Weak gravitational lensing effects on the determination of Ω_m and Ω_Λ from SNeIa

P. VALAGEAS
Service de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette, France

We present an analytical calculation of the probability distribution of the magnification of distant sources due to weak gravitational lensing from non-linear scales, using a realistic description of the non-linear density field. Then, we can directly express the p.d.f. $P(\mu)$ in terms of the p.d.f. of the density contrast realized on non-linear scales (typical of galaxies). Next, we study the effects of weak lensing on the derivation of the cosmological parameters from SNeIa. We show that the inaccuracy introduced by weak lensing is rather small and that observations can unambiguously discriminate between $\Omega_m = 0.3$ and $\Omega_m = 1$, and one can clearly distinguish an open model from a flat cosmology. However, we note that going to high redshift $z_s > 1$ does not decrease much the inaccuracy because weak lensing effects grow with z_s. On the other hand, one may obtain some valuable information on the properties of the underlying non-linear density field from the measure of weak lensing distortions.

1 Probability distribution of the magnification

1.1 Density field

In order to obtain the probability distribution of the magnification of distant sources by gravitational lensing we need the properties of the density field. Hence we briefly recall here the formalism we use to characterize the density fluctuations. It is convenient to express the probability distribution of the density contrast $\delta = (\rho - \overline{\rho})/\overline{\rho}$ at scale R and redshift z (here $\overline{\rho}(z)$ is the mean universe density) in terms of the many-body correlation functions $\xi_p(\mathbf{r}_1, ..., \mathbf{r}_p)$. Thus we define the quantities ($p \geq 2$):

$$S_p = \frac{\overline{\xi}_p}{\overline{\xi}_2^{\,p-1}} = \frac{\langle \delta_R^p \rangle_c}{\langle \delta_R^2 \rangle^{\,p-1}} \qquad \text{with} \qquad \overline{\xi}_p = \int_V \frac{d^3r_1 ... d^3r_p}{V^p} \;\; \xi_p(\mathbf{r}_1, ..., \mathbf{r}_p) \tag{1}$$

where $V = 4\pi/3R^3$ is the volume of a spherical sphere of radius R and $\langle\delta_R^p\rangle_c$ are the cumulants of the density contrast δ at scale R. Next we introduce the generating function

$$\varphi(y) = \sum_{p=1}^{\infty} \frac{(-1)^{p-1}}{p!} \, S_p \, y^{\,p} \tag{2}$$

with $S_1 = 1$. Then, one can show[1] that the probability distribution of the density contrast δ within spheres of size R is:

$$P(\delta) = \int_{-i\infty}^{+i\infty} \frac{dy}{2\pi i \overline{\xi}} \, e^{[(1+\delta)y - \varphi(y)]/\overline{\xi}} \tag{3}$$

where we note $\overline{\xi}_2$ as $\overline{\xi}$. The interest of the generating function $\varphi(y)$ is that in the highly non-linear regime, where the stable-clustering ansatz[2] provides a good description of the density field, it is independent of time. Then, in order to get the properties of the non-linear density field we only need the evolution of the two-point correlation function ξ_2, or of the power-spectrum $P(k)$, which is given by the fits obtained from numerical simulations[3]. This *scaling property* has been checked in details against numerical simulations for various power-spectra and it also characterizes the mass functions of collapsed objects as well as underdense regions [4].

1.2 Magnification

As a photon travels from a distant source towards the observer its trajectory is deflected by density fluctuations along the light path. This produces an apparent displacement of the source as well as a distortion of the image. In particular, the convergence κ (defined as the trace of the shear matrix) will magnify (or demagnify) the source as the cross section of the beam is decreased (or increased). One can show[5] that the convergence κ and the magnification μ are given for small values of $|\kappa|$ by:

$$\mu = 1 + 2\kappa, \quad \kappa = \frac{3}{2}\,\Omega_m \int_0^{\chi_s} d\chi \; w(\chi,\chi_s)\;\delta(\chi) \;\; \text{with} \;\; w(\chi,\chi_s) = \frac{H_0^2}{c^2}\,\frac{\mathcal{D}(\chi)\mathcal{D}(\chi_s-\chi)}{\mathcal{D}(\chi_s)}\;(1+z) \tag{4}$$

where χ is the radial comoving coordinate (and χ_s corresponds to the redshift z_s of the source) while $\mathcal{D}$ is the angular distance. In order to obtain the p.d.f. $P(\mu)$ we simply need to derive the cumulants $\langle\mu^p\rangle_c$. This will provide the parameters $S_{\mu,p}$, similar to Eq. 1, and the generating function $\varphi_\mu(y)$, similar to Eq. 2. From Eq. 4 we can write the cumulants $\langle\mu^p\rangle_c$ in terms of the cumulants $\langle\delta^p\rangle_c$ of the density field and using Eq. 1 we can express these quantities in terms of the coefficients S_p. Next, the resummation of these cumulants $\langle\mu^p\rangle_c$ as in Eq. 2 can be written in terms of the generating function $\varphi(y)$ which characterizes the underlying density field. However, it is convenient to introduce first a “reduced magnification” η by:

