

**MODULAR INFLATION
WITHOUT FLAT DIRECTIONS
AND THE CURVATON**

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Inflation and the Hot Big Bang

The Standard Hot Big Bang

The Cosmological Principle: On large scales the Universe is Homogeneous and Isotropic

CMB Observations: Spatial Flatness & $n_s \approx 1$

flat FRW : $ds^2 = dt^2 - a^2(t) (dr^2 + r^2 d\Omega^2)$

- **Successes:** Hubble expansion, t_0 , BBN, CMB
- **Problems:** Baryons, Horizon & Flatness, LSS & CMB-Anisotropy

Inflation

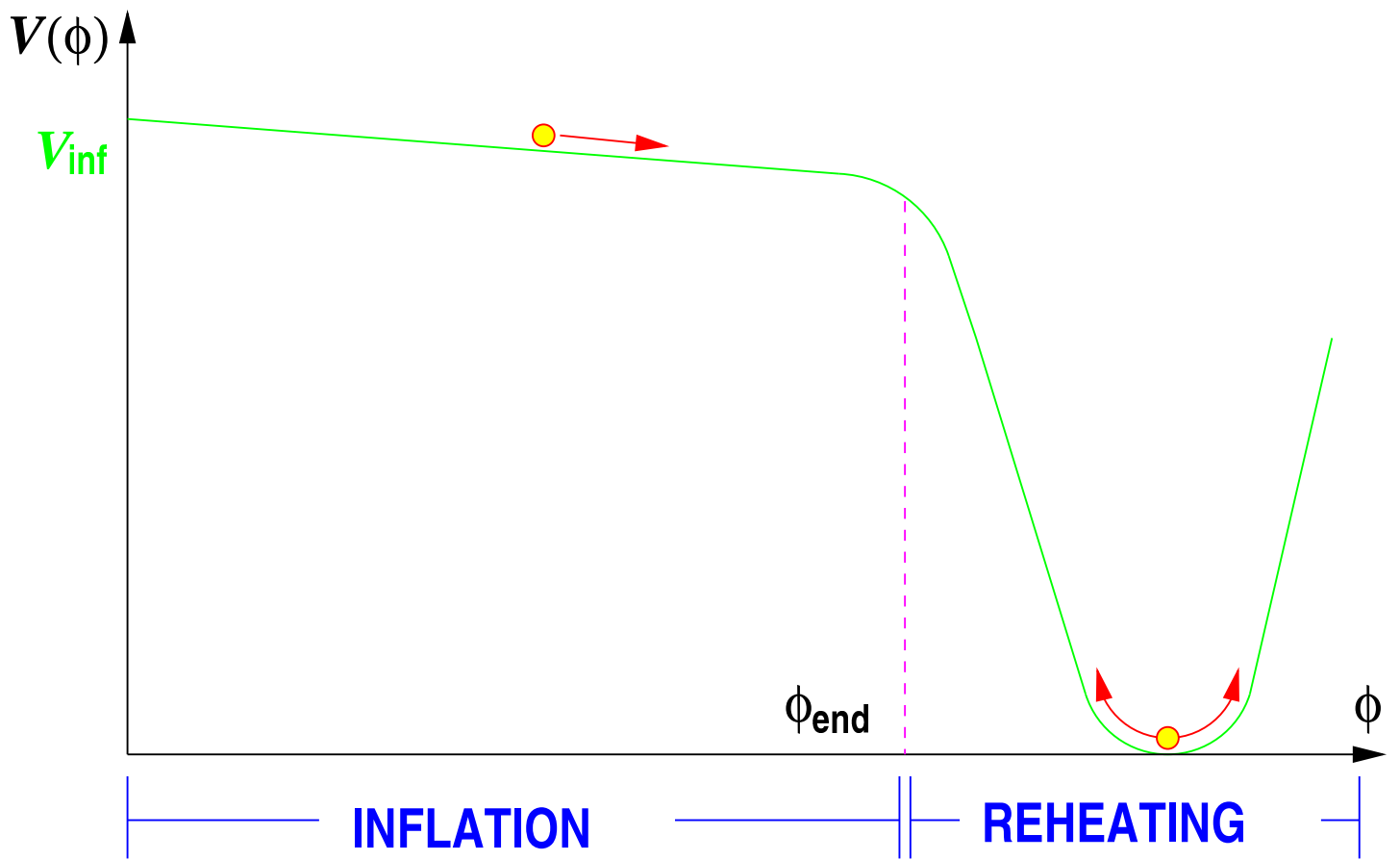
Inflation: A period of accelerated expansion in the Early Universe

