RECONSTRUCTION OF A SCALAR-TENSOR THEORY OF GRAVITY IN AN ACCELERATING UNIVERSE

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We consider a scalar tensor theory of gravity in which our universe is presently accelerating. We show how one can reconstruct the full theory – namely, the field potential and the functional form of the scalar-gravity coupling – along with the present density of dustlike matter using the following cosmological observations: luminosity distance data and linear dustlike matter density fluctuations data, both as functions of redshift.

1 Introduction

There is growing observational evidence for the presence of a(n effective) cosmological constant term [1,2], as well as theoretical motivations for it (see e.g., for recent reviews[3,4]), accounting for about two thirds of the present energy density of the universe. As well known, a pure cosmological constant of this smallness is highly unnatural. A possible way to account for an effective Λ-term is through a scalar field, often coined quintessence[5] or Λ-field, possessing some appropriate potential. We will show how it is possible to recover the full theory *from the observations*. We will start by briefly reviewing the reconstruction program in the framework of general relativity (GR)[6]. Then we will describe its extension when gravity is given by a scalar-tensor theory (ST)[7].

1.1 Reconstruction in GR

The reconstruction program goes as follows. Writing the Friedmann equations for a flat FRW universe, $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$, filled with pressureless matter and a scalar field ϕ minimally coupled to gravity, we get

$$3H^2 = 8\pi G\Big(\rho_m + \frac{\dot{\Phi}^2}{2} + V\Big) , \qquad (1)$$

$$\dot{H} = -4\pi G\Big(\rho_m + \dot{\Phi}^2\Big) . \qquad (2)$$

Changing variables from time t to redshift z, combining Eqs.(1,2), the following equation is obtained

$$8\pi G V(z) = 3H^2(z) - \frac{1+z}{2}\frac{dH^2}{dz} - \frac{3}{2}\Omega_{m,0}H_0^2(1+z)^3 \qquad (3)$$

The first step is to determine $H(z)$ from the luminosity distance data $D_L(z)$, using

$$\frac{1}{H(z)} = \left(\frac{D_L(z)}{1+z}\right)' , \qquad (4)$$

where a prime stands for a derivative with respect to z. As is seen from (3), we get the potential V as a function of z. It is then straightforeward to recover the function $V(\phi - \phi_0)$. An additional independent way to implement the reconstruction program is by using linear dustlike matter density fluctuations data $\delta_m(z) \equiv (\delta\rho_m)/\rho_m + 3Hv$ where v is the peculiar velocity potential of dustlike matter. Indeed, we have on the relevant small scales for which $\frac{k}{a} \gg H$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\, \delta_m \simeq 0\ , \ . \qquad (5)$$

From (5), extracting δ_m from the observations, it is possible to reconstruct $H(z)$. After that, the reconstruction is completely analogous to the preceding case (with one difference: in the first case we need $\Omega_{m,0}$ as an independent input, while in the second case H_0 is needed). Note that the following inequality must be satisfied in this model

$$\frac{dH^2(z)}{dz} \geq 3\Omega_{m,0}H_0^2(1+z)^2 . \qquad (6)$$

This inequality saturates when the Λ-term is exactly constant. It is not clear from the existing data whether (6) is satisfied at all. Actually the opposite holds: an attempt to reconstruct the potential from the supernovae data, as well as fitting of existing data to a model with a linear equation of state $p_\Lambda = w\varepsilon_\Lambda$, with $w < -1$ for the Λ-term, shows that a possible violation of inequality (6), though strongly restricted, is not completely excluded. It is therefore important to consider a more general class of models, like ST theories of gravity, where the inequality (6) is no longer compulsory. Such ST theories also arise in more fundamental theories like M-theory.

1.2 Reconstruction with a ST theory

We consider now the extension of the reconstruction program to ST theories[7]. We consider the following Lagrangian density in the Jordan frame[8]

$$L = \frac{1}{2}\Big(F(\Phi)\ R - g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\Big) - U(\Phi) + L_m(g_{\mu\nu})\ , \qquad (7)$$

where L_m describes dustlike matter and $F(\Phi) > 0$. This corresponds to the Brans-Dicke parameter $\omega = F/(dF/d\Phi)^2 > 0$. We do not introduce any direct coupling between Φ and L_m so that, in particular, fundamental constants do not change with time. We can proceed in a way analogous to the previous case. For a flat FRW universe the background equations in the JF are given by

$$3FH^2 = \rho_m + \frac{\dot{\Phi}^2}{2} + U - 3H\dot{F}\ , \qquad (8)$$

$$-2F\dot{H} = \rho_m + \dot{\Phi}^2 + \ddot{F} - H\dot{F}\ . \qquad (9)$$

Combining Eqs.(8,9) and changing the argument from time t to redshift z, we obtain the following basic equation for $F(z)$:

$$F'' + \left[(\ln H)' - \frac{4}{1+z}\right] F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z}\right] F$$
$$= \frac{2U}{(1+z)^2 H^2} + 3\,(1+z)\left(\frac{H_0}{H}\right)^2 F_0\,\Omega_{m,0}\,, \tag{10}$$

where a prime denotes a derivative with respect to z.

