BRANE COSMOLOGY

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why brane-worlds?

- **GR breaks down**

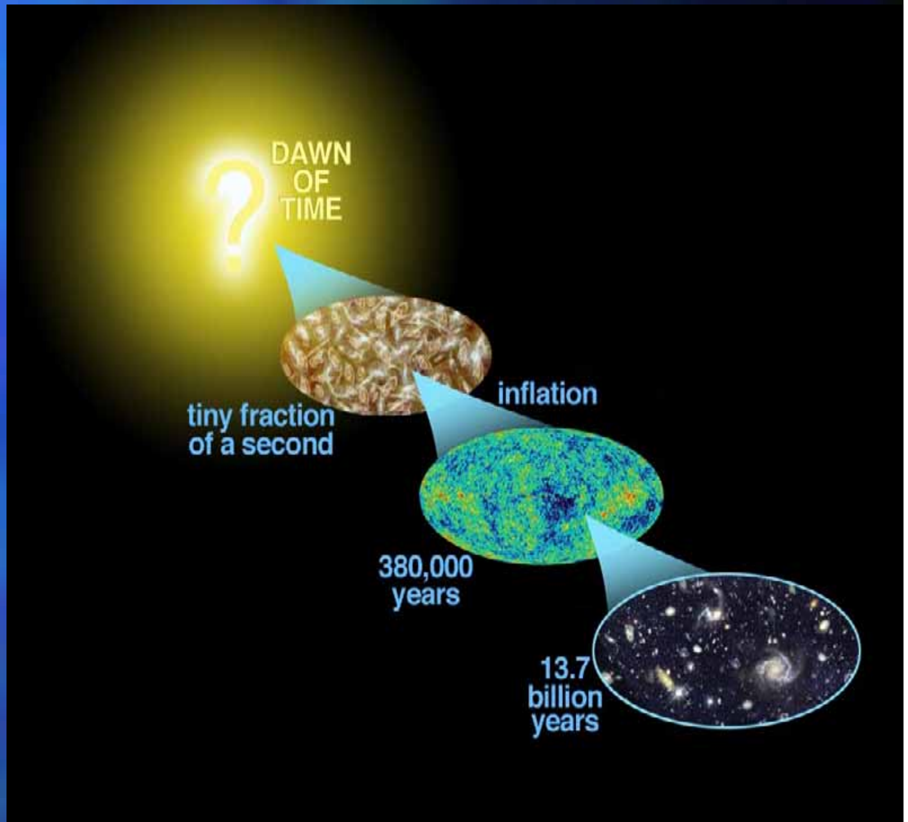
  need quantum gravity in the early universe

- **no QG theory as yet**

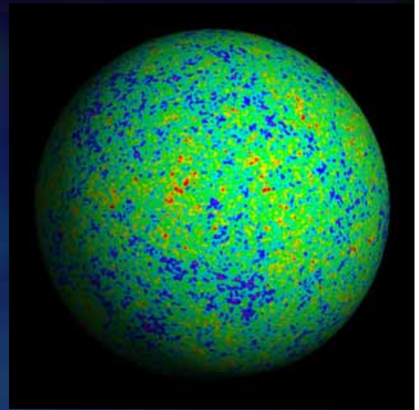
  but M theory is a promising candidate

- **M theory needs extra dimensions + branes**

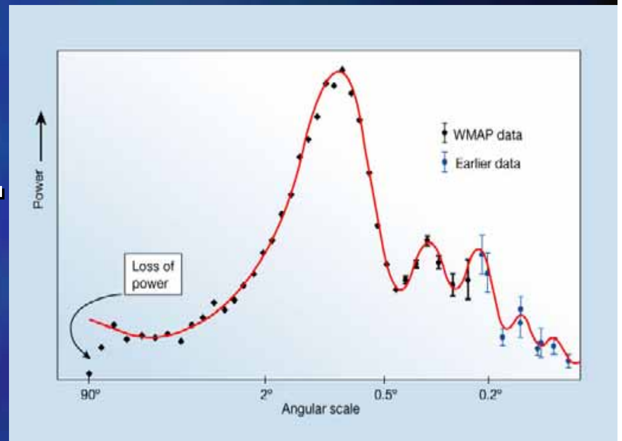
  can lower the Planck scale



- standard cosmology highly successful
  - but – still a paradigm seeking a theory
  - inflation? dark energy? (dark matter?)



- quantum modifications to GR
  - * solve puzzles - inflation, dark energy, low quadrupole?,...
  - * predict new features



- slow progress in M theory cosmology

  ➡ use braneworld phenomenology

GR ➡ phenomenology ➡ QG

2 key aspects

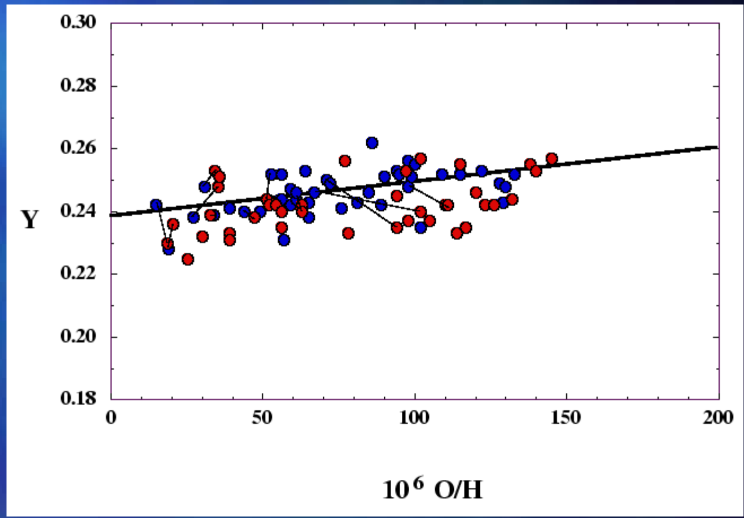
- braneworld gravity brings new features

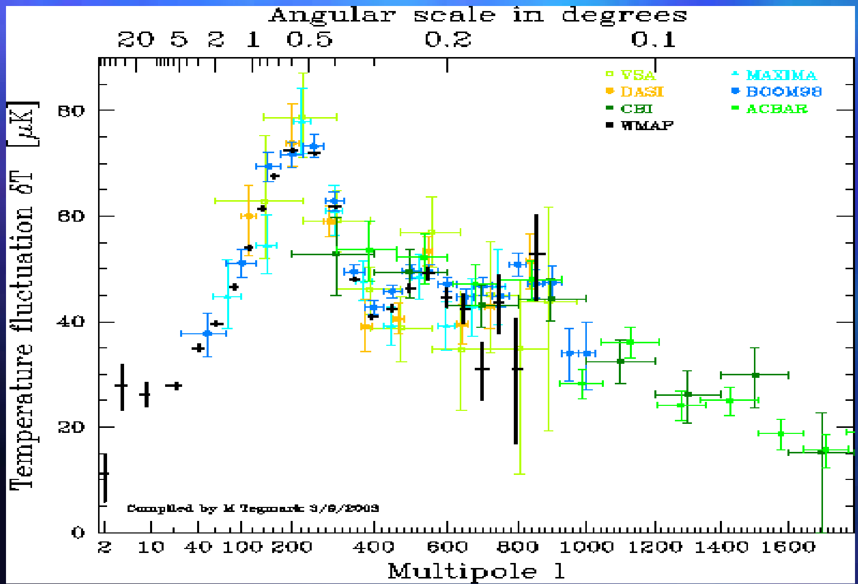
  KK modes, moduli fields, holography, shadow matter ….

