EXTRACTION OF POINT SOURCES FROM CMB MAPS USING WAVELETS

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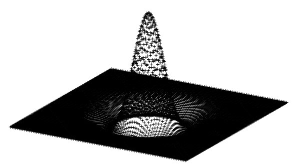

It is shown the power of the wavelet analysis in the detection of point sources in CMB maps, as well as in the estimation of the intensity of such sources. This method produce similar results that the ones proposed by Tegmark & Oliveira-Costa 1999, but with the advantage that in our method nothing is assumed about the underlying signal. We are able to give an estimation of the number of point sources that this method could detect in the different Planck channels, as well as the mean error in the estimation of the flux. Another important consequence of this study is the complementarity of our method with others based on Maximun Entropy Methods.

1 Introduction

In the last years the usage of methods based on wavelets have turned up in the field of Astrophysics and several works have been done on the topic of the Cosmic Microwave Background (CMB). In particular, the detection of non-gaussianity (Ferreira et al. 1997, Hobson et al. 1999, Aghanim & Forni 1999) and denoising (Sanz et al. 1999a, Sanz et al. 1999b, Tenorio et al. 1999) have been considered. An interesting application is the detection and the subtraction of point sources in the CMB maps (Cayón et al. 1999, Tenorio et al. 1999). One of the advantages of the wavelet transform as opposed to the Fourier transform is the spatial information that the first method carry on. A 2D continuous wavelet transform of a function $f(\vec{x})$ is defined as

$$W(\vec{R},\vec{a}) = \int d\vec{x} f(\vec{x}) \Psi(\vec{R},\vec{a};\vec{x}), \qquad (1)$$

where $W(\vec{R},\vec{a})$ is the wavelet coefficient in the scale $\vec{R}$ at the position $\vec{a}$ and the wavelet Ψ is defined through the *mother wavelet* ψ

$$\Psi(\vec{R},\vec{a};\vec{x}) = \frac{1}{|R_1 R_2|^{\frac{1}{2}}}\psi\Big(\frac{x_1-a_1}{R_1},\frac{x_2-a_2}{R_2}\Big). \tag{2}$$

The wavelet function satisfies the following properties:

$$\int d\vec{x}\Psi(\vec{R},\vec{a},\vec{x}) = 0, \qquad \text{compensation;} \tag{3}$$

$$\int d\vec{x}|\Psi(\vec{R},\vec{a},\vec{x})|^2 = 1, \qquad \text{normalization;} \tag{4}$$

$$C_\Psi \equiv (2\pi)^2\int d\vec{k}\frac{1}{|k_1 k_2|^2}|\widehat{\Psi}_{\vec{k}}|^2 < \infty, \qquad \text{admissibility;} \tag{5}$$

where $\widehat{\Psi}_{\vec{k}}$ is the fourier transform of the wavelet function. The last propierty is a necessary and sufficient condition for the synthesis of the function $f(\vec{x})$ through the wavelets coefficients

$$f(\vec{x}) = \frac{1}{C_\Psi}\int d\vec{R}d\vec{a}\frac{1}{|R_1 R_2|^2}W(\vec{R},\vec{a})\Psi(\vec{R},\vec{a};\vec{x}). \tag{6}$$

We have aplied these techniques to detect point sources in CMB maps with the characteristics of the Planck mission, and restricting the study to flat sky patches of 12.8^ox12.8^o. A detailed presentation, development and characteristics of the method are presented in Cayón et al. 1999. Here we point out the main properties and results of that work. We also present estimations for the number of detections and the error in the estimation for all the Planck channels.

2 Detection and estimation

Point sources emission is one of the most important contaminants to the power spectrum of the cosmological signal that instruments onboard of the Planck satellite are going to observe; specially at scales of the antenna beamsize. This can be seen in the Figures 7 and 8 of Toffolatti et al. 1998, where at the highest and lowest frequencies, the contribution of point sources at small scales is one of the main foregrounds. We consider that the point sources are observed with an antenna with a Gaussian rsponse. So, in this situation the signal due to a point source is described by

$$f(\vec{x}) = Ae^{-\frac{(\vec{x}-\vec{x}_o)^2}{2\sigma_a{}^2}}, \tag{7}$$

where A is the flux of the point source and σ_a is the antenna beamsize. Because the spherical simmetry of the problem, we will restrict to isotropic wavelets, i.e. we use a single scale σ. The appropriate isotropic wavelet in this case is the so called Mexican Hat given by

$$\psi(r) = \frac{1}{\sqrt{2\pi}}\left(2-\Big(\frac{r}{\sigma}\Big)^2\right)e^{-\frac{r^2}{2\sigma^2}}, \tag{8}$$

where σ is the wavelet width. It can be shown that this wavelet is optimal when the signal is just a point source function plus white noise.

