Inflation and Creation of Matter in the Universe

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The evolution of the inflationary theory is described, starting from the Starobinsky model and the old inflation scenario, toward chaotic inflation and the theory of eternally expanding self-reproducing inflationary universe. A special attention is devoted to the recent progress in the theory of reheating and creation of matter after inflation.

Inflationary theory was proposed 20 years ago. In the beginning it looked like an interesting piece of science fiction. During the last 20 years it changed quite a lot and became a broadly accepted cosmological paradigm. New versions of this theory do not require any assumptions about initial thermal equilibrium in the early universe. Inflation is no longer based on the mechanism of supercooling and exponential expansion in the false vacuum state. It was proposed in order to resolve various problems of the big bang theory, and in particular to explain the extraordinary homogeneity of the observable part of the universe. However, later we have learned that while making the universe locally homogeneous, inflation can make it extremely inhomogeneous on a vary large scale. According to the simplest versions of inflationary theory the universe is not a single, expanding ball of fire produced in the big bang, but rather a huge eternally growing fractal. It consists of many inflating balls that produce new balls, which in turn produce more new balls, ad infinitum. Even now we continue learning new things about inflationary cosmology, especially about the stage of reheating of the universe after inflation.

In this paper we will briefly describe the history of inflationary cosmology, and then we will give a review of some recent developments.

1 Brief history of inflation

The first model of inflationary type was proposed by Alexei Starobinsky in 1979[1]. It was based on investigation of conformal anomaly in quantum gravity. This model was rather complicated, it did not aim on solving homogeneity, horizon and monopole problems, and it was not easy to understand the beginning of inflation in this model. However, it did not suffer from the graceful exit problem, and in this sense it can be considered the first working model of inflation. The theory of density perturbations in this model was developed in 1981 by Mukhanov and Chibisov[2]. This theory does not differ much from the theory of density perturbations in new inflation, which was developed a year later by Hawking, Starobinsky, Guth, Pi, Bardeen, Steinhardt, and Turner[3].

A much simpler inflationary model with a very clear physical motivation was proposed by Alan Guth in 1981[4]. His model, which is now called "old inflation," was based on the theory of supercooling during the cosmological phase transitions[5]. It was so attractive that even now all textbooks on astronomy and most of the popular books on cosmology describe inflation as exponential expansion of the universe in a supercooled false vacuum state. It is seductively easy to explain the nature of inflation in this scenario. False vacuum is a metastable state without any fields or particles but with large energy density. Imagine a universe filled with such "heavy nothing." When the universe expands, empty space remains empty, so its energy density does not change. The universe with a constant energy density expands exponentially, thus we have inflation in the false vacuum.

Unfortunately this explanation is somewhat misleading. Expansion in the false vacuum in a certain sense is false: de Sitter space with a constant vacuum energy density can be considered either expanding, or contracting, or static, depending on the choice of a coordinate system[6]. The absence of a preferable hypersurface of decay of the false vacuum is the main reason why the universe after inflation in this scenario becomes very inhomogeneous[4]. After a detailed investigation of this problem, Alan Guth and Erick Weinberg concluded that the old inflation scenario cannot be improved[7].

Fortunately, this problem was resolved with the invention of the new inflationary theory [8]. In this theory, just like in the scenario proposed by Guth, inflation may begin in the false vacuum, but this stage of inflation is essentially useless. However, inflation in this scenario continues even when the inflaton field ϕ driving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential. The motion of the field away from the false vacuum is of crucial importance: density perturbations produced during inflation are inversely proportional to $\dot\phi$ [2,3], so these perturbations would be unacceptably large if inflation occurred in the false vacuum with $\phi = const$. Thus the standard description of inflation as an exponential expansion of the universe in the false vacuum is misleading since it misses the main difference between the old inflationary scenario and the new one.

The new inflation scenario was plagued by its own problems. It works only if the effective potential of the field ϕ has a very a flat plateau near $\phi = 0$, which is somewhat artificial. In most versions of this scenario the inflaton field originally could not be in a thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis for old and new inflation, simply did not work in such a situation. Even if the inflaton field is in a state of thermal equilibrium, it may be far away from the minimum of the effective potential at $\phi = 0$ induced by thermal effects. Typically it takes a lot of time for the field ϕ to roll down to $\phi = 0$. During this time temperature drops down and the minimum at $\phi = 0$ ceases to exist. Moreover, thermal equilibrium requires many particles interacting with each other. This means that new inflation could explain why our universe was so large only if it was very large and contained many particles from the very beginning. Finally, even if inflation in this scenario is possible, it can begin only very late, at the time which is many orders of magnitude greater than the Planck time M_p^{-1}. During the preceding epoch the universe could easily collapse or become so inhomogeneous that inflation may never happen[6]. No realistic versions of the new inflationary universe scenario have been proposed so far.