- precision cosmology can constrain braneworld models (and M Theory)

  * via dynamics – BBN, SNe

  * via CMB

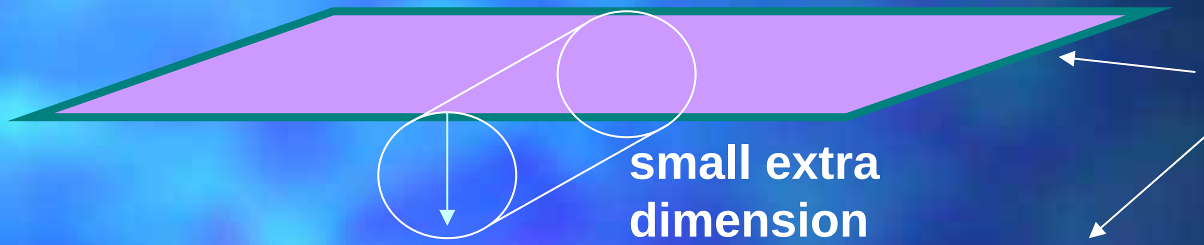




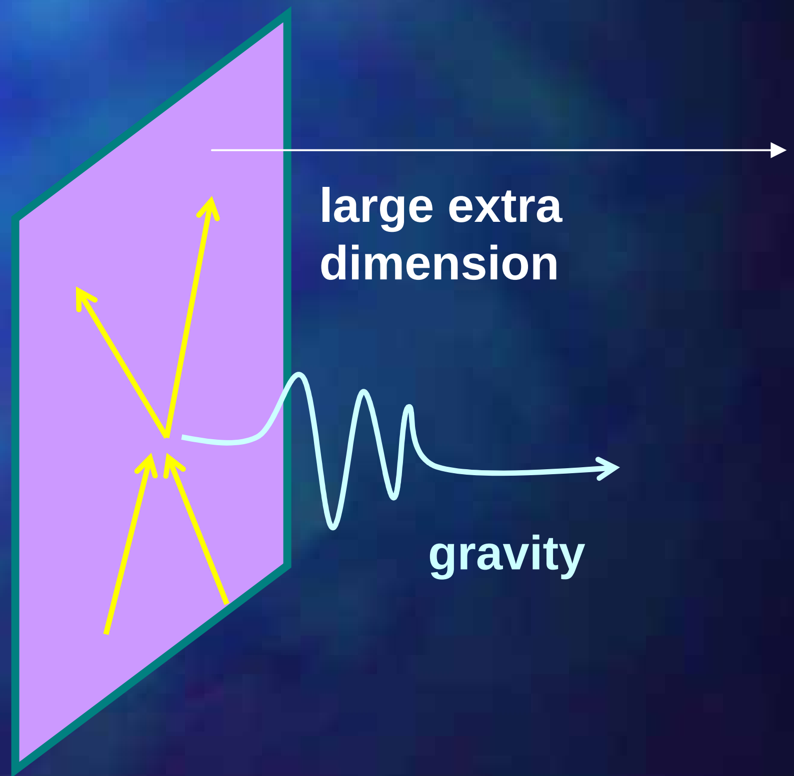
why don't we see the extra dimensions?

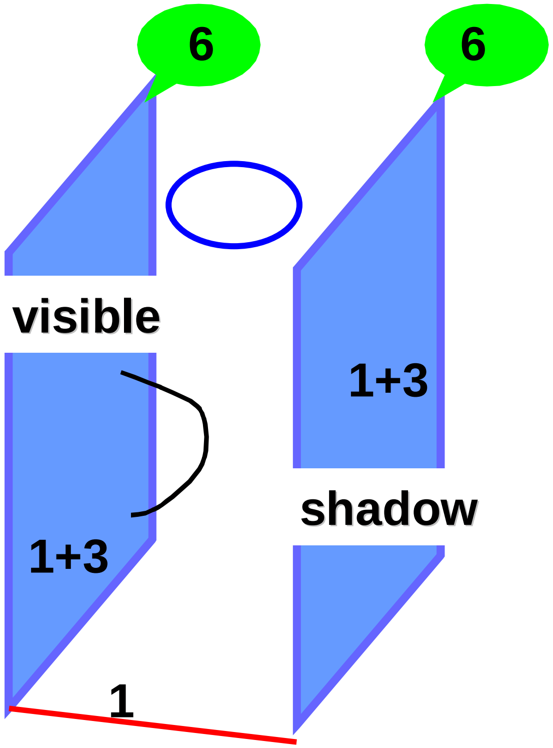
- **conventional Kaluza-Klein idea:**

  internal extra dimension too small to be seen



- **discovery of D-brane**
  - **matter fields** restricted to lower dimensional brane
  - external bulk felt only through **gravity**
  - extra dimension bigger





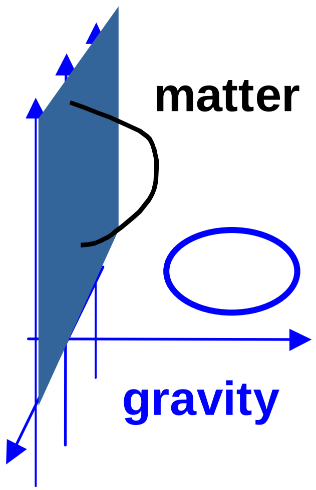
M theory

1 time + 10 space dimensions

1+10 → 1+3+1+(6)