Friedmann : $H^2 \equiv (\dot{a}/a)^2 = \frac{1}{3} \rho / M_P^2$

$\rho \simeq M_P^2 \Lambda_{\text{eff}} \simeq \text{const.} \Rightarrow a \propto e^{Ht}$

- **Horizon:** superluminal expansion rate \Rightarrow superhorizon causal correlations
- **Flatness:** Curvature = inflated away
- **LSS & CMB-A:** Particle Production

Inflationary Paradigm: The Universe inflates due to being dominated by the potential density $V(\phi)$ of a scalar field ϕ (inflaton)



The unbearable lightness of the inflaton

Light Inflaton: $m_{\text{eff}} \equiv \sqrt{V''} \ll H_{\text{inf}}$

A: Inflationary period long enough

There need to be at least $\Delta N_{\text{inf}} \lesssim 70$
e-foldings of Inflation for Horizon & Flatness

B: Particle Production of inflaton perturbations

$m < H \Leftrightarrow$ Compton wavelength $>$ Horizon ($\sim H^{-1}$)

Quantum fluctuations reach and exit Horizon

$\Delta \mathcal{E} \cdot \Delta t \sim 1 \Rightarrow \delta\phi = H/2\pi$ (Hawking - T)

Particle Horizon during Inflation \Leftrightarrow Event Horizon of inverted Black Hole

After Horizon exit: **fluctuation \rightarrow classical object**

Klein - Gordon : $(\ddot{\delta\phi}) + 3H(\dot{\delta\phi}) + m^2(\delta\phi) = 0$

$\Rightarrow \delta\phi \simeq \frac{H}{2\pi} \left[e^{-\frac{1}{3}(\frac{m}{H})^2 H \Delta t} - \frac{1}{9} \left(\frac{m}{H}\right)^2 e^{-3H \Delta t} \right] \simeq H/2\pi$

perturbation freezes \Rightarrow **scale-invariant spectrum**

C: Flatness of curvature perturbation spectrum

$n_s(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$ spectral index of power spectrum

$n_s = 1 + 2\eta - 6\epsilon$: $\epsilon = -\dot{H}/H^2 \ll 1$ &
 $\eta \equiv M_P^2 V''/V = \frac{1}{3}(m/H)^2$

WMAP: $n_s = 0.99 \pm 0.04$

The η -Problem of Inflation

$$V_F(W, K) \simeq e^{K/M_P^2} |W|^2 \times$$

$$\times \left[\sum_{nm} \left(\frac{K_n}{M_P^2} + \frac{W_n}{W} \right) K^{n\bar{m}} \left(\frac{K_m}{M_P^2} + \frac{W_m}{W} \right)^* - 3M_P^{-2} \right]$$

W = Superpotential K = Kähler potential

$$\mathcal{L}_{\text{kin}} = \sum_{nm} K_{nm} \partial_\mu \phi_n \partial^\mu \bar{\phi}_m \Rightarrow \text{Canonical: } \underline{K_{nm} = \delta_{nm}}$$

SUGRA corrections lift flatness of V

$$V_F'' = \frac{K''}{M_P^2} V_F + \dots \Rightarrow \eta = K'' + \dots$$

$$K : \text{Canonical} \Rightarrow \underline{\eta \sim 1 \Rightarrow m \sim H_{\text{inf}}}$$

Famous η -Problem of Inflation

Curvaton to the Rescue

Curvaton: Field responsible for curvature perturbation is other than the inflaton

Curvaton = not associated with physics of inflation
 \Rightarrow can be accommodated in simple extensions of SM

The curvaton can link cosmological observations with collider experiments

e.g. PNGB curvaton = protected by Global U(1)

$$\underline{m_\sigma \ll H} \Rightarrow \delta\sigma = H/2\pi \quad (\text{B})$$

$$\boxed{n_s - 1 = 2(\eta_\sigma - \epsilon)} \ll 1 \quad (\text{C})$$

Only requirement : **Length of Inflation** (A)

Modular Hybrid Inflation

$$V(\Phi, \phi) = \frac{1}{2}m_{\Phi}^2\Phi^2 + \frac{1}{2}\lambda\Phi^2\phi^2 + \frac{1}{4}\alpha(\phi^2 - M^2)^2$$

Φ & ϕ : Flat Directions of Supersymmetry
lifted by Supergravity corrections

$$m_{\Phi} \sim \frac{M_S^2}{m_P} \quad M \sim m_P \quad \alpha \sim \left(\frac{M_S}{m_P}\right)^4$$