$$\eta = \frac{\mu - \mu_{\rm min}}{1-\mu_{\rm min}} \quad \text{with} \quad \mu_{\rm min}(z_s) = 1 - 3\Omega_m F_s(\chi_s) \quad \text{and} \quad F_s(\chi_s) = \int_0^{\chi_s} d\chi \; w(\chi,\chi_s) \tag{5}$$

where $\mu_{\rm min}(z_s)$ is the minimum value of the magnification of a source located at redshift z_s as seen from Eq. 4 (since $\delta \geq -1$). Then, we obtain the simple expressions:

$$\varphi_\eta(y) = \int_0^{\chi_s} d\chi \; \frac{\xi_\eta}{I_\mu}\;\varphi\left(y \frac{w}{F_s}\frac{I_\mu}{\xi_\eta}\right) \;, \quad P(\eta) = \int_{-i\infty}^{+i\infty} \frac{dy}{2\pi i \xi_\eta}\; e^{[\eta y - \varphi_\eta(y)]/\xi_\eta} \quad \text{and} \quad P(\mu) = \frac{P(\eta)}{1-\mu_{\rm min}} \tag{6}$$

for the p.d.f. $P(\eta)$ and $P(\mu)$, where we defined:

$$I_\mu(z) = \pi \int_0^\infty \frac{dk}{k} \frac{\Delta^2(k)}{k} \quad \text{with} \quad \Delta^2(k) = 4\pi k^3 P(k) \quad \text{and} \quad \xi_\eta = \int d\chi \left(\frac{w}{F_s}\right)^2 I_\mu \qquad (7)$$

Thus, Eq. 6 provides the p.d.f. $P(\mu)$ or $P(\kappa)$. In particular, our predictions for $P(\kappa)$ have been shown[6] to agree very well with numerical simulations for a finite smoothing angle $\theta \leq 2'$.

2 Numerical results

In this section we present the numerical results we obtain from the formalism described above, for the case of a low-density flat universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70$ km/s/Mpc, $\sigma_8 = 0.9$ and $\Gamma = 0.21$. This is described with much more details in Valageas[7].

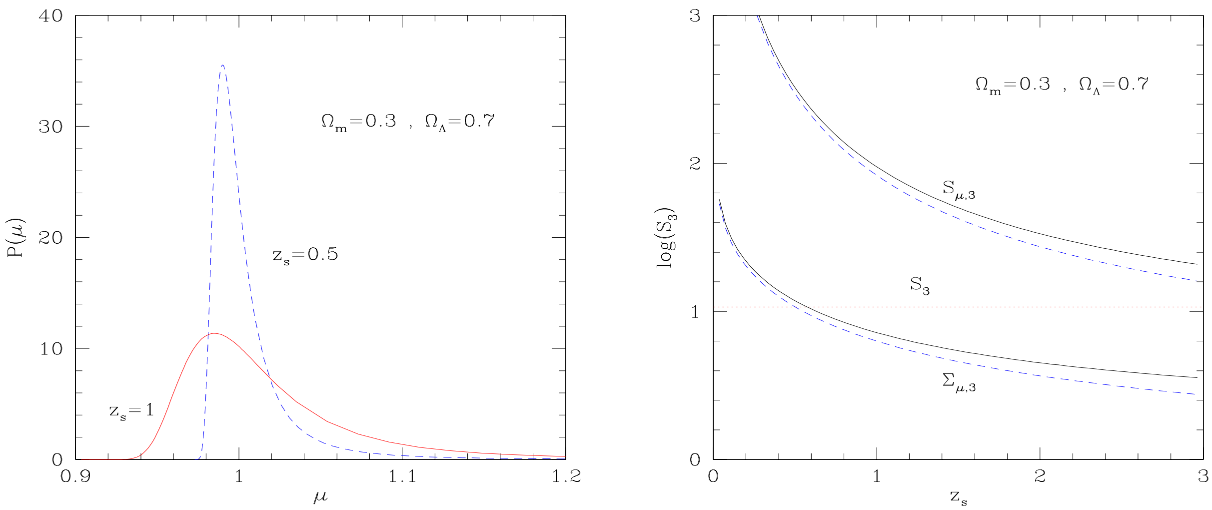

Figure 1: *Left panel:* the p.d.f. $P(\mu)$ for the redshifts $z_s = 0.5$ (dashed line) and $z_s = 1$ (solid line). *Right panel:* the skewness $S_{\mu,3}$ (upper solid line) of the magnification, the quantity $\Sigma_{\mu,3}$ (lower solid line) and the skewness S_3 (dotted line) of the density contrast. The dashed lines show the approximations $S_{\mu,3}^{\rm app}$ and $\Sigma_{\mu,3}^{\rm app}$.