braneworld

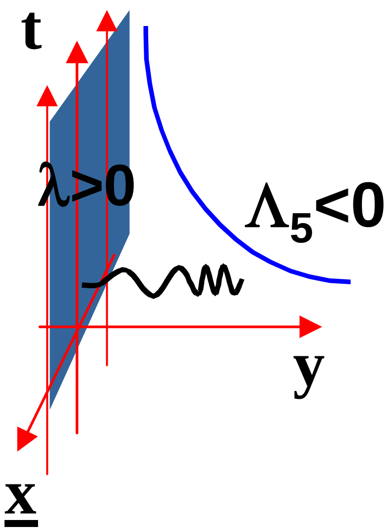
large extra dimension



→ effective 5D braneworld

$$M_5 \sim \left(M_4^2 / L\right)^{1/3} << M_4$$

warped braneworlds

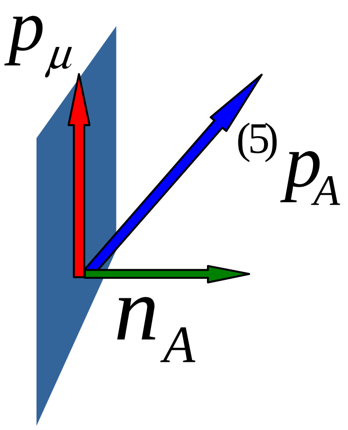


- simple models (Randall-Sundrum)
  - 5D Einstein gravity + vacuum energy
- use curvature to localize gravity
  - brane self-gravity (tension)
  - tension balances bulk vacuum energy
  - brane is Minkowski, bulk 5D AdS
- two models

massive KK modes

background – 5D anti de Sitter

$$^{(5)}ds^2 = dy^2 + e^{-2|y|/\ell}\left(-dt^2 + d\vec{x}^2\right), \quad \Lambda_5 = -6/\ell^2$$



5D gravitons

- **massless in 5D**
- **effective 4D mass**

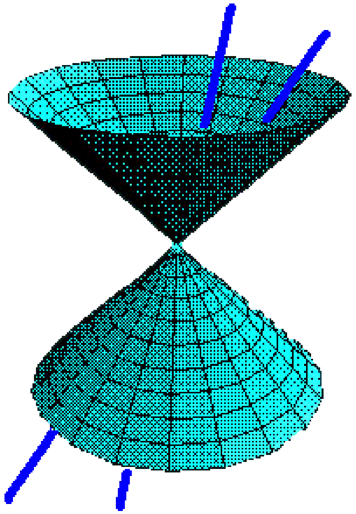
5D metric perturbation

$$^{(5)}g_{AB} \to {}^{(5)}g_{AB} + h_{AB}, \quad |h_{AB}| << 1$$

RS-gauge

$$h_{Ay} = 0, \quad h^{\mu}_{\mu} = 0 = \partial_{\nu} h^{\mu\nu} \Rightarrow 5 \;\; d.o.f.$$

5D spin-2 → 4D spin-2 + spin-1 + spin-0



$$h_{\mu\nu} \quad (5) \rightarrow h_{ij} \quad (2) + \Sigma_i \quad (2) + \beta \quad (1)$$

$$h^{i}_{i} = \partial^{i} h_{ij} = 0 = \partial^{i} \Sigma_{i}$$

linearized 5D field equation

$$\delta^{(5)} R_{AB} = \frac{2}{3} \Lambda_5 h_{AB}, \quad \left[\partial_y h_{AB} \right]_{brane} = 0$$

wave equation

$$\nabla_\mu \nabla^\mu h = e^{-2y/\ell}\left[-h'' + \frac{4}{\ell}h'\right]$$

separate

$$h \to \varphi_m(x) f_m(y), \;\; \nabla_\mu \nabla^\mu \varphi_m = m^2 \varphi_m$$

4D zero-mode – only tensor

m=0: no normalizable scalar or vector

weak-field potential

$$\Phi(r) \propto \frac{1}{r}\left(1 - \frac{2\ell^2}{3r^2}\right) + \ldots$$

m=0 **m>0**

$$\ell < 0.1 mm$$

$$\Rightarrow \lambda > (1 \;\; TeV)^4, \;\; M_5 > 10^5 \;\; TeV$$

cosmological brane

induced 4D field equations

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} + 6\frac{\kappa^2}{\lambda}\left(T^2\right)_{\mu\nu} - E_{\mu\nu}$$

high-energy

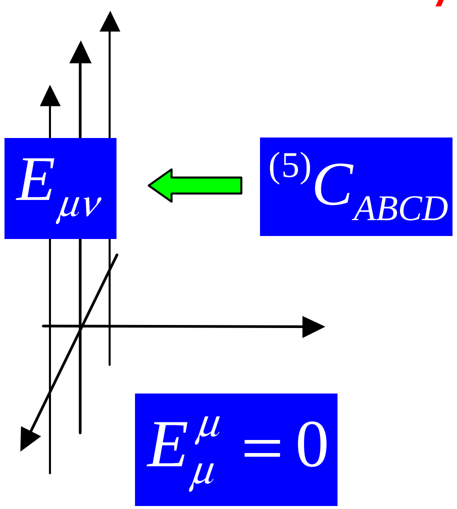
$$\rho_{eff} = \rho\left(1 + \rho / 2\lambda\right)$$

high or low energy

5D graviton - massive KK effects

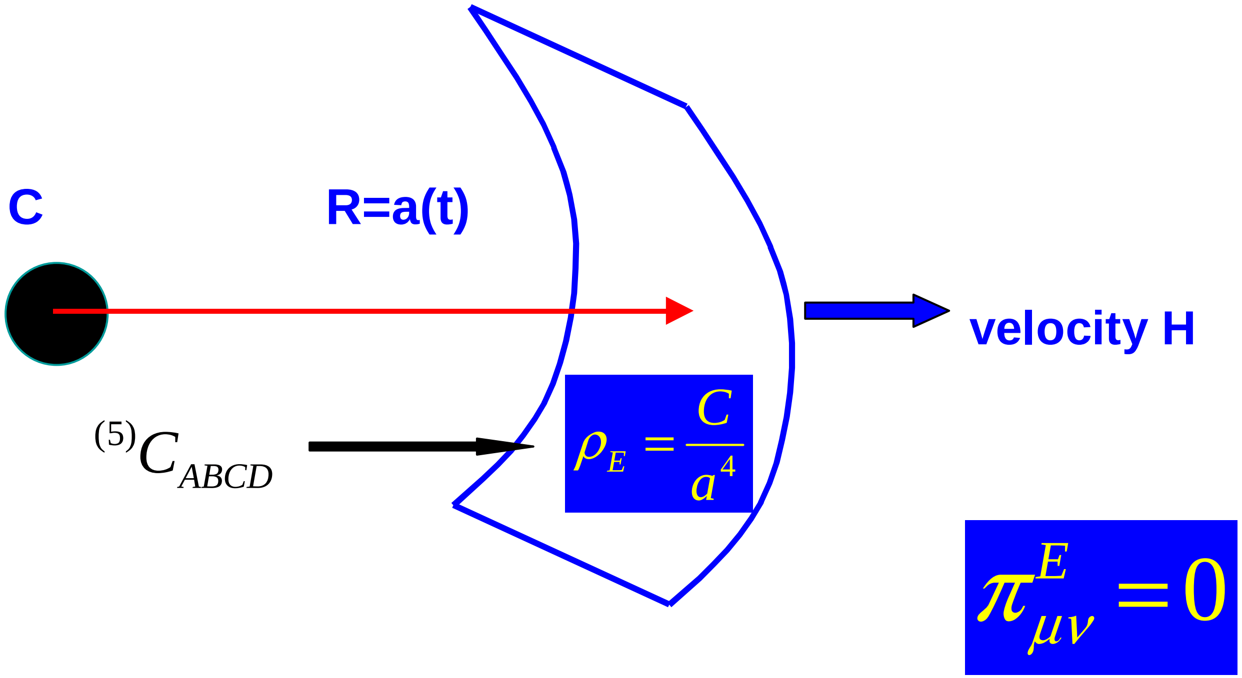
$$E_{\mu\nu} \rightarrow \left(\rho_E , \pi^E_{\mu\nu}\right)$$

