ESTIMATING Ω_m FROM THE RELATIVE MOTIONS OF GALAXY PAIRS

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We have recently proposed a new dynamical estimator of Ω_m – the mean density of the pressureless, nonrelativistic matter in the Universe. The statistic we use is the mean relative peculiar velocity of pairs of galaxies at fixed separation r, called the galaxy streaming velocity, $v_{12}(r)$. The streaming velocity is measured directly from a peculiar velocity-distance indicator survey. Our method is nonlinear and non-perturbative, and it can be used to estimate Ω_m separately from σ_8 – the standard normalization parameter. Our approach is simpler and more direct than reconstruction schemes used in this context; it is free from procedures which amplify observational errors (e.g, taking spatial derivatives of noisy data). We also report the results obtained by applying our technique to the Mark III survey. We find $\sigma_8 \geq 0.75$ and $\Omega_m = 0.35 \pm 0.25$. Fixing $\sigma_8 = 1$ (an "unbiased" model) gives $\Omega_m = 0.35 \pm 0.1$. The unbiased Einstein-de Sitter cosmological model ($\Omega_m = \sigma_8 = 1$) is inconsistent with our data at the 99% confidence level. Our results are consistent with estimates of Ω_m based on distances to the type Ia supernovae, taken together with the MAXIMA and Boomerang measurements of the angular power spectrum of the cosmic microwave background fluctuations.

1. Dynamics of pairwise motions

Most dynamical estimates of the Ω_m parameter use the gravitational effect of departures from a strictly homogeneous distribution of objects such as stars and galaxies considered as test particles. One such dynamical estimator can be constructed by using an equation expressing the conservation of particle pairs in a self-gravitating gas. This equation was derived by Davis and Peebles[1] from the BBGKY theory more than two decades ago, and since then it has successfully resisted theorists' attempts to find a closed form solution. We have recently proposed to apply the weakly nonlinear gravitational instability theory and the strongly nonlinear stable clustering solution as limiting cases to construct an approximate solution of the pair conservation equation. Our approximate solution is given by[2]

$$v_{12}(x,a) \;=\; -\frac{2}{3}\,Hrf(\Omega_m)\bar{\bar{\xi}}(x,a)\,\left[1\,+\,\alpha\bar{\bar{\xi}}(x,a)\right]\;, \qquad (1)$$

where $v_{12}(x,a)$ is the magnitude of the mean (pair-weighted) relative velocity, $v_{12}(x,t)\,\vec{x}/x$, of a pair of particles at a comoving separation vector $\vec{x}$; a is the expansion factor, $r = ax$ is the proper separation, $H(a)$ is the Hubble parameter, while $\bar{\bar{\xi}}(x,a) \equiv \bar{\xi}(x,a)/[1+\xi(x,a)]$, and $\bar{\xi}$ is the two-point correlation funcion of matter density fluctuations, ξ, averaged over a ball of comoving radius x : $\bar{\xi}(x,a) = 3x^{-3}\int_0^x \xi(y,a)y^2\mathrm{d}y$. At the present cosmological time $a = 1$, $x = r$ and $H = 100\,h$ km s^{-1}Mpc^{-1}, where h is the usual dimensionless number parametrizing our uncertainty in the Hubble parameter. The function f is the usual logarithmic derivative of the linear growing mode solution, $D(a)$, $f(\Omega_m) \equiv d\ln D/d\ln a$ (ref.3). For models with a vanishing cosmological constant, as well as for those with zero curvature, $f = \Omega_m^{0.6}$ (e.g, ref. 4). The parameter α is defined by the relation

$$\bar{\xi}^{(2)}(x,a) \;=\; \alpha(x)\left[\bar{\xi}^{(1)}(x,a)\right]^2\;, \qquad (2)$$

where $\bar{\xi}^{(1)}$ and $\bar{\xi}^{(2)} = O\left[\bar{\xi}^{(1)}\right]^2$ are the first two terms in the perturbative expansion for $\bar{\xi}(x,a)$. Equation (3) follows from the Eulerian perturbation theory[5,6,7] . The parameter α depends on the logarithmic slope of the correlation function at the boundary between the linear and the nonlinear regime,

