Dark Energy and Cosmic Speed-Up: M Theory Perspectives

Ishwaree Neupane
TU Kathmandu and NTU Taipei

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- Motivations
- Flux compactification on hyperbolic spaces
- Accelerated cosmologies – possibilities
- Flat and open space cosmologies
- Dark energy, cosmic speed-up and related issues
- Eternal inflation/acceleration

String or M Theory Cosmology: Motivations

Recent interest in string or M-theory cosmology is two fold

(I) **In explaining the observed cosmic acceleration, and also possibly inflation, of the universe by allowing time-dependent extra spaces**

(II) **In deriving a scalar potential from string theory flux compactifications on CY spaces that may allow a metastable de Sitter state (vacuum)**.

If (I) is closer to the reality then dark energy is possibly dynamical, for instance, this could arise due to the slowly varying size of extra dimensions.

If (II) is closer, then "dark energy" may turn out to be a pure vacuum energy, or an honest cosmological constant.

A no-go argument and warped compactification

One of the obstacles for a de Sitter type compactification in supergravity theories is no-go theorem Gibbons (1984), Maldacena-Nunez (2001).

The strong energy condition (SEC) $R_{00}^{(D)} \geq 0$ holds for all D=10 or 11 supergravities. If one allows extra dimensions to be warped and static, then in a compactified theory, one has

$$R_{00}^{(4)} \geq 0$$

But for a four-dimensional metric of the FLRW form

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right),$$

where $k = -1,\ 0,\ +1$, the time-time component of 4D Ricci tensor gives

$$R_{00}^{(4)} = -3 \frac{\ddot{a}(t)}{a(t)}$$

Inflating spacetime implies that $\ddot{a}/a > 0$ and hence, in four dimensions, the SEC must be violated during inflation.

Warped de Sitter compactifications – a no-go argument

How a “no-go” argument hinders the business of a warped string compactification? Consider a D-dimensional metric ansatz

$$\mathbf{ds_D^2 = A^2(y)\, ds_4^2(x) + d\Sigma_m^2(y) \quad (m = D - 4)}$$

$d\Sigma_m^2$ is the metric of some compact non-singular m-manifold $\mathcal{M}$ with coordinates y. For the compact extra dimension(s), we find

$$\left[\int_{\mathcal{M}} \mathbf{A^2}\right] \mathbf{R_{00}^{(4)}} = \int_{\mathcal{M}} \mathbf{A^2\, R_{00}^{(D)}}$$

This implies that

$$\mathbf{R_{00}^{(D)} \geq 0} \quad \text{only} \quad \text{if} \quad \mathbf{R_{00}^{(4)} \geq 0}$$

This holds also in the presence of p-form fields, which are typical in string/M theory. To this end, one might be willing to introduce higher derivative curvature corrections into the effective action and/or take into account some other less well understood non-perturbative effects, like branes or brane-instanton effects. But, in low energy scales, the spacetime region that may violate the SEC with these geometries is very limited and so may not provide a prolonged period of cosmic acceleration.

How to circumvent a "no-go" argument?

Now it is understood that the cosmic acceleration of our universe can arise from supergravity solutions, with or without background fluxes along the extra dimensions, if

1) one gives up the condition of time-independence of internal space, and

2) the internal space is negatively curved (i.e., a hyperbolic space)

Townsend-Wohlfarth (hep-th/0303097) This is an interesting observation. An attempt in this direction was made by E. Kasner in 1921. Look! this was right after T. Kaluza but before O. Klein, although Kasner considered only flat extra dimensions, presumably, due to a mathematical simplicity.