dark radiation **Weyl anisotropic stress**



background cosmology

- Minkowski brane – fixed in AdS_5
- FRW brane – moving in Schw.- AdS_5



- **generalized Friedmann equation**

$$H^2 = \frac{\kappa^2}{3}\rho\left(1+\frac{\rho}{2\lambda}\right)+\frac{C}{a^4}+\frac{\Lambda_4}{3}-\frac{K}{a^2}$$

high-energy term ↑ ↑ **dark radiation**

- **same conservation equation**

$$\dot{\rho}+3H(\rho+p)=0$$

- **solutions (C=0, K=0= Λ_4)**

$$p=\frac{\rho}{3} \Rightarrow a=a_0\left[t(t+t_\lambda)\right]^{1/4} \qquad \left[GR: \quad a=a_0 t^{1/2}\right]$$

$$p=-\rho \Rightarrow a=a_0 e^{Ht}, \qquad H=\kappa\sqrt{\frac{\rho}{3}\left(1+\frac{\rho}{2\lambda}\right)}>H_{GR}$$

- **high energies – new effects**

$$p = \frac{\rho}{3}: \quad \rho >> \lambda \Rightarrow t << t_\lambda \Rightarrow a \sim t^{1/4}$$

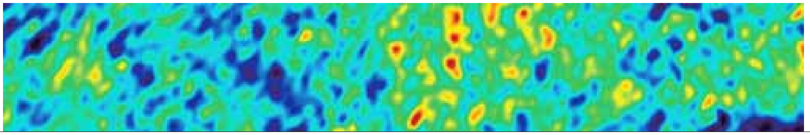
$$p = -\rho: \quad \rho >> \lambda \Rightarrow H \propto \rho \quad \left[GR: \quad H \propto \sqrt{\rho} \right]$$

  - **high-energy inflation**
  - **high-energy reheating**
  - **high-energy early radiation era**

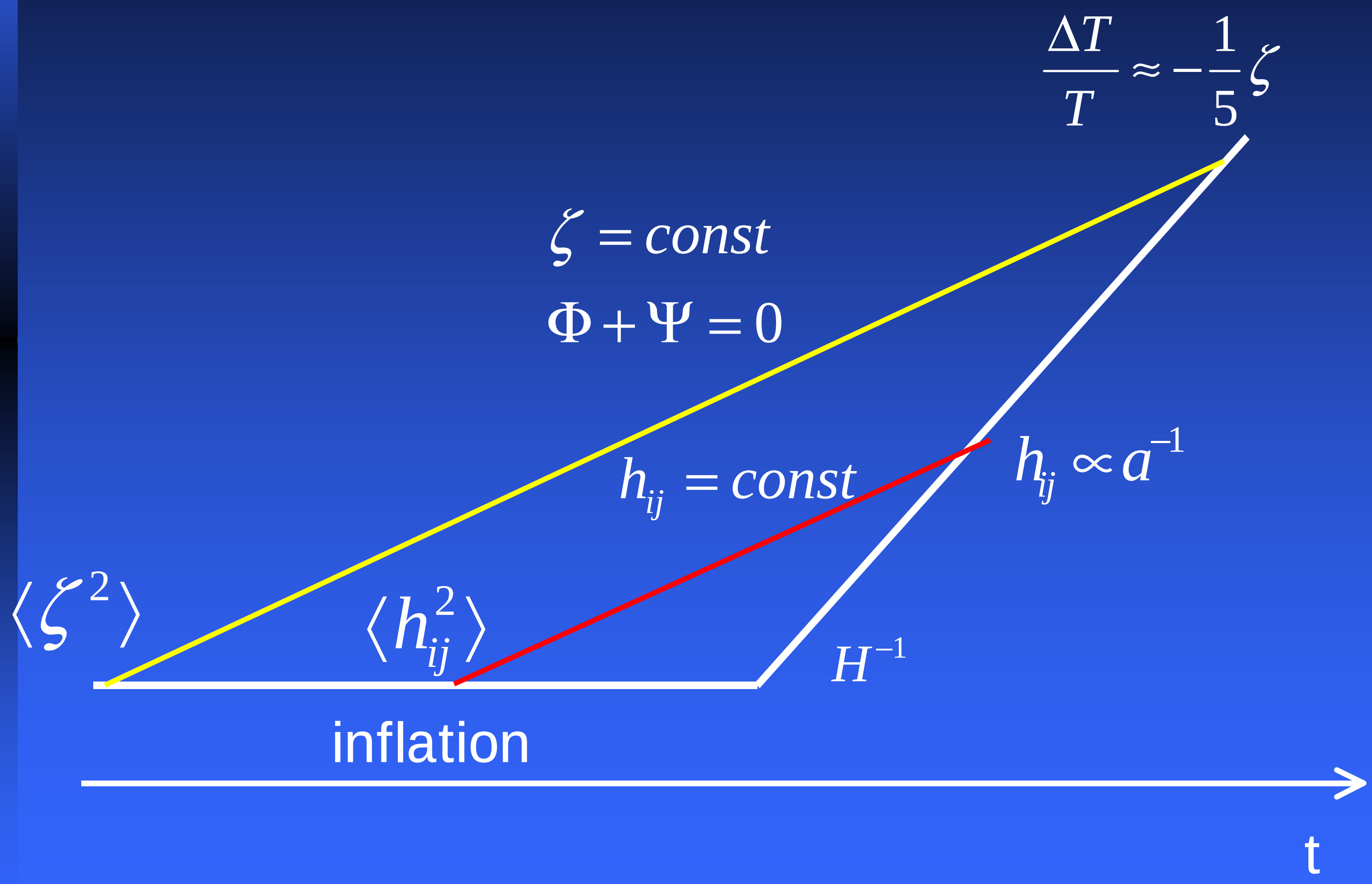
- **low energies – recover GR**

$$\rho << \lambda \;, \quad \lambda > \left(1 \;\; TeV \right)^4$$

  - **below at least electroweak scale**
  - **nucleosynthesis "safe"**
  - **but perturbations will carry 5D effects into CMB**