All problems of old and new inflation were resolved in 1983 with the introduction of the chaotic inflation scenario[9]. In this scenario inflation may occur even in the theories with simplest potentials such as $V(\phi) \sim \phi^n$. It may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved[6].

2 Chaotic inflation

To explain the basic idea of chaotic inflation, let us consider the simplest model of a scalar field ϕ with a mass m and with the potential energy density $V(\phi) = \frac{m^2}{2}\phi^2$. Since this function has a minimum at $\phi = 0$, one may expect that the scalar field ϕ should oscillate near this minimum. This is indeed the case if the universe does not expand. However, one can show that in a rapidly expanding universe the scalar field moves down very slowly, as a ball in a viscous liquid, viscosity being proportional to the speed of expansion.

There are two equations which describe evolution of a homogeneous scalar field in our model, the field equation

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \ , \tag{1}$$

and the Einstein equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_p^2}\left(\dot{\phi}^2 + m^2\phi^2\right) \ . \tag{2}$$

Here $H = \dot{a}/a$ is the Hubble parameter in the universe with a scale factor $a(t)$, $k = -1, 0, 1$ for an open, flat or closed universe respectively, M_p is the Planck mass. The first equation is similar to the equation of motion for a harmonic oscillator, where instead of $x(t)$ we have $\phi(t)$, with a friction term $3H\dot{\phi}$.

If the scalar field ϕ initially was large, the Hubble parameter H was large too, according to the second equation. This means that the friction term in the first equation was very large, and therefore the scalar field was moving very slowly, as a ball in a viscous liquid. Therefore at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and expansion of the universe continued with a much greater speed than in the old cosmological theory. Due to the rapid growth of the scale of the universe and a slow motion of the field ϕ, soon after the beginning of this regime one has $\ddot{\phi} \ll 3H\dot{\phi}$, $H^2 \gg \frac{k}{a^2}$, $\dot{\phi}^2 \ll m^2\phi^2$, so the system of equations can be simplified:

$$3\frac{\dot{a}}{a}\dot{\phi} = -m^2\phi, \qquad \frac{\dot{a}}{a} = \frac{2m\phi}{M_p}\sqrt{\frac{\pi}{3}} \, . \tag{3}$$

The last equation shows that the size of the universe in this regime grows approximately as e^{Ht}, where $H = \frac{2m\phi}{M_p}\sqrt{\frac{\pi}{3}}$.

This stage of exponentially rapid expansion of the universe is called inflation. In realistic versions of inflationary theory its duration could be as short as 10^{-35} seconds. When the field ϕ becomes sufficiently small, viscosity becomes small, inflation ends, and the scalar field ϕ begins to oscillate near the minimum of $V(\phi)$. As any rapidly oscillating classical field, it looses its energy by creating pairs of elementary particles. These particles interact with each other and come to a state of thermal equilibrium with some temperature T. From this time on, the corresponding part of the universe can be described by the standard hot universe theory.

The main difference between inflationary theory and the old cosmology becomes clear when one calculates the size of a typical inflationary domain at the end of inflation. Investigation of this question shows that even if the initial size of inflationary universe was as small as the Plank size $l_P \sim 10^{-33}$ cm, after 10^{-35} seconds of inflation the universe acquires a huge size of $l \sim 10^{10^{12}}$ cm! These numbers are model-dependent, but in all realistic models this size appears to be many orders of magnitude greater than the size of the part of the universe which we can see now, $l \sim 10^{28}$ cm. This immediately solves most of the problems of the old cosmological theory.

Our universe is so homogeneous because all inhomogeneities were stretched $10^{10^{12}}$ times. The density of primordial monopoles and other undesirable "defects" becomes exponentially diluted by inflation. The universe becomes enormously large. Even if it was a closed universe of a size $\sim 10^{-33}$ cm, after inflation the distance between its "South" and "North" poles becomes many orders of magnitude greater than 10^{28} cm. We see only a tiny part of the huge cosmic balloon. That is why nobody has ever seen how parallel lines cross. That is why the universe looks so flat.