Many generalizations of the above arguments can be found in

hep-th/0304177, hep-th/0306291,
hep-th/0311071 hep-th/04mmddd

A hyperbolic space – how it looks like

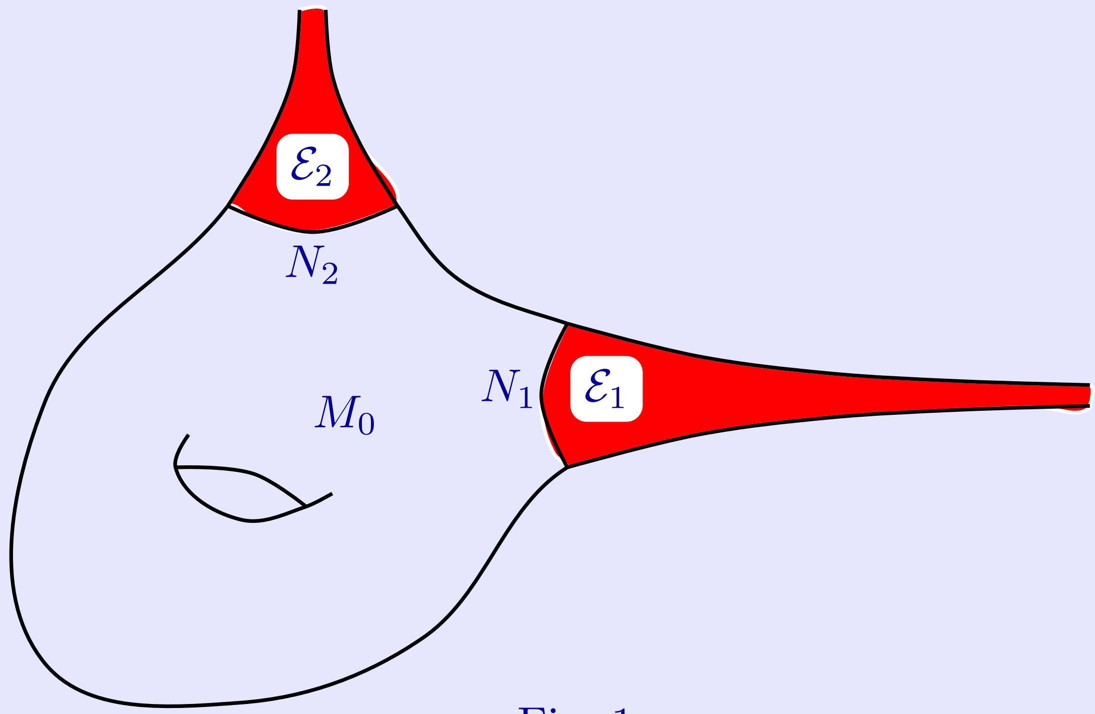
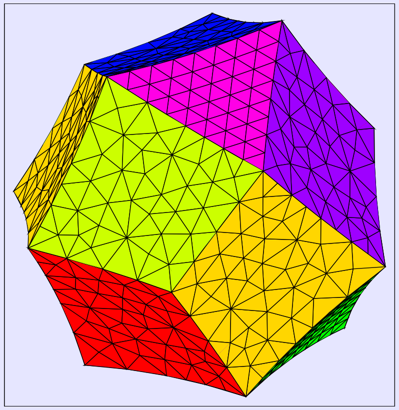


Fig. 1

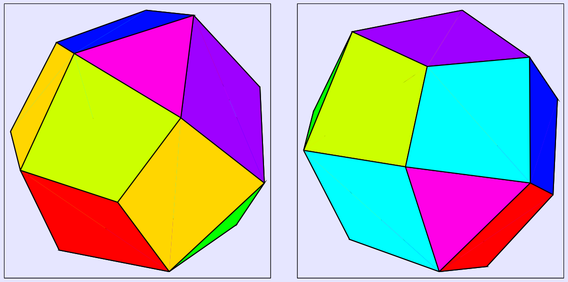
One may decompose a hyperbolic manifold M of finite volume into a relatively compact space M_0 and many other finite disjoint cusps $\mathcal{E}_j$. The connected compact piece N_j has a flat metric g_{N_j}, while $\mathcal{E}_j$ carries a warped product metric $g_{\mathcal{E}_j} = e^{-2r} \cdot g_{N_j} + dr^2$. If M is 2 or 3-dimensional and oriented, then N_j is a circle S^1 or a 2-torus T^2 respectively.

Compact Hyperbolic Three-Manifold

If the extra dimensions are negatively curved, then can they be compact? The answer is, presumably, yes!



