A GENERALIZED INFLATION MODEL WITH COSMIC GRAVITATIONAL WAVES

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We propose a Λ-inflation model which explains a large fraction of the COBE signal by cosmic gravitational waves. The primordial density perturbations fulfil both the contraints of large-scale microwave background and galaxy cluster normalization. The model is tested against the galaxy cluster power spectrum and the high-multipole angular CMB anisotropy.

1 Introduction

The observational reconstruction of the *cosmological density perturbation* (CDP) spectrum is a key problem of the modern cosmology. It provids a dramatic challenge after detecting the primordial CMB anisotropy as the signal found by DMR COBE at 10^0 has appeared to be few times higher than the expected value of $\Delta T/T$ in the most simple and best developed cosmological standard CDM model.

During recent years there were many proposals to improve sCDM (in the simplest term, to remove the discrepancy between the CDP amplitudes at $8h^{-1}\mathrm{Mpc}$ as determined by galaxy clusters, and at large scales, $\sim 1000h^{-1}\mathrm{Mpc}$, according to $\Delta T/T$) by adding hot dark matter, a Λ-term, or considering non-flat primordial CDP spectra. Below, we present another, presumably more natural way to solve the sCDM problem based on taking into account a possible contribution of *cosmic gravitational waves* (CGWs) into the large-scale CMB anisotropy; we will also try to preserve the original near-scale-invariant CDP spectrum. Thus, the problem is reduced to the construction of a simple inflation producing near *Harrison-Zel'dovich* (HZ) spectrum of CDPs ($n_S \simeq 1$) and a considerable contribution of CGWs into the large-scale $\Delta T/T$.

A simple model of such kind is Λ-inflation, an inflationary model with an effective metastable Λ-term[1,2]. This model produces both S (CDPs) and T (CGWs) modes which have a non-power-law spectra, with a shallow minimum in the CDP spectrum, located at a scale k_{cr} (there the Λ-term and the scalar field have equal energies while slowly-rolling at inflation) where the S-slope is exact HZ locally. The S-spectrum is 'red' for $k < k_{cr}$, and 'blue' for $k > k_{cr}$; around the k_{cr} scale T/S is close to its maximum, it is of the order unity depending on the model parameters.

2 Λ-inflation with self-interaction

Let us consider a general potential of Λ-inflation driven by a single scalar field φ:

$$V(\varphi) = V_0 + \sum_{\kappa=2}^{\kappa_{max}} \frac{\lambda_\kappa}{\kappa}\varphi^\kappa, \tag{1}$$

where $V_0 > 0$ and λ_κ are constants, $\kappa = 2, 3, 4, ...$ In the case of massive inflaton ($\kappa = \kappa_{max} = 2$) T/S can be larger than unity only when the CDP spectrum slope in the 'blue' asymptote is very steep, $n_S^{blue} > 1.8$. To avoid such a strong spectral bend on short scales ($k > k_{cr}$) we choose here another simple version of Λ-inflation – the case with self-interaction: $\kappa = \kappa_{max} = 4$, $\lambda_4 \equiv \lambda > 0$; this model is called $\Lambda\lambda$-inflation.

The scalar field φ drives an inflationary evolution if $\gamma \equiv -\dot{H}/H^2 < 1$, where $H \simeq \sqrt{V/3}$ (we assume $8\pi G = c = \hbar = 1$ and $H_0 = 100h\mathrm{km\ s^{-1}Mpc^{-1}}$). This condition holds true for all values of φ if

$$c \equiv \frac{1}{4}\varphi_{cr}^2 = \frac{1}{2}\sqrt{\frac{V_0}{\lambda}} > 1, \tag{2}$$

which we imply hereafter. The fundamental gravitation perturbation spectra q_k and h_k generated in $\Lambda\lambda$-inflation in S and T modes, respectively, are as follows[3,4,2]:

$$q_k = \frac{H}{2\pi\sqrt{2\gamma}} = \frac{\sqrt{2\lambda/3}}{\pi}\left(c^2 + x^2\right)^{3/4}, \quad h_k = \frac{H}{\pi\sqrt{2}} = \frac{2c\sqrt{\lambda/3}}{\pi}\left(1 + \frac{x}{\sqrt{c^2+x^2}}\right)^{-1/2}, \tag{3}$$

where

$$x = \ln\left[\frac{k}{k_{cr}}\left(1 + \left(\frac{x}{c}\right)^2\right)^{1/4}\left(1 + \frac{x}{\sqrt{c^2+x^2}}\right)^{2/3}\right] \simeq \ln(k/k_{cr}). \tag{4}$$

The dimentionless spectrum of density perturbations depends on a transfer function $T(k)$:

$$\Delta_k = 3.6 \times 10^6 \left(\frac{k}{h}\right)^2 q_k T(k). \tag{5}$$

3 CDM cosmology from $\Lambda\lambda$-inflation

Let us consider the CDP spectrum with CDM transfer function, normalized both by $\Delta T/T|_{10^0}$ (including the contribution from CGWs) and the galaxy cluster abundance at $z = 0$, to find the family of the most realistic q-spectra produced in $\Lambda\lambda$-inflation.

In total, we have three parameters entering the function q_k : λ, c and k_{cr}. Constraining them by two observational tests we are actually left with only one free parameter (say, k_{cr}) which may be restricted elsewhere by other observations.

To demonstrate explicitly how the three parameters are mutually related, we first employ a simple analytical estimates for the σ_8 and $\Delta T/T$ tests to derive the key equation relating c and k_{cr}, and then solve it explicitly to obtain the range of interesting physical parameters.