If one considers a universe which initially consisted of many domains with chaotically distributed scalar field ϕ (or if one considers different universes with different values of the field), then domains in which the scalar field was too small never inflated. The main contribution to the total volume of the universe will be given by those domains which originally contained large scalar field ϕ. Inflation of such domains creates huge homogeneous islands out of initial chaos. Each homogeneous domain in this scenario is much greater than the size of the observable part of the universe.

The first models of chaotic inflation were based on the theories with polynomial potentials, such as $V(\phi) = \pm\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$. But the main idea of this scenario is quite generic. One should consider any particular potential $V(\phi)$, polynomial or not, with or without spontaneous symmetry breaking, and study all possible initial conditions without assuming that the universe was in a state of thermal equilibrium, and that the field ϕ was in the minimum of its effective potential from the very beginning[9].

This scenario strongly deviated from the standard lore of the hot big bang theory and was psychologically difficult to accept. Indeed, according to the old and new inflation, the universe initially was hot, then there was a short intermediate stage of inflation, and then the universe became hot again. Chaotic inflation eliminated the necessity of the standard assumption of the hot big bang and thermal equilibrium in the early universe.

Gradually, however, it became clear that the idea of chaotic initial conditions is most general, and that it is much easier to construct a consistent cosmological theory without making unnecessary assumptions about thermal equilibrium and high temperature phase transitions in the early universe.

Many other versions of inflationary cosmology have been proposed since 1983. Most of them are based not on the theory of high-temperature phase transitions, as in old and new inflation, but on the idea of chaotic initial conditions, which is the definitive feature of the chaotic inflation scenario.

3 Quantum Fluctuations and Density Perturbations

The vacuum structure in the exponentially expanding universe is much more complicated than in ordinary Minkowski space. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially during inflation. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more new perturbations of the classical field with wavelengths greater than H^{-1}. The average amplitude of such perturbations generated during a time interval H^{-1} (in which the universe expands by a factor of e) is given by

$$|\delta\phi(x)| \approx \frac{H}{2\pi} = \sqrt{\frac{2V(\phi)}{3\pi M_p^2}} \ . \tag{4}$$

Fluctuations of the field ϕ lead to adiabatic density perturbations $\delta\rho \sim V'(\phi)\delta\phi$, which grow after inflation, and at the stage of the cold matter dominance acquire the amplitude[2,3,6]

$$\frac{\delta\rho}{\rho} = \frac{16\sqrt{6\pi}}{5}\ \frac{V^{3/2}}{V'} \ . \tag{5}$$

Here ϕ is the value of the classical field $\phi(t)$ (4), at which the fluctuation we consider has the wavelength $l \sim k^{-1} \sim H^{-1}(\phi)$ and becomes frozen in amplitude. In the simplest theory of the massive scalar field with $V(\phi) = \frac{m^2}{2}\phi^2$ one has

$$\frac{\delta\rho}{\rho} = \frac{8\sqrt{3\pi}}{5}\, m\phi^2 \ . \tag{6}$$

Taking into account (3) and also the expansion of the universe by about 10^{30} times after the end of inflation, one can obtain the following result for the density perturbations with the wavelength $l(cm)$ at the moment when these perturbations begin growing and the process of the galaxy formation starts:

$$\frac{\delta\rho}{\rho} \sim m \ln l(cm) \ . \tag{7}$$

The definition of $\frac{\delta\rho}{\rho}$ used in[6] corresponds to COBE data for $\frac{\delta\rho}{\rho} \sim 5 \cdot 10^{-5}$. This gives $m \sim 10^{-6}$, in Planck units, which is equivalent to 10^{13} GeV.

An important feature of the spectrum of density perturbations is its flatness: $\frac{\delta\rho}{\rho}$ in our model depends on the scale l only logarithmically. Flatness of the spectrum of $\frac{\delta\rho}{\rho}$ together with flatness of the universe ($\Omega = 1$) constitute the two most robust predictions of inflationary cosmology. It is possible to construct models where $\frac{\delta\rho}{\rho}$ changes in a very peculiar way, and it is also possible to construct theories where $\Omega \neq 1$, but it is not easy to do so. Still such possibility does exist; we will discuss it later.

4 From the Big Bang theory to the theory of eternal inflation

A significant step in the development of inflationary theory which I would like to discuss here is the discovery of the process of self-reproduction of inflationary universe. This process was known to exist in old inflationary theory[4] and in the new one[10], but it is especially surprising and leads to most profound consequences in the context of the chaotic inflation scenario[11]. It appears that in many models large scalar field during inflation produces large quantum fluctuations which may locally increase the value of the scalar field in some parts of the universe. These regions expand at a greater rate than their parent domains, and quantum fluctuations inside them lead to production of new inflationary domains which expand even faster. This surprising behavior leads to an eternal process of self-reproduction of the universe.