Compact Hyperbolic 3-Space – Thurston Manifold



CH 3-Space in Klein Coordinates (using SnapPea)

SnapPea is a computer program useful to study the compact hyperbolic spaces as well as compact hyperbolic orbifolds. It computes the volume, fundamental group, symmetry group, homology, Chern-Simon invariant and length spectrum of such spaces

Cosmic Acceleration and Supergravities

Consider the bosonic part of $(4+m)$ dim supergravity that include the contribution from $(q+2)$-form field strengths:

$$\mathcal{L} = \frac{\mathbf{1}}{\mathbf{16\pi G_{4+m}}}\sqrt{-\mathbf{g_{4+m}}}\left(\mathbf{R} - \frac{\mathbf{8\pi G_{4+m}}}{\mathbf{(q+2)!}}\mathbf{F^2_{[q+2]}}\right)$$

where $F_{[q+2]} = dC_{q+1}$. In $D=11$ ($m=7$), one has 4-form anti-symmetric tensor matter fields as required by supersymmetry.

The metric solution in Einstein conformal-frame reads

$$\mathbf{ds^2_{11}} = \mathbf{e}^{-\mathbf{m}\sigma(\tau)}\,\mathbf{\tilde{g}}_{\mu\nu}\mathbf{dx}^\mu\mathbf{dx}^\nu + \mathbf{R_c^2}\mathrm{e}^{\mathbf{2}\sigma(\tau)}\,\mathbf{d\Sigma^2_{m,k_1}}$$

$$\mathbf{\tilde{g}}_{\mu\nu}\mathbf{dx}^\mu\mathbf{dx}^\nu = -\mathbf{e^{2A(\tau)}}\mathbf{d\tau^2} + \mathbf{e^{2B(\tau)}}\mathbf{d\Omega^2_{3,k=0}}$$

$$\mathbf{e}^{\sigma(\tau)} = (\mathbf{K}\,\mathbf{L})^{\mathbf{1/m-1}}, \quad \mathbf{e^A} = \mathbf{K}^{\frac{\mathbf{m}}{\mathbf{2(m-1)}}}\,\mathrm{e}^{-\frac{\mathbf{(m+2)}}{\mathbf{2(m-1)}}\lambda_0\tau} = \mathbf{e^{3B}}$$

$$\mathbf{K}(\tau) = \begin{cases} \frac{\mathbf{R_c}}{\mathbf{(m-1)}}\,\frac{\lambda_0\,\beta}{\sinh[\lambda_0\beta\,|\tau|\,]}, & \mathbf{k_1} = -\mathbf{1}, \\ \mathrm{e}^{\lambda_0\beta\,\tau}, & \mathbf{k_1} = \mathbf{0}, \end{cases}$$

$$\mathbf{L}(\tau) = \begin{cases} \mathrm{e}^{-\mathbf{3}\lambda_0\,\tau}, & \mathbf{b} = \mathbf{0}, \\ \mathbf{2b}\sqrt{\frac{\mathbf{m-1}}{\mathbf{2m}}}\,\frac{\cosh\mathbf{3}\lambda_0\,\tau}{\lambda_0\,\beta}, & \mathbf{b} \neq \mathbf{0}, \end{cases} \tag{1}$$

where b is the field strength parameter, and $\beta \equiv \sqrt{\frac{3(m+2)}{m}} := \sqrt{3}\,\lambda$.

The proper time t is defined by $dt = e^{3B(\tau)}\, d\tau$. When $m = 7$, the expansion factor is simply

$$\frac{\mathbf{B(t_2)}}{\mathbf{B(t_1)}} = \mathbf{3.04} \quad \text{Too small for inflation}$$

Next, consider that we live in a flat 4d spacetime and the internal space is a product of flat and hyperbolic spaces. The scale factor is

$$\mathbf{B}(\tau) = -\frac{(\mathbf{m_1} + \mathbf{m_2} - \mathbf{4})}{\mathbf{4}} \lambda_{\mathbf{0}}\tau - \frac{\mathbf{m_1}}{\mathbf{2}(\mathbf{m_2} - \mathbf{1})} \alpha_{\mathbf{0}}\tau + \frac{\mathbf{m_2}}{\mathbf{2}(\mathbf{m_2} - \mathbf{1})} \ln\left(\frac{\beta}{\sinh((\mathbf{m_2} - \mathbf{1})\beta\,|\tau|)}\right)$$

There arise many possibilities, e.g., when $m_1 = 1$ and $m_2 = 6$ and $\lambda_0 = 2\alpha_0$, the four-dimensional universe accelerates in the time interval $1.41 > 4\alpha_0\tau > 0.14$. But the expansion factor is simply

$$\frac{\mathbf{S}(\tau_{\mathbf{2}})}{\mathbf{S}(\tau_{\mathbf{1}})} = \mathbf{3.38}$$

Just a small improvement. How to improve sharply? Indeed, one can relax the assumption that $k = 0$ at the on-set of inflation, and look for the possibility of past-eternal (open) inflation with many scalars!