$$\gamma \;\equiv\; -\left(d\ln\xi(x,a)/d\ln x\right)_{\xi=1}\;. \qquad (3)$$

For a γ in the range from 0 to 3, $\alpha(\gamma)$ is well approximated by[2]

$$\alpha \;\approx\; 1.2\;-\;0.65\gamma\;. \qquad (4)$$

Our approximate solution of the pair conservation equation is designed to bridge the weakly nonlinear limiting case, valid for large separations and $|\xi| < 1$, with the so-called stable clustering limit, valid for small separations and $\xi \gg 1$. Indeed, when $\xi \to 0$, eq. (1) agrees exactly with the perturbative solution of the pair conservation equation; while for $r \to 0$, it closely approximates the stable clustering solution, $v_{12}(r) = -Hr$. The expression (1) is valid for all dust-filled cosmological models with Gaussian initial conditios.

Our approximate solution (1) was tested against high-resolution N-body simulations. The numerical experiments agree with the $v_{12}(r)$ profiles, predicted by eq. (1) in the entire dynamical range, probed by the simulations, $0.1 < |\xi| < 10^3$ (refs. 2 and 8).

An important advantage of the present approach is the validity of eq. (1) in the non-linear regime, which breaks the degeneracy between Ω_m and the conventional normalization parameter, σ_8 – the rms matter density contrast in spheres with a radius of $8h^{-1}$Mpc. Note that $\bar{\xi} \propto \sigma_8^2$, and the expression in square brackets in eq. (1) does depend σ_8. However, it does not depend on the density parameter. Hence, in order to separate Ω_m from σ_8, it is sufficient to measure $v_{12}(r)$ at different separations (see Figure 1). In contrast, all dynamical estimates of the density parameter, based on the linear perturbation theory, measure a degenerate quantity: the product $\Omega_m^{0.6}\sigma_8$ rather than Ω_m itself[9].

2. Clustering bias and velocity bias

So far we have assumed that galaxies trace mass, and the mass density contrast equals the galaxy number-density contrast: $\delta(\vec{x},a) = \delta_g(\vec{x},a)$. Then, the galaxy streaming velocity, $v_g(r)$, weighted by the number-density of galaxy pairs, equals the streaming velocity for pairs of dark matter particles, $v_{12}(r)$. For a pair at separation $\vec{r}$, this velocity is given by

$$v_{12}(r) \; = \; \langle (\vec{v}_1 - \vec{v}_2) \cdot \hat{r} \, w_{12} \rangle \; ; \quad w_{12} \; = \; \frac{(1+\delta_1)(1+\delta_2)}{1 \; + \; \xi(r)} \; , \tag{5}$$

where w_{12} is the particle pair density weighting, $\vec{v_i}$ and δ_i are the peculiar velocity and fractional density contrast of matter at position $\vec{r_i}$, $i = 1, 2$; each pair of points in the average is at a fixed separation $r = |\vec{r}_1 - \vec{r}_2|$, the hats denote unit vectors and $\xi(r) = \langle \delta_1 \delta_2 \rangle$. Now, let us allow for biasing. The expression for v_g can be obtained from eq. (6) by formally replacing the weighting function $w_{12}(\delta_1, \delta_2)$ with $w_{12}(\delta_{1g}, \delta_{2g})$. At large r, in the linear limit ($|\xi| \ll 1$) equations (1) and (5) give

$$v_g(r) \; = \; - \frac{2Hr}{r^3} \int_0^r \frac{f(\Omega_m) R(x)}{b(x)} \, \xi_g(x) \, x^2 \, dx \; + \; O\left(\xi_g^2\right) \; , \tag{6}$$

where $R(r) \equiv \langle \delta(0) \delta_g(\vec{r}) \rangle \, [\xi(r) \xi_g(r)]^{-1/2}$ and $b^2(r) \equiv \xi_g(r)/\xi(r)$ are the (possibly separation-dependent) stochasticity and biasing parameters, respectively. $R(r)$ is closely related to the dimensionless cross-correlation coefficient[a], $\mathtt{r}$, recently introduced by Dekel and Lahav[10]. The linear biasing model assumes that $\delta_g(\vec{x}, a)/\delta(\vec{x}, a) \; = \; \sigma_{8g}/\sigma_8 \; = \; b$ is a scale- and time-independent constant. Linear biasing is also deterministic ($R \equiv 1$), and eq. (6) can be rewritten as

