SPINODAL QUINTESSENCE

D. CORMIER[(a)], R. HOLMAN[(b)]
(a) Institute for Physics, University of Dortmund, D-44221 Dortmund, Germany.
(b) Department of Physics, Carnegie Mellon University, Pittsburgh PA 15213 USA

We show how the spinodal instabilities inevitably present in theories with naturally soft boson fields can yield an attractive mechanism for driving the observed accelerated expansion of the Universe. This talk summarizes the results in hep-ph / 0001168.

1 How Can the Universe Accelerate?

The evidence[1] for an accelerated expansion rate of the Universe cannot be ignored or easily disputed. Together with CMB[2] and cluster abundance measurements[3] these results can be interpreted as requiring $\Omega_{\rm matter} \sim 0.3$ and are not inconsistent with a cosmological constant Λ contributing $\Omega_\Lambda \sim 0.7$.

While these results open up a large vista for theorists, there is one problem that has to faced head on at this point: the required cosmological constant is just *too small*!! The value obtained from Ω_Λ would correspond to a vacuum energy $\rho_\Lambda \sim \left(10^{-3}{\rm eV}\right)^4$. The reasons for being extremely concerned about this value are:

- Vacuum energy is quartically divergent in quantum field theories
- All quantum field theories (except possibly for string / M-theory) are effective field theories and so require an ultraviolet cutoff $M_{\rm cutoff}$ to define them.

Given the above points, we would expect that the natural value for the vaccum energy would be $\rho_{\rm vacuum} \sim M_{\rm cutoff}^4$. How do we determine what the cutoff scale should be? We should expect that this scale is related to the domain of validity of the theory, i.e. it should be the scale past which new physics enters the picture. In the standard model, we would expect $M_{\rm cutoff} \sim 1$ TeV; GUT's should lead to $M_{\rm cutoff} \sim 10^{15-16}$ GeV, while theories involving quantum gravity would most likely have $M_{\rm cutoff} \sim M_{\rm Planck}$. In any of these cases, it's clear that the required vacuum energy is too small by many orders of magnitude! Another way to say this is that we *could* use a cosmological constant to drive the acceleration of the scale factor, but it would have to be done at the expense of a fine tuning to 120 decimal places[4].

This seems unreasonable, to say the least!

We can get a hint of what to do from inflation. We should have a dynamical reason for why the energy density driving the accelerated expansion has the requisite equation of state and current value. We are led to try using scalar fields evolving along a potential such that the strong energy condition $\rho + 3p < 0$ is violated.

2 Quintessence,Trackers and Quantum Field Theory

While the use of scalar fields to generate the required energy-momentum tensor to drive an accelerated late time expansion relieves some of the issues discussed above, there is still a fine-tuning/initial condition problem, namely: why is the scalar field energy density of the same order as that of matter *today*? Since they evolve differently as a function of cosmic time, there has to be an adjustment of the initial value of the field energy density (or equivalently the initial field value) so that despite their different time evolution, the ratio $\rho_\phi/\rho_{\rm matter}$ is fixed at the appropriate value today.

There is a class of models known as the tracker models of quintessence[5] where the scalar potential is such that the equations of motion admit attractor solutions that have $\rho_\phi/\rho_{\rm matter}$ fixed at late times, regardless of the initial conditions. A model of this type is discussed by Andy Albrecht in these proceedings[6].

While these models are extremely interesting, they are open to the criticism that they use only *classical* field theory. It is well known that quantum effects can under certain circumstances drastically modify classical field behavior and it is not an unreasonable question to ask whether quantum corrections would disturb the attractor nature of the sought-after tracker solutions.

We'll take a different route below, where we will make essential use of the quantum effects to generate the relevant scale and arrange the dynamics so that the required evolution happens at the appropriate time.

3 Pseudo-Nambu-Goldsone Bosons and Spinodal Instabilities

The class of models we consider are those using pseudo-Nambu-Goldstone bosons (PNGBs) to construct theories with naturally light scalars[7]. Such models have been used for late time phase transitions[8], as well as to give rise to a cosmological "constant" that eventually relaxed to zero[9], not unlike what we want to do here. However, our take on these models will be significantly different from that of ref.[9].

We can write the required energy density as $\rho_{\rm DarkEnergy} \sim \left(10^{-3}{\rm eV}\right)^4$, which is suggestive of a light neutrino mass scale. There is a way to construct models of scalar fields coupled to neutrinos where the scalar field potential naturally incorporates the small mass scale m_ν^4.

Consider a Lagrangian containing a Yukawa coupling of the form[7]:

$$-\mathcal{L}_{\rm Yuk} = \sum_{j=0}^{N-1}\left(m_0 + \varepsilon \exp i\left(\frac{\Phi}{f} + \frac{2\pi j}{N}\right)\right)\overline{\nu}_{jL}\nu_{jR} + {\rm h.c.} \qquad (1)$$

The scale f is the scale at which a the global symmetry that gives rise to the Nambu-Goldstone mode Φ is spontaneously broken. The Lagrangian $\mathcal{L}_{\rm Yuk}$ is to be thought of as part of the low-energy effective theory of Φ coupled to neutrinos at energies below f.