How to get two periods of cosmic acceleration?

In terms of a canonically normalized 4d scalar ϕ, the Lagrangian density is

$$\mathcal{L}_4 = \sqrt{-\mathbf{g_4}}\left[\frac{\mathbf{M_P^2}}{\mathbf{2}}\mathbf{R} - (\partial\phi)^{\mathbf{2}} - \mathbf{2V}(\phi)\right]$$

and the potential $V(\phi)$ arising from M theory flux compactification is

$$\mathbf{V}(\phi) = \frac{\mathbf{M_P^2}}{\mathbf{R_c^2}}\,\mathrm{e}^{-\mathbf{2}\,\lambda\,\frac{\phi}{\mathbf{M_P}}} + \mathbf{M_P^2}\,\frac{\tilde{\mathbf{b}}^{\mathbf{2}}}{\mathbf{2}}\,\mathbf{e}^{-\frac{\mathbf{6}}{\lambda}\,\frac{\phi}{\mathbf{M_P}}}$$

PS: If one prefers a normalization for ϕ such that the kinetic part is $\dot{\phi}^2/2$ and potential is $V(\phi)$, then this is attained by the substitution $\phi \to \phi/\sqrt{2}$ and $\lambda \to \lambda/\sqrt{2}$. We often work in units $M_P^{-1} = 1$.

Next define a new logarithmic time variable τ s.t.

$$\mathbf{d}\tau := \mathrm{e}^{-\lambda\,\phi}\mathbf{dt}, \quad \lambda < \sqrt{\mathbf{3}}$$

$$\mathbf{d}\tau := \mathrm{e}^{-\mathbf{3}\phi/\lambda}\mathbf{dt}, \quad \lambda > \sqrt{\mathbf{3}}$$

$$\mathbf{a(t)} := \mathbf{e}^{\alpha(\tau)}, \quad \phi := \mathbf{x}, \quad \dot{\phi} := \mathbf{y}, \quad \dot{\alpha} := \mathbf{z}$$

The Universe may be non-flat

There is an equation of higher dimensional origin, which is independent of the background fluxes and the number of extra dimensions

$$\mathbf{H^2 - \frac{\ddot{a}}{a} + \frac{k}{a^2} = \dot{\phi}^2}$$

In fact, at some point in phase spaces, one has $\dot{\phi} = 0$ or close to zero. Hence, for $k = 0$, this gives $\dot{H} = \ddot{a}/a - H^2 = 0 \Rightarrow w_\phi = -1$. While, if k=+1, again, at least, in certain spacetime regions, one can satisfy)

$$\mathbf{\ddot{a}/a > H^2 > 0 \Rightarrow \rho + p < 0}$$

$$\mathbf{\Rightarrow N^\mu N^\nu T_{\mu\nu} < 0 \Rightarrow w_\phi = \frac{p}{\rho} < -1}$$

Possible violation of null energy condition! Perhaps the most interesting case is $k < 0$, so $\dot{H} < 0$, and hence $-1 \leq w_\phi < -\frac{1}{3}$

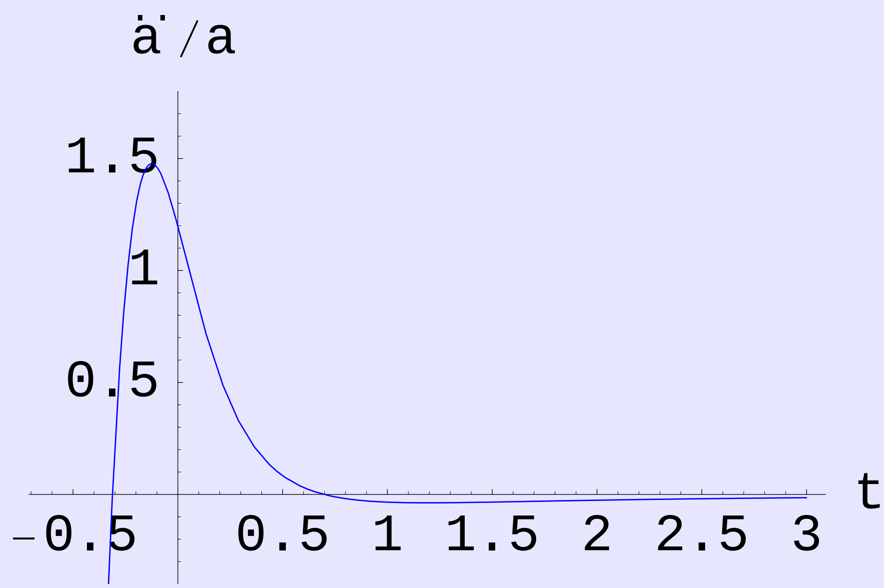
A spatially flat universe and transient acceleration