Global min: $(\Phi, \phi) = (0, \pm M)$ Saddle: $(\Phi, \phi) = (0, 0)$

$M_S \sim \sqrt{m_{3/2}M_P} \sim 10^{11}\text{GeV}$: Gravity mediated
SUSY breaking

Tachyonic mass: $m_{\phi} \sim \sqrt{\alpha}M \Rightarrow m_{\phi} \sim m_{\Phi} \sim m_{3/2}$

$(m_{\phi}^{\text{eff}})^2 = \lambda\Phi^2 - \alpha M^2 \Rightarrow \exists \Phi_c \equiv \sqrt{\alpha/\lambda}M \sim m_{3/2}$:

$$\Phi > \Phi_c \Rightarrow \phi \rightarrow 0 \Rightarrow V \simeq \frac{1}{2}m_{\Phi}^2\Phi^2 + M_S^4$$

$$\Phi < m_P \Rightarrow V_{\text{inf}}^{1/4} \simeq M_S \Rightarrow H_{\text{inf}} \sim m_{3/2}$$

No Flat Direction

● Fast-Roll Inflation ($m_{\Phi} \leq \frac{3}{2}H_{\text{inf}}$)

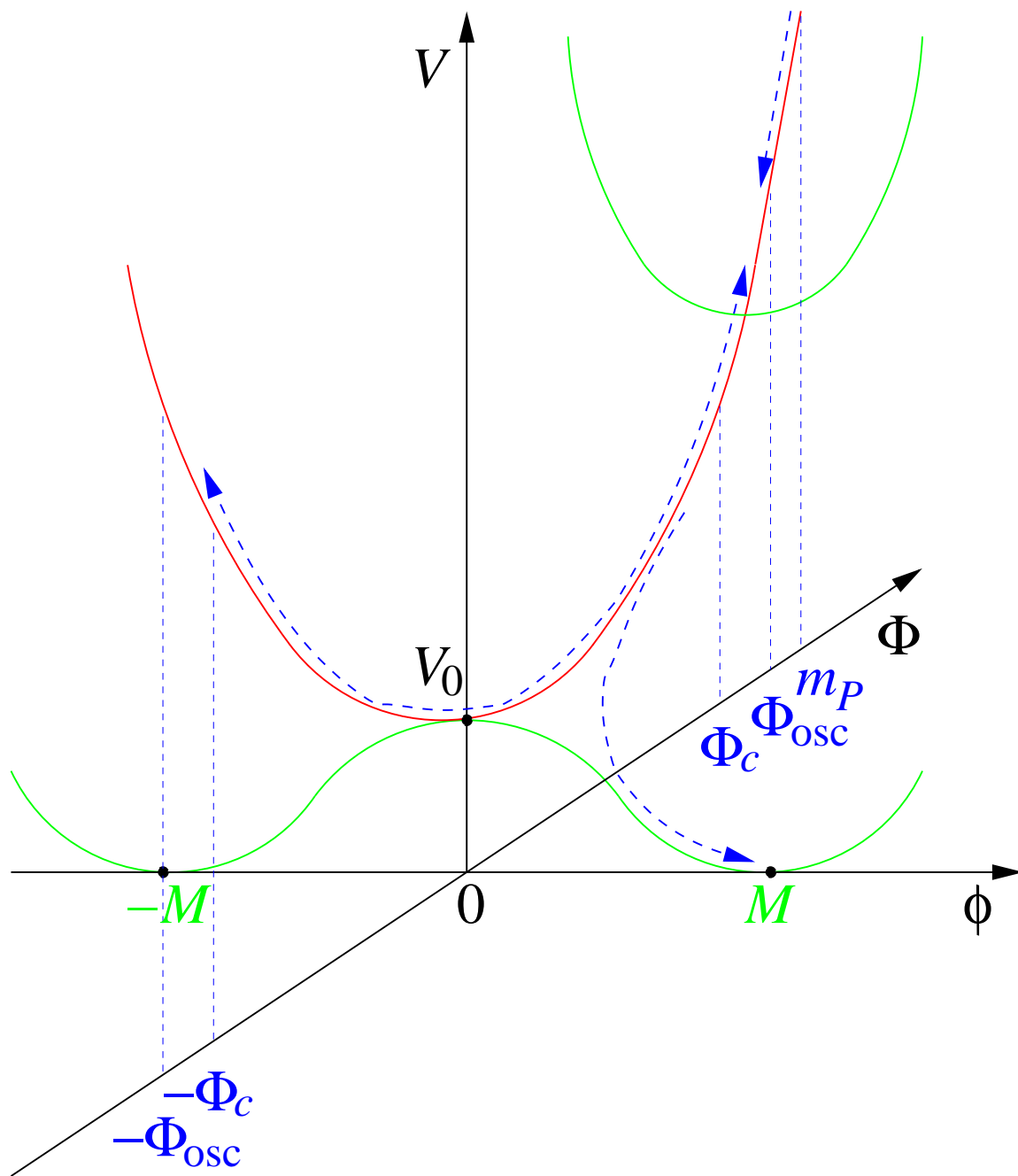
$$\Phi = \Phi_0 \exp(-F_{\Phi}\Delta N) \Rightarrow N_{\text{FR}} \simeq F_{\Phi}^{-1} \ln(m_P/m_{3/2})$$