To understand the mechanism of self-reproduction one should remember that the processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion the distance between any two objects separated by more than H^{-1} is growing with a speed exceeding the speed of light. As a result, an observer in the inflationary universe can see only the processes occurring inside the horizon of the radius H^{-1}.

An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. In this sense any inflationary domain of initial radius exceeding H^{-1} can be considered as a separate mini-universe.

To investigate the behavior of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial radius H^{-1} containing sufficiently homogeneous field with initial value $\phi \gg M_p$. Equation (3) implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by $\Delta\phi = \frac{M_p^2}{4\pi\phi}$. By comparison this expression with $|\delta\phi(x)| \approx \frac{H}{2\pi} = \sqrt{\frac{2V(\phi)}{3\pi M_p^2}} \sim \frac{m\phi}{3M_p}$ one can easily see that if ϕ is much less than $\phi^* \sim \frac{M_p}{3}\sqrt{\frac{M_p}{m}}$, then the decrease of the field ϕ due to its classical motion is much greater than the average amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for $\phi \gg \phi^*$ one has $\delta\phi(x) \gg \Delta\phi$. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1}, the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains (mini-universes) of radius H^{-1}, each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases. This means that the total volume of the universe containing *growing* field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ the situates repeats. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters eternal process of self-reproduction.

This effect is very unusual. Its investigation still brings us new unexpected results. For example, for a long time it was believed that self-reproduction in the chaotic inflation scenario can occur only if the scalar field ϕ is greater than ϕ^* [11]. However, it was shown in[12] that if the size of the initial inflationary domain is large enough, then the process of self-reproduction of the universe begins for all values of the field ϕ for which inflation is possible (for $\phi > M_p$ in the theory $\frac{m^2}{2}\phi^2$). This result is based on the investigation of the probability of quantum jumps with amplitude $\delta\phi \gg H/2\pi$.

Until now we have considered the simplest inflationary model with only one scalar field, which had only one minimum of its potential energy. Meanwhile, realistic models of elementary particles propound many kinds of scalar fields. For example, in the unified theories of weak, strong and electromagnetic interactions, at least two other scalar fields exist. The potential energy of these scalar fields may have several different minima. This means that the same theory may have different “vacuum states,” corresponding to different types of symmetry breaking between fundamental interactions, and, as a result, to different laws of low-energy physics.

As a result of quantum jumps of the scalar fields during inflation, the universe may become divided into infinitely many exponentially large domains that have different laws of low-energy physics. Note that this division occurs even if the whole universe originally began in the same state, corresponding to one particular minimum of potential energy.

If this scenario is correct, then physics alone cannot provide a complete explanation for all properties of our part of the universe. The same physical theory may yield large parts of the universe that have diverse properties. According to this scenario, we find ourselves inside a four-dimensional domain with our kind of physical laws not because domains with different dimensionality and with alternate properties are impossible or improbable, but simply because our kind of life cannot exist in other domains.

This consideration is based on the anthropic principle, which was not very popular among physicists for two main reasons. First of all, it was based on the assumption that the universe was created many times until the final success. Second, it would be much easier (and quite sufficient) to achieve this success in a small vicinity of the Solar system rather than in the whole observable part of our universe.

Both objections can be answered in the context of the theory of eternal inflation. First of all, the universe indeed reproduces itself in all its possible versions. Second, if the conditions suitable for the existence of life appear in a small vicinity of the Solar system, then because of inflation the same conditions will exist in a domain much greater than the observable part of the universe. This means that inflationary theory for the first time provides real physical justification of the anthropic principle[6,12,13,14,15].

Thus during the last ten years inflationary theory changed considerably. It has broken an umbilical cord connecting it with the old big bang theory, as well as with old and new inflation, and acquired an independent life of its own. For the practical purposes of describing the observable part of our universe one may still speak about the big bang, just as one can still use Newtonian gravity theory to describe the Solar system with very high precision. However, if one tries to understand the beginning of the universe, or its end, or its global structure, then some of the notions of the big bang theory become inadequate. Instead of one single big bang producing a single-bubble universe, we are speaking now about inflationary bubbles producing new bubbles, producing new bubbles, *ad infinitum*. In the new theory there is no end of the universe evolution, and the notion of the big bang loses its dominant position, being removed to the indefinite past.

