domains of the wavelet transform, as opposed to using only those regions that represent structure in the maps on the same scale in the horizontal and vertical directions.

Perhaps the most important point addressed in the analysis presented here, however, is the fact that non-Gaussianity tests based on any orthogonal compactly-supported wavelet decomposition are sensitive to the orientation of the input map. As an illustration of this point, we therefore present the results of two separate analyses, in which the relative orientations of the input maps differ by 180 degrees. Nevertheless, the general conclusions concerning non-Gaussianity of the COBE data do not depend on orientation.

2 The wavelet non-Gaussianity test

The two-dimensional discrete wavelet transform (DWT) performs the decomposition of a planar digitised $2^{J_1} \times 2^{J_2}$ image of side L into the sum of a set of two-dimensional planar (digitised) wavelet basis functions[8,9]

$$\frac{\Delta T}{T}(\mathbf{x}_i) = \sum_{j_1=0}^{J_1-1} \sum_{j_2=0}^{J_2-1} \sum_{l_1=0}^{2^{j_1}-1} \sum_{l_2=0}^{2^{j_2}-1} b_{j_1,j_2;l_1,l_2} \psi_{j_1,j_2;l_1,l_2}(\mathbf{x}_i). \tag{1}$$

The scale indices j_1 and j_2 correspond to the scales $L/2^{j_1}$ and $L/2^{j_2}$ in the x- and y-directions respectively, whereas the location indices l_1 and l_2 correspond to the (x, y)-position $(Ll_1/2^{j_1}, Ll_2/2^{j_2})$ in the image. Since each wavelet basis function $\psi_{j_1,j_2;l_1,l_2}(x, y)$ is localised at the relevant scale / position, the corresponding wavelet coefficient $b_{j_1,j_2;l_1,l_2}$ measures the amount of signal in the image at this scale and position. We also define the integer variable $k = 2^{j_1} + 2^{j_2}$, which serves as a measure of inverse scale length, and is constant within each (j_1, j_2) region of the wavelet domain.

For each value of k, we use the wavelet coefficients to calculate estimators of the skewness $\hat{S}(k)$, (excess) kurtosis $\hat{K}(k)$ and scale-scale correlation $\hat{C}^2(k)$ of the parent distribution from which the coefficients were drawn[10]. We therefore obtain $\hat{S}(k)$, $\hat{K}(k)$ and $\hat{C}^2(k)$ 'spectra' of the wavelet coefficients of the COBE map. We then calculate the corresponding spectra for 5000 equivalent 4-year COBE maps in which the CMB signal is a realisation of an inflationary / CDM model with parameters $\Omega_{\rm m} = 1$, $\Omega_\Lambda = 0$, $h = 0.5$, $n = 1$ and $Q_{\rm rms-ps} = 18\ \mu$K (although the our results are insensitive to the assumed form of the CMB power spectrum). Since the orthogonal wavelet decomposition is sensitive to the orientation of the input map, we also repeat the entire non-Gaussianity test for the case where Face 0 and Face 5 are both rotated through 180 degrees.

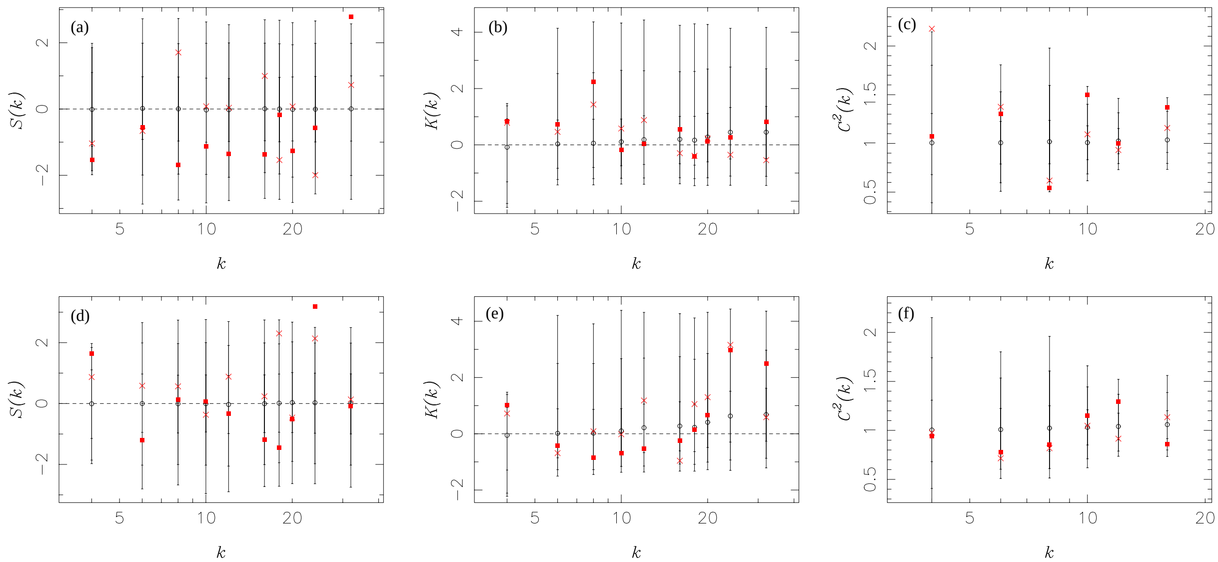

Figure 1: The $\hat{S}(k)$, $\hat{K}(k)$ and $\hat{\mathcal{C}}^2(k)$ spectra for Face 0 [plots (a), (b) and (c)] and Face 5 [plots (d), (e) and (f)] of the DMR-DSMB map. The crosses correspond to the orientation used by PVF (orientation A), whereas the solid squares correspond to the map rotated through 180 degrees (orientation B). The open circles show the mean value for each statistic obtained from 5000 simulated COBE observations of CDM realisations, and the error bars denote the 68, 95 and 99 per cent limits of the corresponding distribution. For convenience, the $\hat{S}(k)$ and $\hat{K}(k)$ spectra have been normalised at each value of k such that the variance of the distribution obtained from the 5000 CDM realisations is equal to unity.