$$v_g(r) \; = \; b \, v_{12}(r) \; = \; - \frac{2}{3} f(\Omega_m) b \; H r \, \bar{\xi}(r) \; = \; - \frac{2}{3} \frac{f(\Omega_m)}{b} \; H r \, \bar{\xi}_g(r) \; , \tag{7}$$

in agreement with Fisher et al.[11] In other words, the linear clustering bias propagates into streaming velocity bias. Note that this will not be the case at small r, in the strongly-nonlinear regime. Indeed, in this limit $w_{12}(\delta_{1g}, \delta_{2g}) \rightarrow b^2 \delta_1 \delta_2 / b^2 \xi = w_{12}(\delta_1, \delta_2)$. The biasing factors in the denominator and numerator cancel out and when $\xi \gg 1$, the streaming velocity is unbiased:

$$v_g(r) \; = \; v_{12}(r) \; . \tag{8}$$

Equations (6) and (7) show that the measurements of $v_g(r)$ can provide us with a powerful tool to study the problem of biasing observationally. In particular, if the elliptical galaxies are significantly more clustered than the spirals, eq. (7) implies that the estimates of $v_g(r)$, derived from two subsamples, containing these two morphological types of galaxies, should give different results, too. Indeed, $v_{12}{}^{(\mathrm{E})}/v_{12}{}^{(\mathrm{S})} = b^{(\mathrm{E})}/b^{(\mathrm{S})}$, where the superscripts refer to elliptical and spiral galaxies, respectively. Some observations give $b^{(\mathrm{E})}/b^{(\mathrm{S})} \approx 2$ (for an excellent review, see ref.9). However, as we will show below, most recent, direct measurements from peculiar velocity-distance data are consistent with $v_{12}{}^{(\mathrm{E})}/v_{12}{}^{(\mathrm{S})} = 1$ and $R \approx b \approx 1$.

3. A new dynamical estimator of Ω_m

We will now describe our measurements. The mean difference between radial velocities of a pair of galaxies is $\langle \, (s_1 - s_2) \, w_{12} \, \rangle \; = \; v_{12} \, \hat{r} \cdot (\hat{r}_1 + \hat{r}_2)/2$, where $s_i = \hat{r}_i \cdot \vec{v}_i$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$. To estimate v_{12}, we minimize the quantity $\chi^2(v_{12}) \; = \; \sum_{ij} \left[(s_i - s_j) - p_{ij} \, v_{12}/2 \right]^2$, where $p_{ij} \equiv \hat{r} \cdot (\hat{r}_i + \hat{r}_j)$ and the sum is over all pairs at fixed separation $r = |\vec{r_i} - \vec{r}_j|$. The resulting statistic is[12]

$$v_{12}(r) \; = \; \frac{2 \sum (s_i - s_j) \, p_{ij}}{\sum {p_{ij}}^2} \; . \tag{9}$$

[a]For unsmoothed fields, $\mathtt{r} = R(0)$. For velocity and density fields, spatially smoothed on scale r_s, $R(r_s) \approx \mathtt{r}(r_s)$, the exact relationship depending on the nature of the spatial filter applied, the shapes of ξ and ξ_g, etc.