¿From this new perspective many old problems of cosmology, including the problem of initial conditions, look much less profound than they seemed before. In many versions of inflationary theory it can be shown that the fraction of the volume of the universe with given properties (with given values of fields, with a given density of matter, etc.) does not depend on time, both at the stage of inflation and even after it. Thus each part of the universe evolves in time, but the universe as a whole may be stationary, and the properties of its parts do not depend on the initial conditions[12].

Of course, this happens only for the (rather broad) set of initial conditions which lead to self-reproduction of the universe. However, only finite number of observers live in the universes created in a state with initial conditions which do not allow self-reproduction, whereas infinitely many observers can live in the universes with the conditions which allow self-reproduction. Thus it seems plausible that we (if we are typical, and live in the place where most observers do) should live in the universe created in a state with initial conditions which allow self-reproduction. On the other hand, stationarity of the self-reproducing universe implies that an exact knowledge of these initial conditions in a self-reproducing universe is irrelevant for the investigation of its future evolution[12].

5 Towards stringy inflation

Whereas the main principles of inflationary cosmology by now are well understood, it remains difficult (though not impossible) to implement it in such popular theories as supergravity and string theory. The main problem here is that the effective potential for the inflaton field in supergravity typically is too curved, growing as $\exp\frac{C\phi^2}{M_p^2}$ at large values of ϕ. The typical value of the parameter C is $O(1)$, which makes inflation impossible because the inflaton mass in this theory becomes of the order of the Hubble constant. The situation is not much better in string theory. Typical shape of the scalar field potential in string theory is $\exp\frac{C\phi}{M_p}$ with $C = O(1)$, which also prevents inflation for $\phi > M_p$.

As a result, after many years of development of supersymmetric theories we still do not have a consistent cosmological theory based on supergravity and superstrings. Of course, interpretation of string theory (or M-theory?) changes every year, so one may simply postpone investigation of this problem and study only globally supersymmetric models.

Hopefully, this problem is only temporary; it may be related to our limited understanding of string theory (or M-theory), which rapidly changes every year. It might be possible also that inflation can be implemented in the simplest versions of string theory, but in a rather nontrivial way.

One of the often discussed possibilities is the pre-big-bang scenario developed in[16]. This scenario assumes that inflation occurred in a stringy phase prior to the big bang. The possibility that the big bang is not a beginning but a phase transition is very exciting. Unfortunately, it is extremely difficult to match the stage of the pre-big-bang inflation and the subsequent post-big-bang evolution[17]. It is difficult to obtain density perturbations with a nearly-flat spectrum in this scenario. Recently it was found also that this scenario requires the universe to be exponentially large, flat and homogeneous prior to the stage of the pre-big-bang inflation [18,19]. For example, in the standard big bang theory the initial size of the homogeneous part of the universe must be approximately 10^{30} times greater than the Planck scale. This constitutes the flatness problem. This problem is easily resolved in chaotic inflation scenario because in this scenario one should not make any assumptions about initial properties of the universe on the scale greater than the Planck length. Meanwhile, to solve the flatness problem (i.e. to explain the origin of the large number 10^{30}) in the context of the pre-big bang theory one must introduce two independent large dimensionless parameters, $g_0^{-2} > 10^{53}$, and $B > 10^{91}$ [19]. To address this problem the authors of the PBB scenario changed it a bit and assumed that the universe must be flat and homogeneous from the very beginning; small inhomogeneities must be correlated in a very particular way over infinite distance. The spectrum of inhomogeneities must differ from the spectrum of quantum fluctuations because otherwise the probability of realization of the PBB scenario is exponentially small[20].

We do not see any reason why these unusual conditions should be satisfied in the early universe. The main advantage of the standard inflationary theory is that none of these assumptions is necessary. Thus it seems very difficult to replace the usual post-big-bang inflation by the pre-big-bang one. And if the post-big-bang inflation is indeed necessary, then the pre-big-bang stage will have no observational manifestations.

Therefore it is very desirable to find those versions of string theory and supergravity where a consistent post-big-bang inflationary theory can be developed. This is a difficult problem, but not hopeless. Perhaps the easiest way to find a consistent inflationary model in supersymmetric theories is based on the hybrid inflation scenario[21]. The simplest version of this scenario is based on chaotic inflation in the theory of two scalar fields with the effective potential

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2\ . \qquad (8)$$

The effective mass squared of the field σ is equal to $-M^2 + g^2\phi^2$. Therefore for $\phi > \phi_c = M/g$ the only minimum of the effective potential $V(\sigma,\phi)$ is at $\sigma = 0$. The curvature of the effective potential in the σ-direction is much greater than in the ϕ-direction. Thus at the first stages of expansion of the universe the field σ rolled down to $\sigma = 0$, whereas the field ϕ could remain large for a much longer time.