with $F_{\Phi} \equiv \frac{3}{2}\left(1 - \sqrt{1 - \frac{4}{3}|\eta_{\Phi}|}\right) < \frac{3}{2}$ and $|\eta_{\Phi}| \simeq \frac{1}{3}(m_{\Phi}/H_{\text{inf}})^2$

● Locked Inflation ($m_{\Phi} > \frac{3}{2}H_{\text{inf}}$)

$$\Phi = \Phi_0 \exp\left(-\frac{3}{2}\Delta N\right) \times \cos(m_{\Phi}\Delta t)$$

Oscillation $(\Delta t)_s \sim \bar{\Phi}^{-1} < m_{\phi}^{-1} \Rightarrow N_{\text{lock}} \simeq \frac{2}{3} \ln(m_P/m_{3/2})$



● Tachyonic Fast-Roll Inflation

$$\Phi < \Phi_c \Rightarrow V \simeq V_{\text{inf}} - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4}\alpha\phi^4$$

$$\phi = \phi_0 \exp(F_\phi \Delta N) \Rightarrow N_\phi \simeq F_\phi^{-1} \ln(m_P/m_{3/2})$$

with $F_\phi \equiv \frac{3}{2} \left(\sqrt{1 + \frac{4}{3}|\eta_\phi|} - 1 \right)$ and $|\eta_\phi| \simeq \frac{1}{3}(m_\phi/H_{\text{inf}})^2$

Total e-folds: $N_{\text{tot}} = N_\Phi + N_\phi \simeq \left(\frac{1}{F_\Phi} + \frac{1}{F_\phi} \right) \ln \left(\frac{m_P}{m_{3/2}} \right)$

$$N_\Phi = \begin{cases} N_{\text{FR}} & \text{when } m_\Phi \leq \frac{3}{2}H_{\text{inf}} \quad (F_\Phi > 3/2) \\ N_{\text{lock}} & \text{when } m_\Phi > \frac{3}{2}H_{\text{inf}} \quad (F_\Phi = 3/2) \end{cases}$$

Horizon & Flatness $\Rightarrow N_{\text{cosmo}} = 72 - \ln \left(\frac{m_P}{V_{\text{inf}}^{1/4}} \right) - \frac{1}{3} \ln \left(\frac{V_{\text{inf}}^{1/4}}{T_{\text{reh}}} \right)$

$$T_{\text{reh}} \sim \sqrt{\Gamma m_P} \quad \& \quad \Gamma \simeq g^2 m_\phi \quad \text{with} \quad \frac{m_{3/2}}{m_P} \leq g \leq 1$$

$$\frac{m_\phi}{H_{\text{inf}}} < \frac{3}{2} \left\{ \left[\left(\frac{108 + \ln \sqrt{g}}{\ln(m_P/m_{3/2})} - \frac{7}{4} \right)^{-1} + 1 \right]^2 - 1 \right\}^{1/2} = (2 - 3)$$

Enough inflation is achieved without flat directions

$$m_\Phi \sim m_\phi \sim m_{3/2} \sim H_{\text{inf}}$$

Summary and Conclusions

- Inflation offers an elegant solution to most of the problems of the Hot Big Bang and, in particular, explains LSS formation and the CMB-anisotropy.
- Inflation amplifies the quantum fluctuations of light fields, generating thereby a scale-invariant super-horizon spectrum of curvature perturbations.
- However, flat directions are typically lifted by excessive supergravity corrections (η -problem).
- The curvaton ameliorates the tuning problems of inflation model-building and allows inflation to be achieved without the use of flat directions.
- The only remaining requirement is the total duration of inflation, which has to solve the horizon and flatness problems.
- Modular hybrid inflation is a concrete example that inflation without flat directions is indeed possible.