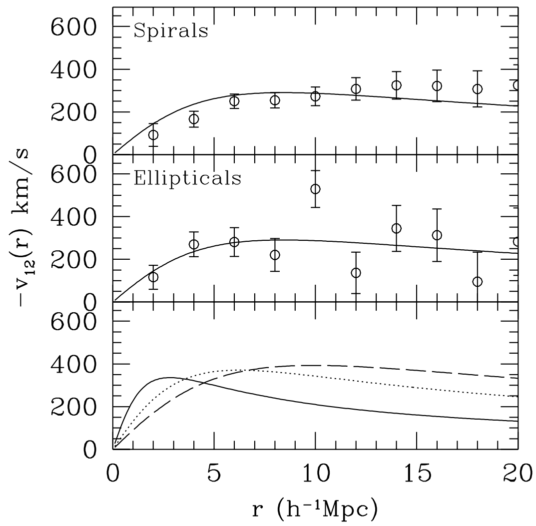

Figure 1: The streaming velocities of 2437 spiral galaxies (top panel) and 544 elliptical galaxies (center panel) estimated from the Mark III catalogue. To guide the eye, and show that although the two samples have different noise levels (because of much smaller number of galaxies in the elliptical sample), the $v_{12}(r)$ signal in both cases is similar, we also plot $v_{12}(r)$ calculated from eq. (1) for a $\xi \propto r^{-1.75}$ power-law model with $\sigma_8 = 1.25$ and $\Omega_m = 0.3$. Three theoretical $v_{12}(r)$ profiles are plotted (bottom panel) with $\xi \propto r^{-1.75}$, $\sigma_8 \Omega_m^{0.6} = 0.7$ and $(\sigma_8, \Omega_m) = (0.5, 1.75)$ (solid line), $(1, 0.55)$ (dotted line) and $(1.5, 0.28)$ (dashed line). This plot shows how measurements of $v_{12}(r)$ break the degeneracy between Ω_m and σ_8.

Monte-Carlo simulations show that this estimator is remarkably insensitive to biases in the way galaxies are selected from the sky and can be easily corrected for biases due to errors in the estimates of the radial distances[12]. The survey used here is the Mark III standardized catalogue of distances and peculiar velocities of 2437 spiral 544 elliptical galaxies[13,14]. The standard Tully-Fisher (TF) distance estimates were used for spirals; the distances to the ellipticals were estimated using the $D_n - \sigma$ method (for a comprehensive and pedagogical description of all redshift-independent distance estimators currently in use, see ref. 15). The total survey depth is over 120 h^{-1}Mpc, with homogenous sky coverage up to 30 h^{-1}Mpc. The inverse TF and IRAS density field corrections for inhomogeneous Malmquist bias in the spiral sample agree with each other and give similar streaming velocities, with lognormal distance errors of order $\sigma_{\ln d} \approx 23\%$. For the elliptical sample, $\sigma_{\ln d} \approx 21\%$ and the distances assume a smooth Malmquist bias correction[16].

In Fig. 1, reproduced here from ref. 17, we plot the estimate of the streaming velocities over a range of scales for both spiral and elliptical galaxies. All theoretical $v_{12}(r)$ curves assume a power-law model for $\xi(r) \propto \xi_g(r) \propto r^{-\gamma}$, with $\gamma = 1.75$. Our choice is motivated by observational estimates of $\xi_g(r)$, based on the APM data. The observations[18] give $\gamma = 1.75 \pm 0.1$ for separations in the range $r = 0.1 - 10\,h^{-1}$Mpc. The error bars in Fig. 1 are the one-sigma uncertainties due to: (i) the lognormal errors in the distance estimates, (ii) sparse sampling of the density field (shot noise) and (iii) finite sample volume (cosmic variance). The small sample volume also introduces correlations between measurements of $v_{12}(r)$ at different values of r. These errors were estimated from mock catalogues described in Ferreira et al.[12]

The most striking feature in Figure 1 is that the estimates from the two very different classes of objects are remarkably consistent with each other, unlike previous comparisons using the velocity correlation tensor.[19,20] This leads us to believe that, although galaxies may be subject to clustering bias they respond the same way to the underlying density field: **there is no velocity**

bias. For a velocity ratio $q \equiv v_{12}^{(\mathrm{E})}/v_{12}^{(\mathrm{S})} = 1$, we obtain $\chi^2 \simeq 1$, while for $q = 2$ the χ^2 is greater than 2.

The complete lack of velocity bias in our data is consistent with recent numerical simulations, performed by Kauffmann et al.[21]. Unlike "plain" N-body codes, these simulations include a model for dissipative processes. Their results show significant clustering bias, $b(r) > 1$, on nonlinear scales ($\xi(r) \gg 1$) but no velocity bias: $v_{12}(r) = v_g(r)$ for all separations r. It is interesting to note that this would be exactly the kind of behavior one would expect from our simplistic velocity bias model, given by equations (7) and (8) if $b(r)$ was greater than unity at small r, while at large separations $b(r) = 1$.