We display in the left panel in Fig.1 the p.d.f. $P(\mu)$ of the magnification μ for the redshifts $z_s = 0.5$ and $z_s = 1$ of the source. One can clearly see the asymmetry due to the lower cutoff at $\mu_{\rm min}$ and the extended large μ tail, as well as the shift of the maximum of $P(\mu)$ below the mean $\langle\mu\rangle = 1$. This agrees with the behaviour obtained in numerical simulations[6]. Note that the p.d.f. $P(\mu)$ is very different from a gaussian at both redshifts. A convenient way to quantify the departure from a gaussian is to consider the skewness $S_{\mu,3} = \langle\mu^3\rangle_c/\langle\mu^2\rangle_c^2$ of the magnification, or the quantity $\Sigma_{\mu,3} = \langle\mu^3\rangle_c/\langle\mu^2\rangle_c^{3/2}$. These are shown in the right panel in Fig.1, together with the skewness S_3 of the density field and the approximations $S_{\mu,3}^{\rm app} = S_3/(3\Omega_m F_s)$ and $\Sigma_{\mu,3}^{\rm app} = S_{\mu,3}^{\rm app} \langle\mu^2\rangle_c^{1/2}$. We can check that at larger redshift the p.d.f. $P(\mu)$ becomes "closer" to a gaussian as the parameters $S_{\mu,3}$ and $\Sigma_{\mu,3}$ decrease. However, it is interesting to note that these parameters can be fairly large (higher than S_3) and even diverge for $z_s \to 0$. Hence at low z_s the p.d.f. $P(\mu)$ is strongly non-gaussian. Thus, if the intrinsic magnitude dispersion of the sources is sufficiently small, one might observe these non-gaussian features and check that they agree with the usual models of the density field. Moreover, from Eq. 6 we see that one could get an estimate of the properties of the underlying density field itself (e.g., its skewness S_3) from $P(\mu)$.

In practice[8], one uses the Type Ia supernovae as standard candles in order to derive the cosmological parameters Ω_m and Ω_Λ from the observed relation redshift $\leftrightarrow$ luminosity distance.