At the moment when the inflaton field ϕ becomes smaller than $\phi_c = M/g$, the phase transition with the symmetry breaking occurs. If $m^2\phi_c^2 = m^2M^2/g^2 \ll M^4/\lambda$, the Hubble constant at the time of the phase transition is given by $H^2 = \frac{2\pi M^4}{3\lambda M_p^2}$. If one assumes that $M^2 \gg \frac{\lambda m^2}{g^2}$ and that $m^2 \ll H^2$, then the universe at $\phi > \phi_c$ undergoes a stage of inflation, which abruptly ends at $\phi = \phi_c$.

One of the advantages of this scenario is the possibility to obtain small density perturbations even if coupling constants are large, $\lambda, g = O(1)$. This scenario works if the effective potential has a relatively flat ϕ-direction. But flat directions often appear in supersymmetric theories. This makes hybrid inflation an attractive playground for those who wants to achieve inflation in supergravity.

Another advantage of this scenario is a possibility to have inflation at $\phi \ll M_p$. This helps to avoid problems which may appear if the effective potential in supergravity and string theory blows up at $\phi > M_p$. Several different models of hybrid inflation in supergravity have been proposed during the last few years (F-term inflation [22], D-term inflation [23], etc.) A detailed discussion of various versions of hybrid inflation in supersymmetric theories can be found in [24].

6 Reheating after inflation

The theory of reheating of the universe after inflation is the most important application of the quantum theory of particle creation, since almost all matter constituting the universe was created during this process.

At the stage of inflation all energy is concentrated in a classical slowly moving inflaton field ϕ. Soon after the end of inflation this field begins to oscillate near the minimum of its effective potential. Eventually it produces many elementary particles, they interact with each other and come to a state of thermal equilibrium with some temperature T_r.

Elementary theory of this process was developed many years ago[25]. It was based on the assumption that the oscillating inflaton field can be considered as a collection of noninteracting scalar particles, each of which decays separately in accordance with perturbation theory of particle decay. However, recently it was understood that in many inflationary models the first stages of reheating occur in a regime of a broad parametric resonance. To distinguish this stage from the subsequent stages of slow reheating and thermalization, it was called *preheating*[26]. The energy transfer from the inflaton field to other bose fields and particles during preheating is extremely efficient.

To explain the main idea of the new scenario we will consider first the simplest model of chaotic inflation with the effective potential $V(\phi) = \frac{m^2}{2}\phi^2$, and with the interaction Lagrangian $-\frac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\chi$. We will take $m = 10^{-6}M_p$, as required by microwave background anisotropy[6], and in the beginning we will assume for simplicity that χ particles do not have a bare mass, i.e. $m_\chi(\phi) = g|\phi|$.

In this model inflation occurs at $|\phi| > 0.3M_p$ [6]. Suppose for definiteness that initially ϕ is large and negative, and inflation ends at $\phi \sim -0.3M_p$. After that the field ϕ rolls to $\phi = 0$, then it grows up to $10^{-1}M_p \sim 10^{18}$ GeV, and finally rolls back and oscillates about $\phi = 0$ with a gradually decreasing amplitude.

We will assume that $g > 10^{-5}$ [26], which implies $gM_p > 10^2 m$ for the realistic value of the mass $m \sim 10^{-6}M_p$. Thus, immediately after the end of inflation, when $\phi \sim M_p/3$, the effective mass $g|\phi|$ of the field χ is much greater than m. It decreases when the field ϕ moves down, but initially this process remains adiabatic, $|\dot{m}_\chi| \ll m_\chi^2$.