The effective value of Newton's gravitational constant G_N in Eqs.(8,9) is given by the formula $G_N = 1/8\pi F$. Its present value $G_{N,0}$ is used in the definition of the critical density $\varepsilon_{\rm crit}$. For a massless dilaton, the effective gravitational constant between two test masses is given by

$$G_{\rm eff} = \frac{1}{8\pi F}\left(\frac{2F + 4(dF/d\Phi)^2}{2F + 3(dF/d\Phi)^2}\right)\,. \tag{11}$$

In our case, the dilaton is massive, so (11) will hold for physical scales R such that

$$R^{-2} \gg \max\left(\left|\frac{d^2U}{d\Phi^2}\right|, H^2, H^2\left|\frac{d^2F}{d\Phi^2}\right|\right)\,. \tag{12}$$

Let us now list the restrictions of the theory (7) which follow from solar-system and cosmological tests. The post-Newtonian parameters β and γ for this theory are:

$$\gamma = 1 - \frac{(dF/d\Phi)^2}{F + 2(dF/d\Phi)^2} \qquad\qquad \beta = 1 + \frac{1}{4}\,\frac{F\,(dF/d\Phi)}{2F + 3(dF/d\Phi)^2}\,\frac{d\gamma}{d\Phi}\,. \tag{13}$$

Using the upper bounds on $(\gamma - 1)$ from solar system measurements[9,10], we get

$$\omega_0^{-1} = F_0^{-1}\,(dF/d\Phi)_0^2 < 4\times 10^{-4}\,. \tag{14}$$

So, $G_{N,0}$ and $G_{\rm eff,0}$ coincide with better than 2×10^{-4} accuracy. On the other hand, the difference between G_N and $G_{\rm eff}$ may be larger at redshifts $z \sim 1$ since neither the upper limit on β, nor the present experimental bound $|\dot G_{\rm eff}/G_{\rm eff}| < 6\times10^{-12}/{\rm yr}$[10] significantly restrict $(d^2F/d\Phi^2)_0$. Also, our theory should satisfy the following requirements: 1) The Λ-term satisfies $\Omega_{\Lambda,0} \sim 0.7 \sim 2\Omega_{m,0}$; 2) $2U_0 > (\rho_m + 2\dot\Phi^2 + 3\ddot F + 3H\dot F)_0$ so that our universe is presently accelerating; 3) The dark matter described by the Λ-term remains unclustered at scales up to $R \sim 10h^{-1}(1+z)^{-1}$ Mpc. To achieve this, it is *sufficient* to assume that the inequality (12) is satisfied for all scales of interest.

The first step is purely kinematical: like in GR, we determine $H(z)$ from $D_L(z)$ using Eq.(4). However, in contrast to GR, Eq.(10) is no longer sufficient to determine $U(z)$; one should know $F(z)$, too. For this purpose we will use $\delta_m(z)$. We consider the perturbation equations in the longitudinal gauge $ds^2 = -(1+2\phi)dt^2 + a^2(1-2\psi)d\mathbf{x}^2$, (see[7] for details). The idea is that, in the short wavelength limit, the leading terms are either those containing k^2, or those with δ_m. The equation for dustlike matter density perturbation is then standard:

$$\ddot\delta_m + 2H\dot\delta_m + k^2a^{-2}\phi \simeq 0\,. \tag{15}$$

We also obtain

$$\delta\Phi \simeq (\phi - 2\psi)\,\frac{dF}{d\Phi} \simeq -\phi\,\frac{F\,dF/d\Phi}{F + 2(dF/d\Phi)^2}\,. \tag{16}$$

Hence, unlike in GR where matter producing the Λ-term is not gravitationally clustered at small scales, in scalar-tensor gravity the dilaton remains partly clustered for arbitrarily small scales, this clustering being small only because ω is large. In the same short wavelength limit, Poisson's equation has the same form as in GR, however with Newton's constant replaced by $G_{\rm eff}$, defined in (11) above. So, Eq.(15) finally becomes

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\,\rho_m\;\delta_m \simeq 0\;, \qquad (17)$$

In terms of z, (17) reads:

$$H^2\;\delta_m'' + \left(\frac{(H^2)'}{2} - \frac{H^2}{1+z}\right)\delta_m' \simeq \frac{3}{2}(1+z)H_0^2\frac{G_{\rm eff}(z)}{G_{N,0}}\;\Omega_{m,0}\;\delta_m\;. \qquad (18)$$

Extracting $H(z)$ (through $D_L(z)$) and $\delta_m(z)$ from observations with sufficient accuracy, we can reconstruct $G_{\rm eff}(z)/G_{N,0}$ analytically. Since, as follows from Eq.(14), the quantities $G_{\rm eff,0}$ and $G_{N,0}$ coincide with better than 0.02% accuracy, Eq.(18) taken at $z=0$ gives also the value of $\Omega_{m,0}$ with the same accuracy. Thus, in principle, no independent measurement of $\Omega_{m,0}$ is required.

The resulting equation $G_{\rm eff}(z) = p(z)$, where $p(z)$ is a given function following from observational data, can be transformed into a nonlinear second order differential equation for $F(z)$ if we exclude $d\Phi$ (which appears in $dF/d\Phi$) using the background equation (9), which reads

$$\Phi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z}\right]\;F' + \frac{2(\ln H)'}{1+z}\;F - 3(1+z)\frac{H_0^2}{H^2}F_0\;\Omega_{m,0}\;. \qquad (19)$$

Therefore, $F(z)$ can be determined by solving that equation provided F_0 $(=1/8\pi G_{N,0})$ and F_0' are known. Finally, using Eq. (19) we find $\Phi(z)$ by simple integration. After that, both unknown functions $F(\Phi)$ and $U(\Phi)$ are completely fixed as functions of $\Phi - \Phi_0$ in that range probed by the data. Equations (18), (10) and (19) giving the subsequent steps of the reconstruction constitute our fundamental result.

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