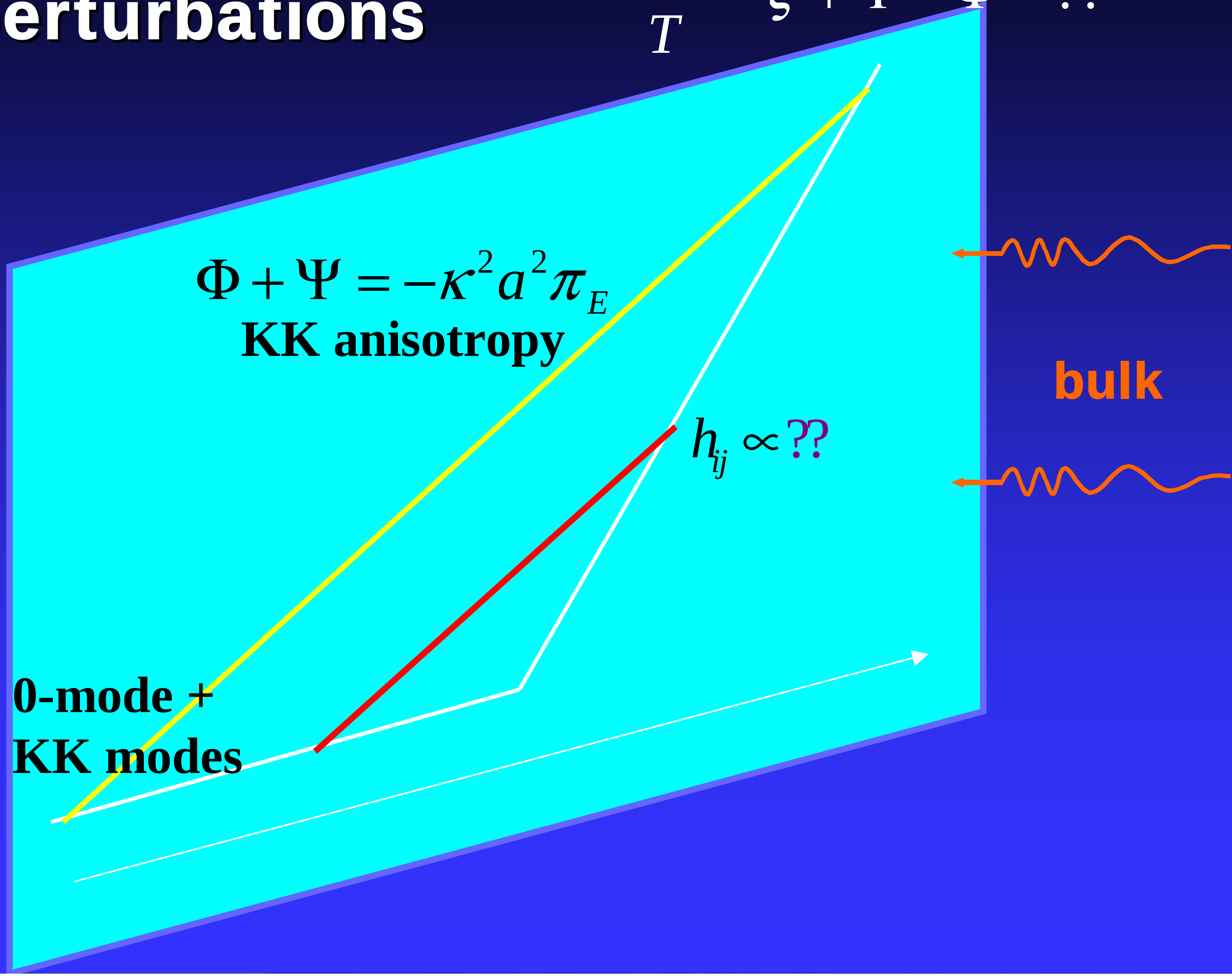


standard 4D perturbation picture



brane-world perturbations

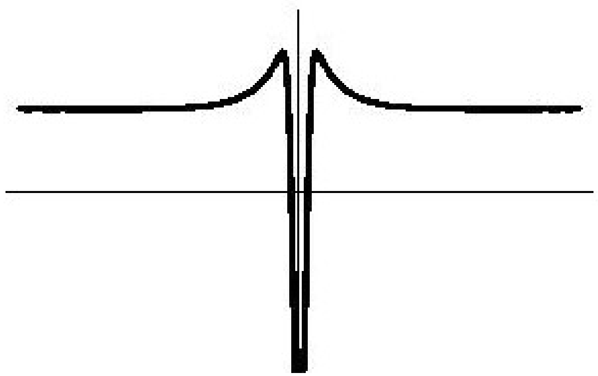
$$\frac{\Delta T}{T} \approx \zeta + \Psi - \Phi = ??$$



perturbations from inflation

de Sitter brane in AdS_5 bulk

- **like Minkowski brane:**
  - **tensor zero mode**
  - **no scalar / vector 0-mode from 5D graviton**
- **unlike Minkowski:**
  - **mass gap for KK modes**



$$m > \frac{3}{2} H$$

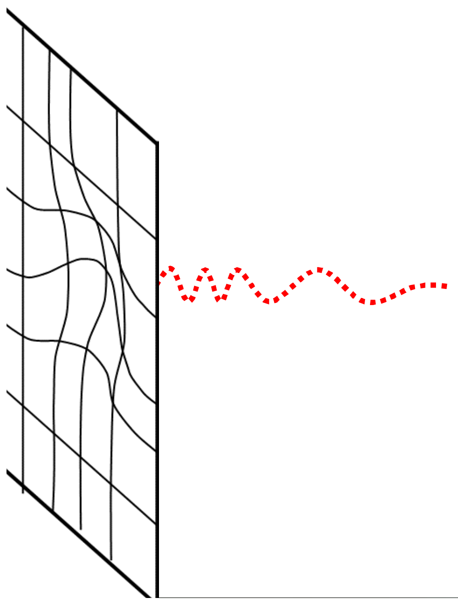
massive modes not excited during inflation

tensor 0-mode frozen until horizon re-entry

scalar perturbations

- **no** 5D graviton contribution to lowest order
- **only** from density perturbations - decouple from 5D perturbations (large scales)
- curvature perturbation can be found **without** knowledge of bulk (large scales)

$$\zeta_{tot} = \Phi + \frac{(\delta\rho + \delta\rho_E)}{3(\rho + p)} = \zeta_m + \frac{\delta C / a^4}{3(\rho + p)}$$