With $k = 0$ and $\lambda > 1$, only a transient acceleration is possible

$$a(t) = \mathbf{e}^{\alpha} = \left(\cosh \frac{\mathbf{q}\tau}{\mathbf{R_c}}\right)^{\delta_-} + \left(\sinh \frac{\mathbf{q}\tau}{\mathbf{R_c}}\right)^{\delta_+} + \mathbf{C_1}$$

$$\mathbf{e}^{\phi/\sqrt{\mathbf{3}}} = \left(\cosh \frac{\mathbf{q}\tau}{\mathbf{R_c}}\right)^{\delta_-} - \left(\sinh \frac{\mathbf{q}\tau}{\mathbf{R_c}}\right)^{\delta_+} + \mathbf{C_2}$$

$$\delta_{\pm} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}(\sqrt{\mathbf{3}} \pm \lambda)}, \quad \mathbf{q} = \sqrt{\frac{\mathbf{3} - \lambda^{\mathbf{2}}}{\mathbf{2}}}$$



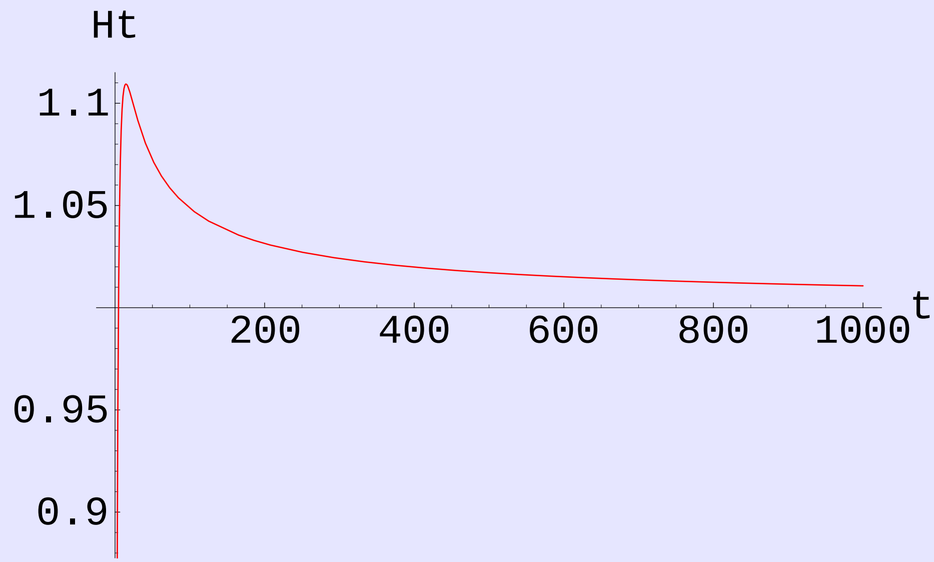
An example of transient acceleration (with $c = 3/\sqrt{7}$ and $k = 0$)

Open Universe and eternal acceleration

To lowest order in cosmological perturbations, the zero-flux solution is

$$\mathbf{a}(\mathbf{t}) = \frac{\lambda}{\sqrt{\lambda^2 - 1}}\,\mathbf{t} + \beta\,\mathbf{t}^{\mathbf{n}}\,, \quad \phi(\mathbf{t}) = \frac{1}{\lambda}\ln\left(\frac{\lambda \mathbf{t}}{\mathbf{R_c}}\right) + \beta\,\frac{3(1-\mathbf{n})}{4}\sqrt{\lambda^2 - 1}\,\mathbf{t}^{\mathbf{n}-1}$$

where β is undetermined (but a small constant) and $n^2 = (2/\lambda)^2 - 3$. This gives an eternally accelerating solution for $1 < \lambda < \sqrt{4/3}$.