The term proportional to ε could be obtained by a coupling to a Higgs field χ that acquires an expectation value $\langle\chi\rangle = f/\sqrt{2}\exp i\frac{\Phi}{f}$. Note that in the absence of m_0 this Yukawa term possesses a continuous chiral $U(1)$ symmetry. The term proportional to m_0 breaks this symmetry explicitly to a residual discrete Z_N symmetry given by:

$$\nu_j \to \nu_{j+1}, \nu_{\rm N-1} \to \nu_0, \Phi \to \Phi + \frac{2\pi {\rm f}}{\rm N}. \qquad (2)$$

This interaction can generate an effective potential for the Nambu-Goldstone mode Φ which must vanish in the limit that $m_0 \to 0$ which is equivalent to the vanishing of the neutrino masses. Since Φ is an angular degree of freedom, the effective potential is periodic:

$$V(\Phi) = M^4\left(1 + \cos\frac{N\Phi}{f}\right). \qquad (3)$$

Here M should be associated with a light neutrino mass $m_\nu \sim 10^{-3}$ eV.

At finite temperature we have[10]:

$$c\left(T\right)M^4\cos\frac{N\Phi}{f}, \tag{4}$$

for $N \geq 3$, where $c\left(T\right)$ vanishes at high temperature T so that the Φ potential becomes exactly flat with value $M^4 \sim m_\nu^4$; this cosmological constant contribution will have no effect during nucleosynthesis and through the matter dominated phase until $T \sim M$. At this time $c\left(T\right)$ reaches its asymptotic value of unity and we have the potential in eq.(3).

When $\cos N\Phi/f > 0$, $V\left(\Phi\right)$ has regions of spinodal instability, i.e. where the effective mass squared is negative. If Φ is in this region, modes of sufficiently small comoving wavenumber follow an equation of motion that at least for early times is that of an inverted harmonic oscillator. This instability will then drive the non-perturbative growth of quantum fluctuations until they reach the spinodal line where $\cos N\Phi/f = 0$[11]. Since the quantum fluctuations grow non-perturbatively large, we have to resum perturbation theory to regain sensible behavior and this is done by the Hartree truncation[11]. The prescription for this is:

- Separate expectation value from fluctuations: $\Phi\left(\vec{x},t\right) = \phi\left(t\right) + \psi\left(\vec{x},t\right)$, with $\phi\left(t\right) = \left\langle\Phi\left(\vec{x},t\right)\right\rangle$.
- Hartree factorization:
$$\cos\frac{N\psi}{f} \longmapsto \left(1 - \frac{N^2\left(\psi^2 - \left\langle\psi^2\right\rangle\right)}{2f^2}\right)\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}, \quad \sin\frac{N\psi}{f} \longmapsto \frac{N\psi}{f}\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}. \tag{5}$$
- Quantum equations of motion:
$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{NM^4}{f}\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}\sin\frac{N\phi}{f} = 0, \tag{6}$$
$$\ddot{f}_k + 3\frac{\dot{a}}{a}\dot{f}_k + \left(\frac{k^2}{a^2} - \frac{N^2M^4}{f^2}\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}\cos\frac{N\phi}{f}\right)f_k = 0, \ \left\langle\psi^2\right\rangle = \int\frac{d^3k}{\left(2\pi\right)^3}\left|f_k\right|^2 \tag{7}$$
- Semiclassical Friedmann equation:
$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_p^2}\left[\begin{array}{c}\rho_m\left(t\right) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\langle\dot{\psi}^2\rangle + \frac{1}{2a^2}\left\langle(\vec{\nabla}\psi)^2\right\rangle + \\ M^4\left(1 + \cos(N\phi/f)\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}\right)\end{array}\right], \tag{8}$$
with $\rho_m(t) = \rho_m(t_i)\frac{a^3(t_i)}{a^3(t)}$ being the matter energy density.

Since the potential is flat at high T, $\phi\left(t\right)$ is equally likely to be above the spinodal line as below, and if there was an epoch of inflation at earlier times, it is reasonable to suppose that the zero mode is high up enough so that the fluctuations can grow sufficiently large. The scenario is:

1. $\phi\left(t\right)$ starts above the spinodal line
2. When $3H \sim m_\Phi \sim \frac{m_\nu^2}{f}$, $\phi\left(t\right)$ starts to roll.
3. With prior inflation have: $\frac{\left\langle\psi^2\right\rangle(t_i)}{f^2} \approx \frac{H^2_{\rm inflation}}{4\pi^2 f^2}\left(N_{\rm e-folds} - 60\right)$
4. Fluctuations grow and kill off terms with $\exp-\frac{N^2\left\langle\psi^2\right\rangle}{2f^2}$ in the equations of motion, leaving a remnant cosmological constant m_ν^4 at late times. This drives the acceleration observed today.

4 Conclusions

There is no shortage of models to explain the accelerating expansion of the universe. However, most options are lacking in motivation and require significant fine tuning of initial conditions or the introduction of a fine tuned small scale into the fundamental Lagrangian. We too have a fine tuned scale: the neutrino mass. However, we can take solace in the fact that this fine tuning is related to a particle that can be found in the Particle Data Book, with known mechanisms to produce the required value, and experiments dedicated to its measurement.

The model itself is also relatively benign, not requiring invocations of String or M-theory to justify its potential. Chiral symmetry breaking leading to PNGB's is not unheard of in nature (pions do exist after all!), and should probably be expected in GUT or SUSY symmetry breaking phase transitions involving coupled scalars. This, together with the dynamical effects of backreaction allow the present model to be successful in explaining the data with only minor tuning of initial conditions.

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