Particle production occurs at the time when the adiabaticity condition becomes violated, i.e. when $|\dot{m}_\chi| \sim g|\dot{\phi}|$ becomes greater than $m_\chi^2 = g^2\phi^2$. This happens only when the field ϕ rolls close to $\phi = 0$. The velocity of the field at that time was $|\dot{\phi}_0| \approx mM_p/10 \approx 10^{-7}M_p$. The process becomes nonadiabatic for $g^2\phi^2 < g|\dot{\phi}_0|$, i.e. for $-\phi_* < \phi < \phi_*$, where $\phi_* \sim \sqrt{\frac{|\dot{\phi}_0|}{g}}$ [26]. Note that for $g \gg 10^{-5}$ the interval $-\phi_* < \phi < \phi_*$ is very narrow: $\phi_* \ll M_p/10$. As a result, the process of particle production occurs nearly instantaneously, within the time

$$\Delta t_* \sim \frac{\phi_*}{|\dot{\phi}_0|} \sim (g|\dot{\phi}_0|)^{-1/2}. \tag{9}$$

This time interval is much smaller than the age of the universe, so all effects related to the expansion of the universe can be neglected during the process of particle production. The uncertainty principle implies in this case that the created particles will have typical momenta $k \sim (\Delta t_*)^{-1} \sim (g|\dot{\phi}_0|)^{1/2}$. The occupation number n_k of χ particles with momentum k is equal to zero all the time when it moves toward $\phi = 0$. When it reaches $\phi = 0$ (or, more exactly, after it moves through the small region $-\phi_* < \phi < \phi_*$) the occupation number suddenly (within the time Δt_*) acquires the value[26]

$$n_k = \exp\left(-\frac{\pi k^2}{g|\dot{\phi}_0|}\right), \tag{10}$$

and this value does not change until the field ϕ rolls to the point $\phi = 0$ again.

The main idea of the scenario of broad parametric resonance is that each time when the field ϕ approaches the point $\phi = 0$, new χ particles are being produced. Bose statistics implies, roughly speaking, that the number of particles produced each time will be proportional to the number of particles produced before. This leads to explosive process of particle production[26].

Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. Explosive reheating leads to many interesting effects. For example, specific nonthermal phase transitions may occur soon after preheating, which are capable of restoring symmetry even in the theories with symmetry breaking on the scale $\sim 10^{16}$ GeV[27]. These phase transitions are capable of producing topological defects such as strings, domain walls and monopoles[28]. Strong deviation from thermal equilibrium and the possibility of production of superheavy particles by oscillations of a relatively light inflaton field may resurrect the theory of GUT baryogenesis[29] and may considerably change the way baryons are produced in the Affleck-Dine scenario[30], and in the electroweak theory[31].

Usually only a small fraction of the energy of the inflaton field $\sim 10^{-2}g^2$ is transferred to the particles χ when the field ϕ approaches the point $\phi = 0$ for the first time[32]. The role of the parametric resonance is to increase this energy exponentially within several oscillations of the inflaton field. But suppose that the particles χ interact with fermions ψ with the coupling $h\bar{\psi}\psi\chi$. If this coupling is strong enough, then χ particles may decay to fermions before the oscillating field ϕ returns back to the minimum of the effective potential. If this happens, parametric resonance does not occur. However, something equally interesting may occur instead of it: The energy density of the χ particles at the moment of their decay may become much greater than their energy density at the moment of their creation. This may be sufficient for a complete reheating. We called it "instant preheating"[32].

Indeed, prior to their decay the number density of χ particles produced at the point $\phi = 0$ remains practically constant[26], whereas the effective mass of each χ particle grows as $m_\chi = g\phi$ when the field ϕ rolls up from the minimum of the effective potential. Therefore their total energy density grows. One may say that χ particles are "fattened," being fed by the energy of the rolling field ϕ. The fattened χ particles tend to decay to fermions at the moment when they have the greatest mass, i.e. when ϕ reaches its maximal value $\sim 10^{-1}M_p$, just before it begins rolling back to $\phi = 0$.

At that moment χ particles can decay to two fermions with mass up to $m_\psi \sim \frac{g}{2}10^{-1}M_p$, which can be as large as 5×10^{17} GeV for $g \sim 1$. This is 5 orders of magnitude greater than the masses of the particles which can be produced by the usual decay of ϕ particles. As a result, the chain reaction $\phi \to \chi \to \psi$ considerably enhances the efficiency of transfer of energy of the inflaton field to matter.

More importantly, superheavy particles ψ (or the products of their decay) may eventually dominate the total energy density of matter even if in the beginning their energy density was relatively small. For example, the energy density of the oscillating inflaton field in the theory with the effective potential $\frac{\lambda}{4}\phi^4$ decreases as a^{-4} in an expanding universe with a scale factor

$a(t)$. Meanwhile the energy density stored in the nonrelativistic particles ψ (prior to their decay) decreases only as a^{-3}. Therefore their energy density rapidly becomes dominant even if originally it was small. A subsequent decay of such particles leads to a complete reheating of the universe.