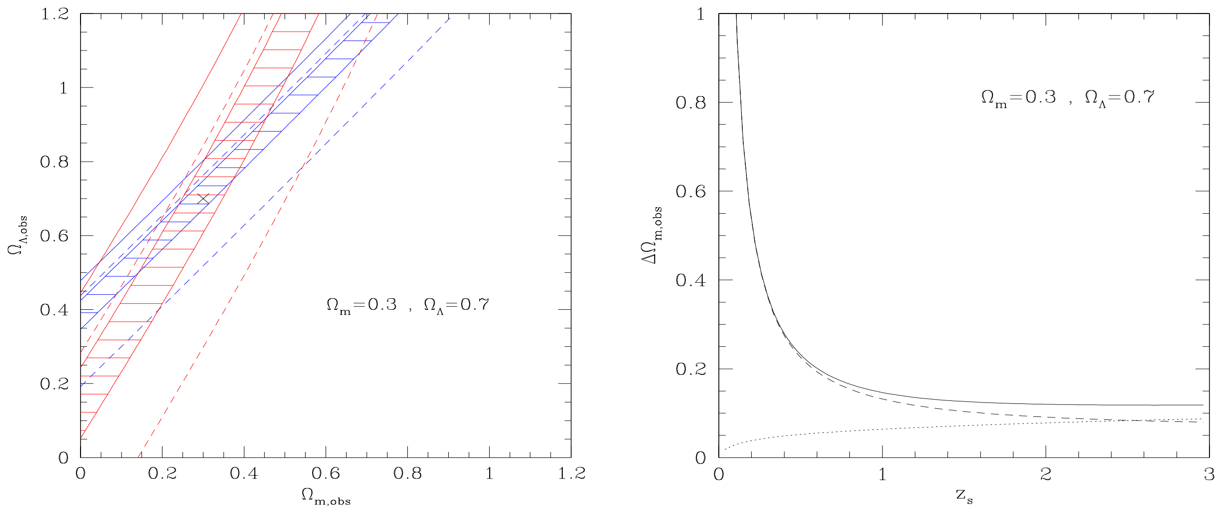

Figure 2: *Left panel:* The "confidence regions" $\mathcal{R}_\mathcal{P}$ for the observed parameters $\Omega_{m,obs}$ and $\Omega_{\Lambda,obs}$ for the redshifts $z_s = 0.5$ and $z_s = 1$. The dashed region within two solid lines corresponds to $\mathcal{P} = 68\%$: within 68% of the cases the cosmology will be observed to lie within this domain in the $(\Omega_{m,obs}, \Omega_{\Lambda,obs})$ plane. The region within the two dashed lines corresponds to $\mathcal{P} = 95\%$. The solid line $\mathcal{L}_{\min}$ on the left corresponds to the lower bound $\mu_{\min}$: the observed parameters $(\Omega_{m,obs}, \Omega_{\Lambda,obs})$ cannot lie to the left of this curve. The boundary lines obtained for a larger redshift are steeper (i.e. closer to the vertical). *Right panel:* the uncertainty $\Delta\Omega_{m,obs}$ of the observed parameter Ω_m from sources at redshift z_s. The solid line takes into account both the intrinsic dispersion σ_B of supernovae and the fluctuation $\delta\mu$ of the magnification by weak lensing. The dashed curve shows the influence of σ_B alone (case with $\delta\mu = 0$) while the dotted curve represents the influence of weak lensing alone (case with $\sigma_B = 0$).

However, even if these sources are perfect candles with no intrinsic dispersion nor instrumental noise, the weak lensing effects discussed above will introduce some dispersion and some bias as the supernovae will be randomly magnified by the density fluctuations located along the line of sight. Thus, for a given redshift z_s of the source and a peculiar cosmology $(\Omega_m, \Omega_\Lambda)$ of the actual universe we can obtain "confidence regions" in the $(\Omega_{m,obs}, \Omega_{\Lambda,obs})$ plane for the observed parameters $\Omega_{m,obs}$ and $\Omega_{\Lambda,obs}$. This means that for any $\mathcal{P}$ with $0 \leq \mathcal{P} \leq 1$, the "observed universe" has a probability $\mathcal{P}$ to lie within the domain $\mathcal{R}_\mathcal{P}$. Of course, a measure at a single redshift only provides a value for the distance $\mathcal{D}_{obs}$ (hence for the deceleration parameter q_0 at low redshift $z_s \to 0$). Hence the parameters $\Omega_{m,obs}$ and $\Omega_{\Lambda,obs}$ are only constrained to lie on a line in the $(\Omega_{m,obs}, \Omega_{\Lambda,obs})$ plane. As a consequence, the regions $\mathcal{R}_\mathcal{P}$ are unbounded strips in the $(\Omega_{m,obs}, \Omega_{\Lambda,obs})$ plane. We display in the left panel in Fig.2 these "confidence regions" $\mathcal{R}_\mathcal{P}$ for the redshifts $z_s = 0.5$ and $z_s = 1$ and the confidence levels $\mathcal{P} = 68\%$ and by $\mathcal{P} = 95\%$. Thus, we can see that two observations at redshifts $z_s = 0.5$ and $z_s = 1$ only determine Ω_m within an interval $\Delta\Omega_m > 0.3$. However, one can clearly discriminate a critical density universe from a low-density flat scenario and the accuracy improves with more observations. In order to compare the effect of weak gravitational lensing on the derivation of the cosmological parameters with the inaccuracy due to the intrinsic magnitude dispersion σ_B of the sources, we show in the right panel in Fig.2 the uncertainty $\Delta\Omega_{m,obs}$ of the observed parameter Ω_m (assuming a flat universe) due to σ_B alone, to the weak lensing alone, and to the sum of both effects. We can see that the error due to the intrinsic dispersion σ_B dominates up to $z_s \leq 2$ (but for a critical density universe the weak lensing contribution already dominates for $z_s \geq 1.4$). Note also that going to high redshift $z_s > 1$ does not increase the accuracy of the determination of Ω_m by much.

References

1. Balian R., Schaeffer R., A&A **220**, 1 (1989).

2. Peebles P.J.E. in *The large scale structure of the universe*, Princeton University Press, 1980.
3. Peacock J.A., Dodds S.J., MNRAS **280**, L19 (1996).
4. Valageas P., Lacey C., Schaeffer R., 2000, MNRAS **311**, 234 (2000).
5. Bernardeau F., van Waerbeke L., Mellier Y., A&A **322**, 1 (1997).
6. Valageas P., accepted by A&A, astro-ph 9911336 (2000)
7. Valageas P., accepted by A&A, astro-ph 9904300 (2000)
8. Perlmutter S., Aldering G., Goldhaber G., et al., ApJ **517**, 565 (1999).