3 Results

The $\hat{S}(k)$, $\hat{K}(k)$ and $\hat{\mathcal{C}}^2(k)$ spectra for Face 0 and Face 5 of the DSMB map are plotted in Fig. 1 (see figure caption for details). We see that for orientation A (crosses), all the points in the skewness and kurtosis spectra lie comfortably within their respective Gaussian probability distributions for both faces. In the scale-scale correlation spectrum, however, we confirm PVFs finding of a point at $k = 4$ that lies slightly outside the 99 per cent confidence limit. On the other hand, for orientation B (solid squares) we obtain two skewness detections someway beyond the 99 per cent confidence limit. These occur in Face 0 at $k = 32$ and in Face 5 at $k = 24$, and correspond to wavelet basis functions on small (pixel-to-pixel) angular scales. Thus it is unlikely that this non-Gaussianity is cosmological in origin. In order to investigate the robustness of these high-k outliers, we repeated the entire analysis for the DSMB map with all multipoles above $\ell = 40$ removed. A similar filtering was also applied to the 5000 CDM realisations. Since the 7-degree FWHM COBE beam essentially filters out all modes beyond $\ell = 40$, we would expect these modes to contain no contribution from the sky and consist only of instrumental noise or observational artefacts. The corresponding $\hat{S}(k)$, $\hat{K}(k)$ and $\hat{\mathcal{C}}^2(k)$ spectra for two orientations of Face 0 and Face 5 of the filtered DSMB map are plotted in Fig. 2. We see immediately that the high-k skewness detections that were present in the unfiltered map have now disappeared. Thus we find no strong evidence for cosmological non-Gaussianity in the filtered DSMB map in orientation B. For orientation A, however, as we might expect, the level of significance of the $\hat{\mathcal{C}}^2$ detection at $k = 4$ was only slightly reduced by the filtering process.

4 Discussion and conclusions

In assessing the significance of the $\hat{\mathcal{C}}^2(4)$ outlier in orientation A, we must take into account the fact that (for the filtered maps) no other outliers are found any of the other statistics we have calculated[11]. Since the $2 \times 2 \times 26 = 104$ statistics presented here are not independent of one

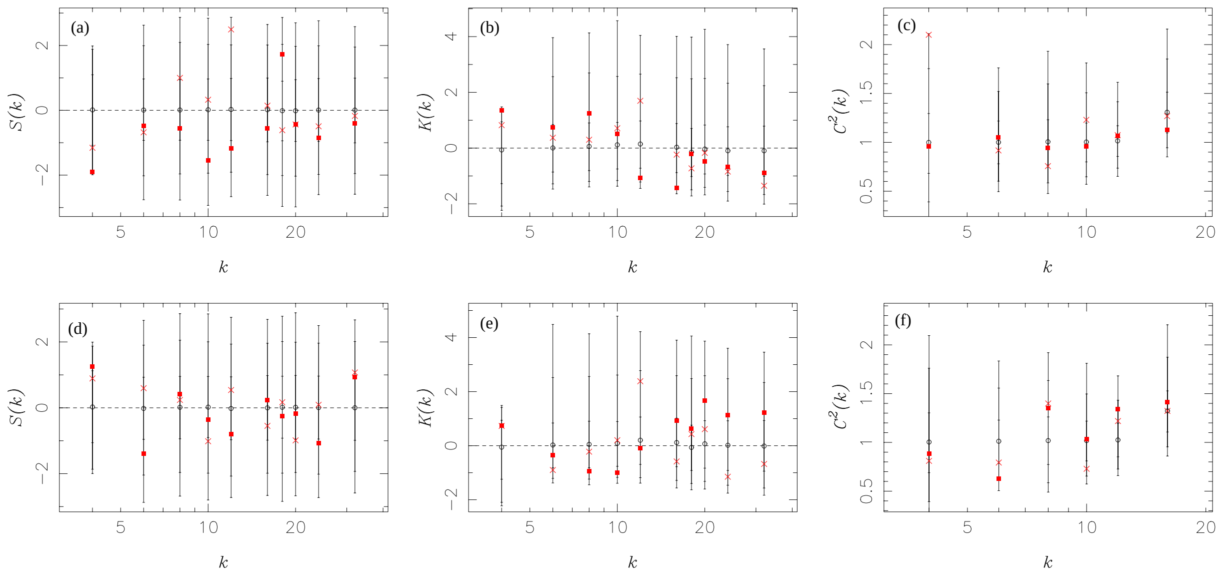

Figure 2: As in Fig. 1, but for the DMR-DSMB map with all mutlipoles above $\ell = 40$ removed.

another and generally do not possess Gaussian one-point functions, the only way of obtaining a meaningful estimate of the significance of our results is by Monte-Carlo simulation. We use the 5000 CDM realisations to estimate the probability of obtaining at least one robust outlier at > 99 percent level in *any* of our 104 statistics, even when the underlying CMB signal is Gaussian. This probability is found to be 0.59. Therefore planar orthogonal wavelet analysis of the 4-year COBE DMR-DSMB data can only rule out Gaussianity at the 41 per cent level Thus, we conclude that this method does *not* provide strong evidence for non-Gaussianity in the CMB.

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