The wavelet coefficient at the position of the peak source is given by

$$\frac{W(R,\vec{x}_o)}{R} = 2\sqrt{2\pi}A\frac{(\frac{R}{\sigma_a})^2}{\Big(1+(\frac{R}{\sigma_a})^2\Big)^2}, \tag{9}$$

this ratio is maximun at scale $R \equiv \sigma_a$.

The wavelet transform has the following general behaviour: it amplifies the component of the signal that resembles the wavelet, whereas the characteristics of the signal quite different from the wavelet practically vanish. So, in our case, when we convolve the point sources with the wavelet at the right scale (σ_a), we obtain coefficients with a significant contribution. On the other hand signals as noise, CMB, dust (whose profiles are quite different from the wavelet one) have smaller wavelet coefficients. The main effect of this is that we are *amplifying* the amplitude of the point source with respect the map dispersion

$$\frac{W(R=\sigma_a)}{\sigma_{W(R=\sigma_a)}} > \frac{A}{\sigma_{\text{Real Map}}}. \tag{10}$$

This is the idea of the filter developed by Tegmark & Oliveira-Costa 1998 (T&O-C), but in that case is necessary to assume that the foregrounds and the cosmological signal are represented by Gaussian fields and that we know the spectral dependence of these signals. In Cayón et al. 1999, it has been shown that isotropic wavelets such as the Mexican Hat give similar or even better results in the number of detections than the ones given by T&O-C; with the main advantage than nothing is assumed about the underlying signal.

3 Results

We have performed two simulations with the Planck channels characteristics at 857 GHz (dust dominated) and 100 GHz (CMB dominated). The signal at the highest frequency is the sum of dust emission (Finkbeiner et al. 1999 model), white noise and point sources emission (Toffolatti et al. 1998 model). The other signal is the sum of Gaussian CMB ($\Omega_m = 1; \Omega_\Lambda = 0$), white noise and point sources emission (Toffolatti et al. 1998). We have tested both the Mexican Hat wavelet and the T&O-C filter and, as we can see in Table 1, the wavelet technique obtain similar or better results than T&O-C filter.

Table 1: Detections over 5σ using Mexican Hat, T&O-C filter and the real space

Frequency (GHz)	Wavelet space	(T&O-C)	Real Space
857	35	32	1
100	3	3	0

For the 857 GHz channel the 80% of point sources with flux above 1.58 Jy are detected with errors in the flux estimation below 20% (absolute error), whereas the 100% of point sources above 0.36 Jy in the 100 GHz channel are localized with errors in the flux estimation below 25% (absolute error).

It is important to point out that this method can be complementary to those based in Maximun Entropy presented in Hobson et al. 1999. This method has an important contribution coming from bright sources (the ones well subtracted by the Mexican Hat wavelet) in the residual maps. So, the bright point sources could be removed from the maps before MEM is applied.

We have made a primary estimation of the number of point sources that the Mexican Hat wavelet can detect in each Planck channel. The results are presented in Table 2. In these simulations, all foregrounds are present except synchrotron emission.

Finally, we have carried out more detailed simulations with all emissions and in several sky areas, to give a significant prediction for the number of point sources that an experiment as Planck could detect. Also we are able to determine espectral indexes for the brightest sources. Results will be given in a future paper.

Table 2: Primary predictions from Point Sources detection in Planck with Mexican Hat

Frequency (GHz)	PS's detected above 5σ in Real Space (*)	PS's detected above 5σ in Wavelet Space	Amplification	Flux (Jy) for wich 100% of PS's are detected	Maximum error in the amplitude estimation for previus flux (%)
857	10(0)	40	11.90	1.80	20
545	12(2)	14	5.33	0.91	35
353	2(2)	10	3.15	0.37	20
217	1(1)	4	3.49	0.25	10
143	0	3	2.47	0.23	12
100(HFI)	0	2	2.36	0.54	28
100(LFI)	0	4	2.22	0.36	15
70	1(1)	4	2.84	0.41	25
44	0	2	1.52	0.78	20
30	0	2	1.81	0.56	15

(*) Numbers in brackets indicate how many of the detections above 5σ are real PS's (above 0.92 Jy in the 857 PS simulation; 0.39 Jy in the 545 PS simulation; 0.13 Jy in the 353 PS simulation; 0.05 Jy in the 217, 100(HFI), 100(LFI) & 70 PS simulations; 0.03 Jy in the 143 PS simulation; 0.12 Jy in the 44 PS simulation and 0.13 Jy in the 30 PS simulation)

4 Conclusions

The wavelet method is a new and powerful tool to study the CMB field. In particular, the Mexican Hat wavelet is an efficient one to detect and susbtract point sources from CMB maps. The main advantage of this method is that nothing is assumed about the underlying signal, and the results obtained are similar or better than the ones obtained by T&O-C filter. This method represents a complementary tool to the Maximun Entropy Methods concerning the detection and subtraction of the brightest point sources.

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