We will now obtain an estimate of σ_8 and Ω_m from the shape of the $v_{12}(r)$, measured from the Mark III data.[17] For our estimate, we assume a power-law $\xi(r)$ with $\gamma = 1.75$ for $r \leq 10\ h^{-1}$ Mpc. Given the depth of the Mark III catalogue we expect the covariance between estimates of $v_{12}(r)$ to be uncorrelated at $r < 10\ h^{-1}$Mpc; we use N-body simulations to determine the covariance of the estimates over this range of scales and use a simple χ^2 minimization to obtain the 1-σ constraints:

$$\sigma_8 \ \geq \ 0.75\ ;\ \Omega_m \ = \ 0.35 \ \pm \ 0.25\ . \tag{10}$$

Fixing $\sigma_8 = 1$ we obtain

$$\Omega_m \ = \ 0.35 \ \pm \ 0.1\ . \tag{11}$$

We can get a more conservative constraint on both σ_8 and Ω_m by examining a v_{12} at a single separation, $r = 10\,h^{-1}$Mpc. The measured value, $-\,v_{12} \ = \ 280^{+68}_{-53}\ \mathrm{km\,s^{-1}}$, can be now compared with v_{12}, evaluated from eq. (1) assuming $\sigma_8 = 1$ and $\Omega_m = 1$. The observed v_{12} is inconsistent with the prediction at the 99% confidence level.

4. Discussion

Our results agree with the estimates from the location of the first maximum in the angular power of the cosmic microwave background (cmb) anisotropy, reported recently by two groups – Boomerang[22] and MAXIMA.[23] The spherical harmonic number of the first peak is sensitive to the value of the sum $\Omega = \Omega_m + \Omega_\Lambda$, where the second component is the vacuum energy density. When the recent cmb results, consistent with $\Omega \approx 1$, are combined with measurements of high-redshift supernovae,[24,25] the resulting Ω_m is compatible with our measurements. For example, the MAXIMA experiment, combined with the supernova data, gives $0.4 < \Omega_\Lambda < 0.76$ and $0.25 < \Omega_m < 0.50$, at the 99% confidence level.

Our results are also compatible with a number of earlier dynamical estimates of the parameter $\beta \equiv \Omega_m^{0.6}/b$, if the biasing parameter is $b \approx 1$. A technique, based on the action principle[26] gives $\beta = 0.34 \pm 0.13$; comparisons of peculiar velocity fields with redshift surveys based on the integral form of the continuity equation (called velocity-velocity, or v-v comparisons) typically give[27,28,29,30] $\beta = 0.5 - 0.6$.

However, our results disagree with the IRAS-POTENT estimate[31], $\beta = 0.89 \pm 0.12$ (unless the IRAS galaxies are "antibiased", and $b < 1$, in conflict with all v-v studies of the IRAS survey). The IRAS-POTENT analysis is based on the continuity equation in its differential form; it uses a rather complicated reconstruction technique to recover the full velocity field from its radial component. The reason for the disagreement is not clear at present. One can think of at least two possible sources of systematic errors: (i) the reconstruction scheme itself: by design it contains a powerful noise amplifier – the reconstruction requires taking spatial derivatives of noisy data, and (ii) the nonlinear corrections adopted. The latter diverge like $\Omega_m^{-1.8}$ in the limit of small Ω_m – a disturbing problem, in particular because the entire POTENT scheme has been tested with N-body simulations and mock catalogues only for the $\Omega_m = 1$ model. By contrast, the accuracy of the nonlinear corrections for the v-v analysis is Ω_m-independent. The v-v approach is also simpler than IRAS-POTENT because it does not involve the reconstruction of the full velocity vector from its radial component measurements. Moreover, the POTENT reconstruction requires taking second derivatives of noisy data.

The advantages of the new statistic we have used here can be summarized as follows. First, v_{12} can be estimated directly from velocity-distance surveys, without subjecting the observational data to multiple operations of spatial smoothing, integration and differentiation, used in various reconstruction schemes. Second, unlike cosmological parameter estimators based on the acoustic peaks, expected to appear in the cosmic microwave background power spectrum,[32] the Ω_m estimate based on v_{12} is model-independent. Finally, our approach offers the possibility to break the degeneracy between Ω_m and σ_8 simply by measuring the $v_{12}(r)$ at different separations.

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