Instead of taking the σ-integral numerically we may estimate the spectrum amplitude on cluster scale ($k = k_1 \simeq 0.3h/\mathrm{Mpc}$):

$$q_{k_1} \simeq 4.5 \times 10^{-7} \frac{h^2 \sigma_8}{k_1^2 T(k_1)} \ . \tag{6}$$

On the other hand, the spectrum amplitude on large scale ($k_2 = k_{COBE} \simeq 10^{-3} h/\mathrm{Mpc}$) can be taken from $\Delta T/T$ due to the Sachs-Wolfe effect[5]:

$$\langle \left(\frac{\Delta T}{T}\right)^2 \rangle_{10^0} = \mathrm{S} + \mathrm{T} \simeq 1.1 \times 10^{-10}, \quad \mathrm{S} = 0.04 \, \langle q^2 \rangle_{10^0} \simeq 0.06 q_{k_2}^2. \tag{7}$$

The relation between the variance of the q potential averaged in 10^0-angular-scale and the power spectrum at COBE scale, involves a factor of the effective interval of spectral wavelengths proportional to $\ln\left(\frac{k_2}{k_{hor}}\right) \simeq 1.6$. To estimate T/S, we use the following approximation formula at $x_2 = x_{COBE}$:

$$\frac{\mathrm{T}}{\mathrm{S}} \simeq -6 n_T = \frac{6}{\sqrt{c^2 + x_2^2}} \left(1 - \frac{x_2}{\sqrt{c^2 + x_2^2}}\right) . \tag{8}$$

Evidently, both normalizations, (6) and (7), determine essentially the corresponding q_k amplitudes at the locations of cluster (k_1) and COBE (k_2) scales. Taking their ratio we get the key equation relating c and k_{cr} (see eqs.(3),(4),(8)):

$$\left(\frac{q_{k_1}}{q_{k_2}}\right)^2 \simeq D \left(1 + \frac{\mathrm{T}}{\mathrm{S}}\right) , \tag{9}$$

Eq.(9) has a clear physical meaning: the ratio of the S-spectral powers at cluster and COBE scales is proportional to σ_8^2 and inversly proportional to the fraction of the scalar mode contributing to the large-scale temperature anisotropy variance, S/(S+T). It provides quite a general constraint on the fundamental inflation spectra in a wide set of dark matter models using only two basic measurement (the cluster abundance and large scale $\Delta T/T$). The DM information is contained in the D-coefficient which can be calculated using the same equation (9) for a simple inflationary spectrum (preserving the given DM model). For CDM with $h = 0.5$ we have:

$$D \simeq \frac{0.6 \sigma_8^2}{1 - 3.1 \Omega_b} \ , \qquad \Omega_b < 0.2. \tag{10}$$

The solution of eq.(9) has been obtained in the plane $x_2 - c$ numerically. For the whole range $0.1 < D < 0.5$, it can be analytically approximated with a precision better that 10% as follows:

$$\ln^2\left(\frac{k_0}{k_{cr}}\right) \simeq E\,(c_0 - c)\,(c + c_1) \ , \quad 2 < c \leq c_0 \ . \tag{11}$$

Notice there exists no solution of eq.(9) for $c > c_0$. We have found the following best fit coefficients $E, \ k_0[h/\mathrm{Mpc}]$ and $c_{0,1}$:

$$E \simeq 1, \quad \ln k_0 \simeq 49 D^2 + 1.3, \quad c_0 \simeq 61 D^2 + 6.2, \quad c_1 \simeq 44 D^2 + 4.0.$$

The tensor-mode-contribution is approximated similarly (k_{cr} is measured in $[h/\mathrm{Mpc}]$):

$$\frac{\mathrm{T}}{\mathrm{S}} \simeq \frac{2.53 - 4.3D}{(\ln k_{cr} + 4.65)^{2/3}} + \frac{1}{3} \ . \tag{12}$$

4 Discussion

We have presented a new inflationary model predicting a near scale-invariant spectrum of density perturbations and large amount of CGWs. Our model is consistent with COBE $\Delta T/T$ and cluster abundance data. The perturbation spectra depend on one free scale-parameter, k_{cr}, which can be found in further analysis by fitting other observational data. At the location of k_{cr}, the CDP spectrum transfers smoothly from the red ($k < k_{cr}$) to the blue ($k > k_{cr}$) parts.

Today we seriously discuss a nearly flat shape of the dimensionless CDP spectrum within the scale range encompassing clusters and superclusters,

$$\Delta_k^2 \sim k^{(0.9\pm 0.2)}\,, \quad k \in (0.04, 0.2)h\,\mathrm{Mpc}^{-1}, \tag{13}$$

(with a break towards the HZ slope on higher scales) which stays in obvious disagreement with the sCDM prediction. The arguments supporting eq.(13) came from the analysis of large-scale galaxy distribution[6] and the discovery of large quasar groups[7,8], a higher statistical support was brought by recent measurements of the galaxy cluster power spectrum[9,10].

A possible explanation of eq.(13) could be a fundamental red power spectrum established on large scales, then the transition to the spectrum (13) at $\sim 100\mathrm{Mpc/h}$ would be much easier understood with help of a traditional modification of the transfer function $T(k)$ (e.g. for mixed hot+cold dark matter). The rediness may be not too high, remaining in the range (0.9, 1). A way to enhance the power spectrum at Mpc scale could be the identification of k_{cr} within a cluster scale ($k_{cr} \sim k_1$).

Notice that one of the problems for the matter-dominated models is a low number of σ_8: if $\sigma_8 < 0.6$, then the first acoustic peak in $\Delta T/T$ cannot be as high as $\gtrsim 70\ \mu$K.

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