This is exactly what happens in the theories where the post-inflationary motion of the inflaton field occurs along a flat direction of the effective potential. In such theories the standard scenario of reheating does not work because the field ϕ does not oscillate. Until the invention of the instant preheating scenario the only mechanism of reheating discussed in the context of such models was based on the gravitational production of particles[33]. The mechanism of instant preheating is much more efficient. After the moment when χ particles are produced their energy density grows due to the growth of the field ϕ. Meanwhile the energy density of the field ϕ moving along a flat direction of $V(\phi)$ decreases extremely rapidly, as $a^{-6}(t)$. Therefore very soon all energy becomes concentrated in the particles produced at the end of inflation, and reheating completes[32,34].

Creation of fermions after inflation may work even without an intermediate stage of creation of bosons[35,36]. Indeed, the process of creation of bosons and fermions is most efficient at the moment when their mass vanishes. For fermions with mass m interacting with the scalar field with a constant g it occurs at the moment when $m + g\phi = 0$. Then the masses of fermions created at $\phi \approx -m/g$ grow and become equal to m after inflation (if the effective potential has a minimum at $\phi = 0$). This process, just as the process of fermion production in the original version of the instant preheating scenario, can produce fermions with masses up to 10^{18} GeV[36]. The number of fermions produced in each moment when the mass of the fermions vanishes in this scenario is smaller than in the scenario of Ref.[32], but the possibility of a direct production of supermassive fermions is rather attractive.

Recently it was discovered that not only usual fermions strongly coupled to the inflaton field can be produced during inflation. Rather surprisingly, gravitinos can be abundantly produced, which may lead to severe cosmological problems[37,38]. This result was quite unexpected; one could think that the gravitino production should be suppressed by the small gravitational coupling constant. However, this is not the case for the massive gravitinos with helicity 3/2. They are produced at the same rate as chiral fermions in global SUSY. One can qualitatively understand this effect in the following way.

Gravitino become massive because they eat the goldstino. One can study gravitino production in the gauge when they are massless, investigate production of chiral fermions, identify those fermions that play the role of goldstino after reheating, and in the end of the calculations use the gauge where each of these goldstinos are eaten by gravitino. As a result, one obtains the number of gravitinos which (in the first approximation) is equal to the number of goldstinos, and this number is not suppressed by the gravitational coupling.

Another interesting effect is the moduli production. If the moduli fields have sufficiently small mass, they can be produced during inflation by the same mechanism as the long-wavelength fluctuations of the inflaton field. The energy density of the moduli fields produced by this mechanism can be very large, which leads to a new version of the cosmological moduli problem[39,38,40].

7 Conclusions

During the last 20 years inflationary theory gradually became the standard paradigm of modern cosmology. But this does not mean that all difficulties are over and we can relax. First of all, inflation is still a scenario which changes with every new idea in particle theory. Do we really know that inflation began at Planck density 10^{94} g/cm^3? What if our space has large internal dimensions, and energy density could never rise above 10^{25} g / cm^3, see[41]? Was there any stage before inflation? Is it possible to implement inflation in string theory / M-theory?

Inflationary theory evolved quite substantially, from old inflation to the theory of chaotic eternal self-reproducing universe. We learned that in most versions of inflationary theory the universe must be flat, and the spectrum of density perturbations should be also flat ("flat paradigm"). But we also learned that there are models where the spectrum of density perturbations is not flat. If necessary, it is possible to obtain not only adiabatic perturbations, but also isocurvature nongaussian perturbations with a very complicated spectrum. It is even possible to have inflation with $\Omega \neq 1$.

Still inflationary models are falsifiable. Each particular inflationary model can be tested, and many of them have been already ruled out by comparison of their predictions with observational data. A new generation of precision experiments in cosmology are going to make our life even more complicated and interesting. However, it is difficult to disprove the basic idea of inflation.

Typical lifetime of a new trend in high energy physics and cosmology nowadays is about 5 to 10 years. If it survived for a longer time, the chances are that it will be with us for quite a while. Inflationary theory by now is 20 years old, and it is still very much alive. It is the only theory which explains why our universe is so homogeneous, flat, and isotropic, and why its different parts began their expansion simultaneously. It provides a mechanism explaining galaxy formation and solves numerous different problems at the intersection between cosmology and particle physics. It seems to be in a good agreement with observational data, and it does not have any competitors. Thus we have some reasons for optimism.

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