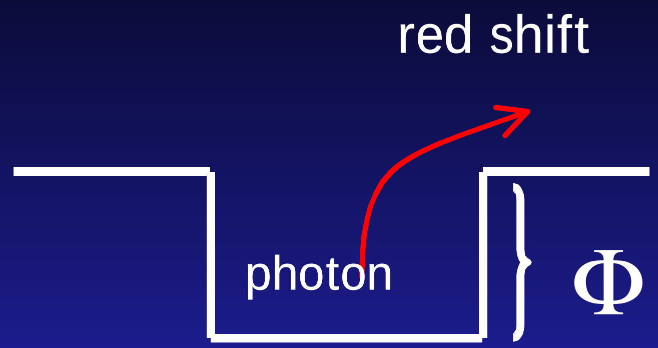


matter curvature perturbation conserved as in GR

$$\left(\frac{\delta\rho}{\rho}\right)^2 \approx \left(\frac{\delta\rho}{\rho}\right)^2_{GR} \left(\frac{V}{2\lambda}\right)^3 \Rightarrow \varphi_{COBE} << M_4$$

- **Sachs-Wolfe effect**

$$\frac{\Delta T}{T} = \zeta_m + \Psi - \Phi$$



- **metric perturbations**

$$\zeta_{tot} = \Phi - \frac{H}{\dot{H}}\left(\frac{\dot{\Phi}}{H} - \Psi\right),$$

$$\Phi + \Psi = -\kappa^2 \ell^{-1} k^{-2} a^2 \pi_{\varepsilon}$$

cannot predict CMB anisotropies unless Weyl anisotropic stress is known

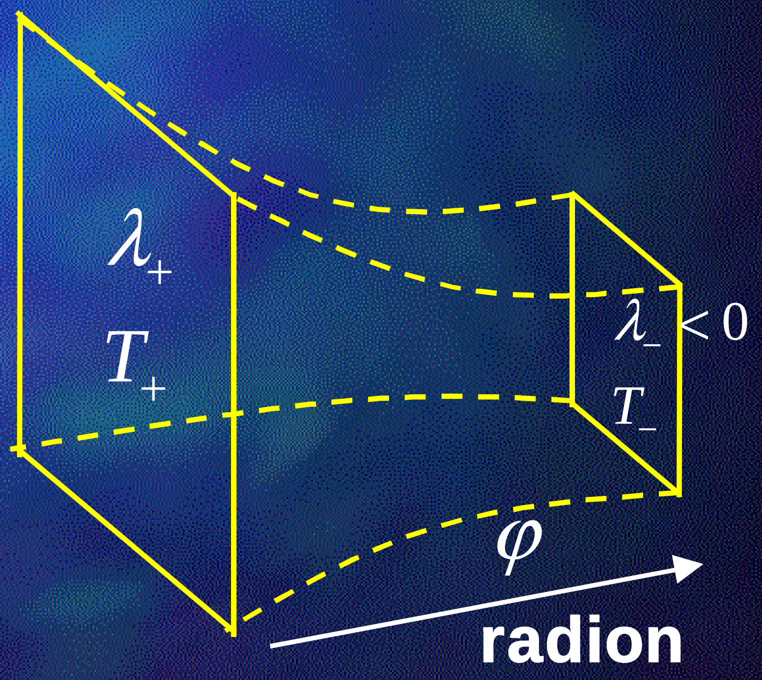
π_E - low energy approximation

structure formation - very low energy

- bulk curvature scale < 1mm << brane's
- use gradient expansion to solve 5D field equations

$$|\partial_y F| << |\nabla_\mu F|$$

- need 2 boundary conditions



effective equations on +ve tension brane

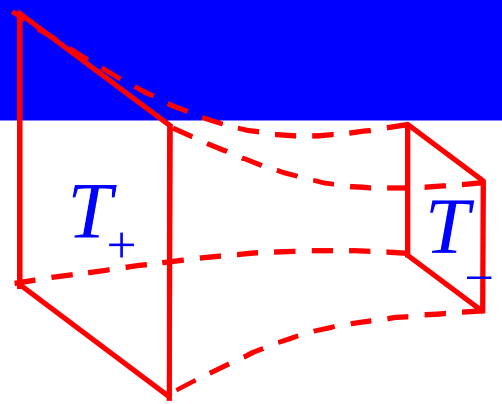
➡ **scalar-tensor theory**

$$G^{\mu}{}_{\nu} = \kappa^2 \frac{1}{\varphi}\left[T_{+}{}^{\mu}{}_{\nu} + (1-\varphi)^2 T_{-}{}^{\mu}{}_{\nu}\right] + \frac{1}{\varphi}(\nabla^{\mu}\nabla_{\nu}\varphi - \delta^{\mu}{}_{\nu}\nabla^2\varphi)$$

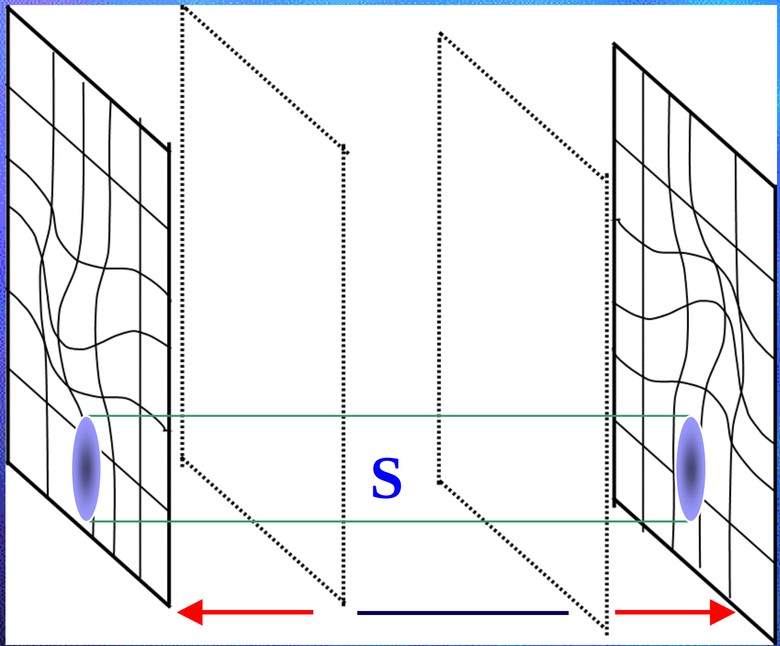
$$+ \frac{\omega(\varphi)}{\varphi^2}\left(\nabla^{\mu}\varphi\nabla_{\mu}\varphi - \frac{1}{2}\delta^{\mu}{}_{\nu}(\nabla\varphi)^2\right)$$