An example of eternal acceleration (with $\lambda = 3/\sqrt{7}$ and $k = -1$)

Scalar Potential = Cosmological Constant ?

For many (if not all) classical compactifications of supergravities, only $\lambda \gtrsim 1$ arises in practice. In particular, for the hyperbolic compactification, since $\lambda = \sqrt{\frac{m+2}{m}}$, one has $1 \lesssim \lambda < \sqrt{3}$ when $m \geq 2$.

In the M theory case $m = 7$, and so $\lambda = 3/\sqrt{7}$, we find ($R_c^{-1} \equiv M$)

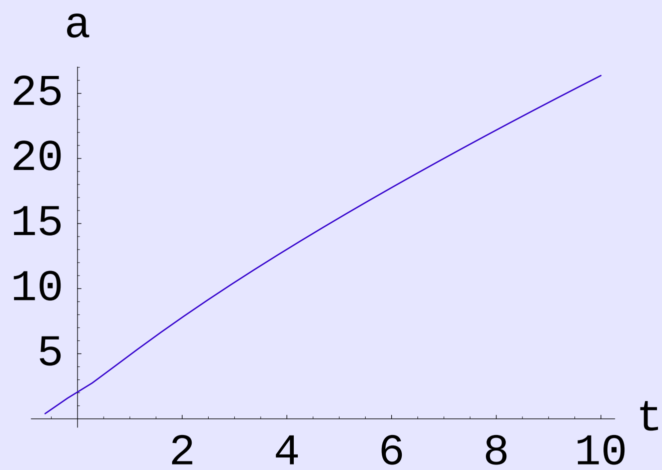
$$\mathbf{V}(\phi) = \mathbf{M_P^2 M^2 \, e^{-2\sqrt{9/7}\,\frac{\phi}{M_P}}} + \frac{\mathbf{M_P^2 \, \tilde{b}^2}}{\mathbf{2}} \, \mathbf{e^{-2\sqrt{7}\,\frac{\phi}{M_P}}}$$

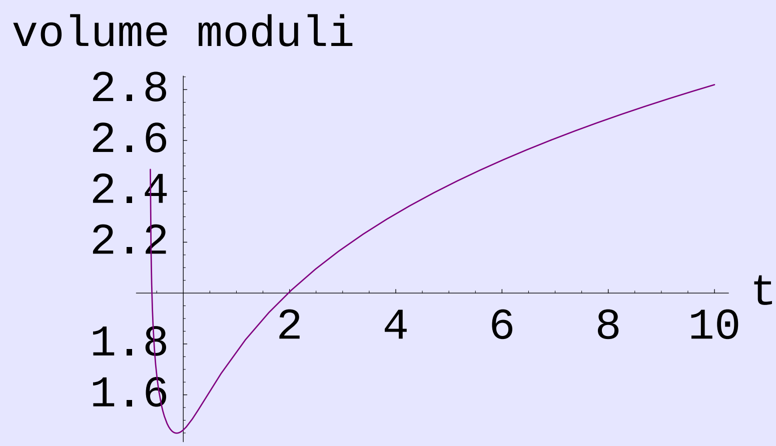
• The late-time cosmology is less affected by the flux term when $\phi >> M_P$. The first exponent $2\sqrt{9/7} \approx 2.267$ is within the limit where astronomical data may be relevant, $\lambda_* \lesssim \sqrt{6}$.

The compactification radius $R_c^{-1} \equiv M$ is not fixed by the field equations. At any rate, it is possible to tune the above potential to the present value of the cosmological constant, that is, $V(\phi) \sim 10^{-120}$, in 4d Planck units, given that $\phi - \phi_0 \sim 115.7$ and $M \sim \; 10^{15}$ GeV.

Just a coincident, in the string theory flux compactification, so called KKLT approach, the inflaton field ϕ must roll within the range $\phi - \phi_0 \sim 116$

Flat Universe, Bouncing moduli and decompactification





The $k = 0$ case. The scale factor of 3-space could grow much faster with t as compared to the volume moduli (size of the internal manifold). In fact, the decompactification is a slow process.