$$\nabla^{\mu}\nabla_{\mu}\varphi = f(\varphi, T_{\pm})$$

where $$\omega(\varphi) = \frac{3\varphi}{2(1-\varphi)}$$



scalar perturbations



φ_+ φ φ_-

$$\zeta_\pm = \zeta_{m\pm} - \frac{\kappa^2}{6}\frac{\delta C a_\pm^{-4}}{\dot{H}_\pm}$$

$$\ddot{S} + \left(3H_+ + \frac{\dot{\varphi}}{\varphi}\right)\dot{S} = f(\varphi_+, \varphi_-)$$

$$\pi_{\mathbf{E}} = f(\varphi_\pm, S)$$

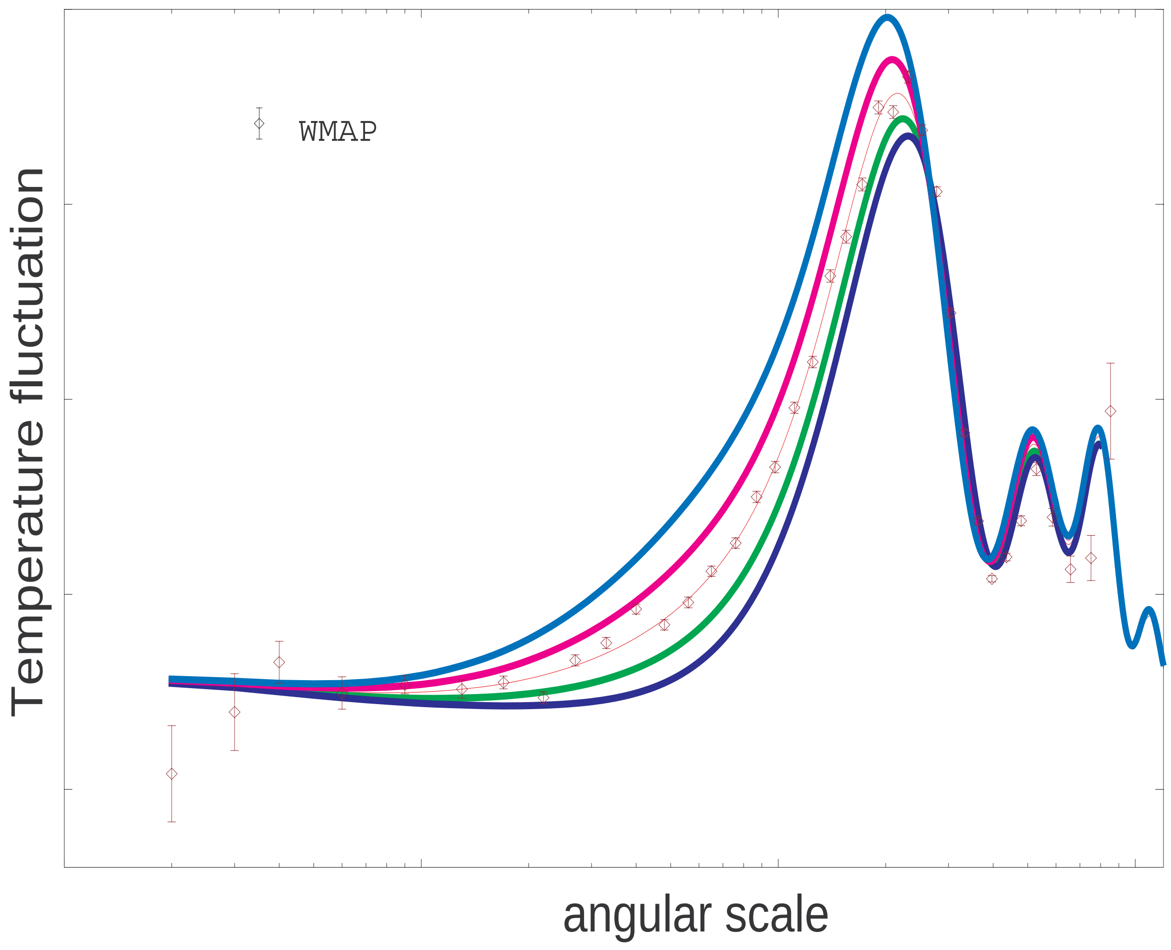
brane-world CMB anisotropies

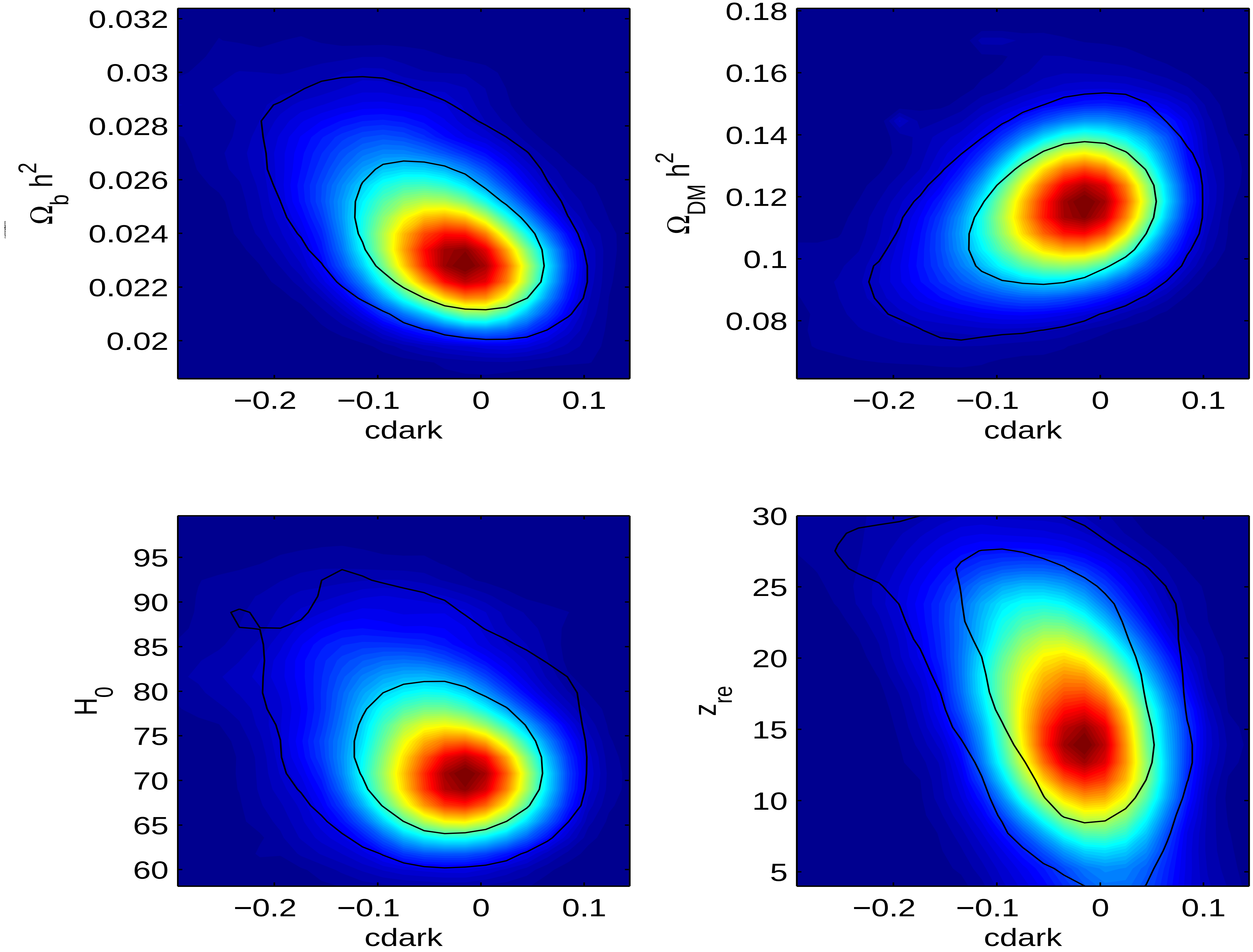
model with most simple background

- **radion fixed**
- **no dark radiation in background**

$$cdark = \frac{\delta\rho_E / \rho_r}{4\zeta_m}$$

$$\pi_E = f(cdark, \zeta_m)$$

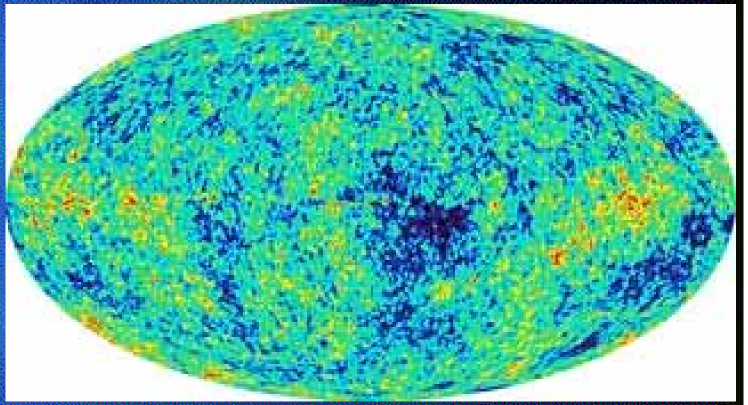


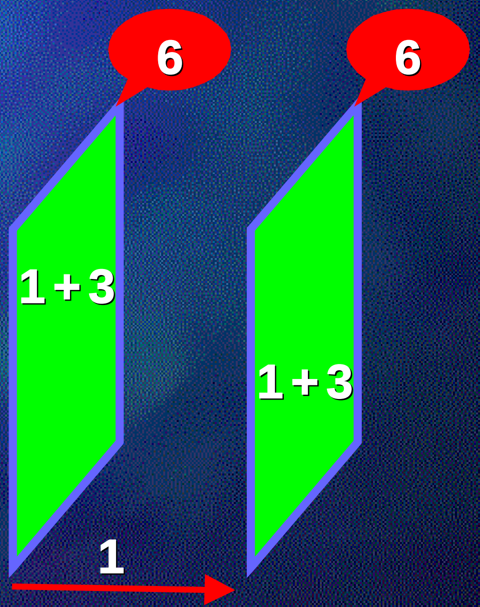


0.032
0.03
0.028
0.026
0.024
0.022
0.02
$\Omega_b h^2$
−0.2
−0.1
0
0.1
cdark
0.18
0.16
0.14
0.12
0.1
0.08
$\Omega_{DM} h^2$
−0.2
−0.1
0
0.1
cdark
95
90
85
80
75
70
65
60
H_0
−0.2
−0.1
0
0.1
cdark
30
25
20
15
10
5
z_{re}
−0.2
−0.1
0
0.1
cdark

further work

- simple RS model OK so far
- compute CMB for
  - more realistic background
  - one-brane model
  - models with bulk dilaton / moduli field
  - models with quantum corrections
  - M theory models?

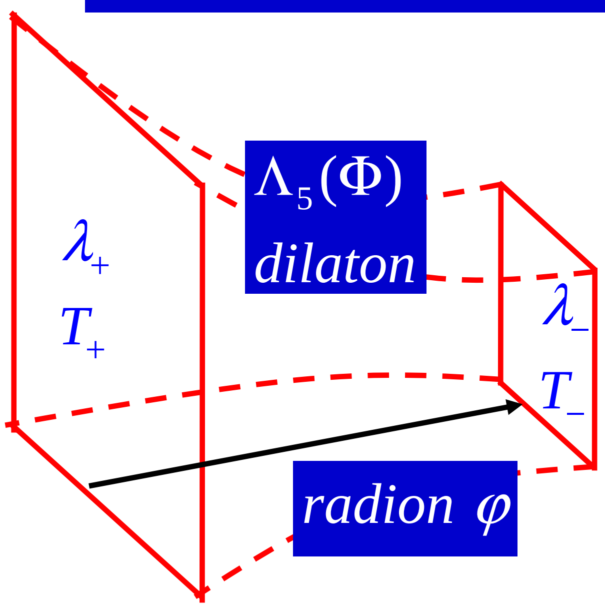




more developed models

bulk scalar field (dilaton / moduli)

$$S_{grav} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-{}^{(5)}g}\left[{}^{(5)}R - 2\Lambda_5(\Phi) - \kappa_5^2(\partial\Phi)^2\right]$$
$$- \int d^4x\sqrt{-g}\,\lambda_\pm(\Phi)$$



- *dilaton can drive brane inflation
- *dilaton + shadow matter
  - ? dark matter
  - ? dark energy
- *brane-brane collision
  - ? big bang (ekpyrotic)

quantum curvature corrections

Gauss-Bonnet (early universe)

$$S_{grav} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-{}^{(5)}g}\left[{}^{(5)}R - 2\Lambda_5 + \alpha\left\{{}^{(5)}R^2 + \ldots\right\}\right] - \int d^4x\sqrt{-g}\,\lambda$$

induced gravity (late universe)

$$S_{grav} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-{}^{(5)}g}\left[{}^{(5)}R - 2\Lambda_5\right] - \int d^4x\sqrt{-g}\left[\lambda - \mu^2 R\right]$$