$\mathcal{M}_0 \times \mathcal{M}_1 \times \mathcal{M}_2$ spaces

Can inflation and observed cosmic acceleration both arise from M theory compactification ? This is plausible.

Consider that the internal manifold is a product of two (or more) non-trivial curved spaces $\mathcal{M}_{k_1,m_1} \times \mathcal{M}_{k_2,m_2} \times \cdots$ of dimension m_1, m_2, $\cdots$, so $m = \sum_i m_i$ and total spacetime dimensions $D = 4 + m$.

For canonically normalized scalars ϕ_i, the kinetic term is

$$\frac{1}{2} \sum_i \dot{\phi}_i^2$$

The potential term (in the units $M_p^{-1} = \sqrt{8\pi G} = 1$) is

$$\mathbf{V}(\phi_1, \phi_2) = \mathbf{M}_\mathbf{P}^2 \left(\mathbf{\Lambda_1}\, \alpha^{-\lambda\, \phi_1} + \mathbf{\Lambda_2} \alpha^{-\, \sigma \phi_1 - \rho\, \phi_2} \right)$$

If needed, one may replace Λ_i by $-\epsilon_i \Lambda_i$ so that $\epsilon_i = -1, 0$ or $+1$, respectively for hyperbolic, flat and spherical internal spaces. The couplings λ, σ, ρ are of order unity (see below) and

$$\mathbf{\Lambda_i} \sim \frac{\mathbf{1}}{\mathbf{R_i^2}}, \quad \mathbf{R_i} \gtrsim \mathcal{O}(\mathbf{10})\, \mathbf{M_P^{-1}}$$

A model with two scalar fields

The equations of motion we need to solve are ($i = 1, 2$)

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{dV(\phi_1, \varphi_2)}{d\phi_i} = 0$$

along with the Friedman equation

$$H^2 = \frac{\rho_\phi}{3M_P^2} - \frac{k}{a^2}$$

where

$$\rho_\phi \equiv \frac{1}{2}\sum_i^2 \dot{\phi}_i^2 + V(\phi)$$

.

The term k/a^2 decays rapidly compared to the energy density in the scalar fields, but there is no need to take $k = 0$ at the on set of inflation.

Regardless of initial conditions or/and possible quantum correction terms, the attractor solution following from the above set of equations satisfies

$$a(t) = \begin{cases} a_0 \, (t/t_0)^p \,, & k = 0, \\ \sqrt{\frac{1}{1-p}}\, t, & k = -1\,. \end{cases}$$

An open universe is more preferred over a closed universe. In addition, a solution with $\Lambda_1 > 0$ and $\Lambda_2 < 0$ (or vice versa) exists only if $\lambda < \sigma$, $\sigma^2 + \rho^2 > \lambda\sigma$ (or $\lambda > \sigma$, $\sigma^2 + \rho^2 < \lambda\sigma$). This is however not obtainable from time-dependent classical compatification of supergravities , since

$$\lambda = 2/\sigma = \sqrt{\frac{2(m_1 + 2)}{m_1}} > \sqrt{2}$$

$$\rho = 2\sqrt{\frac{m_1 + m_2 + 2}{m_2(m_1 + 2)}}, \quad p = \frac{2}{\lambda^2} + \frac{2(\lambda - \sigma)^2}{\lambda^2 \rho^2} < 1$$

Remarks:

(1) A solution with $\epsilon_i > 0$ and real volume scalars can exist only if $p < 1/3$ and so a spherical compactification, in general, cannot (eternally) reproduce inflationary universes.

(2) Only a period of transient acceleration is possible when $k = 0$. A very short period of acceleration is possible even if $k = +1$ but in this case the universe goes either a collapsing phase or shows an oscillatory behavior.

How do we see that there is inflation?

The four-dimensional universe inflates when one of the field is trapped around its extremum and the second field is rolling faster; especially, when $\phi_1 \sim 0$ and $\phi_2 \sim ln(t/t_2)$, the solution is :

$$\mathbf{a(t)} = \begin{cases} \mathbf{a_0}\, \alpha^{\mathbf{H_0\, t}}, & \mathbf{k = 0}, \\ \mathbf{H_0^{-1}} \cosh \mathbf{H_0\, t}, & \mathbf{k = +1}, \\ \mathbf{H_0^{-1}} \sinh \mathbf{H_0\, t}, & \mathbf{k = -1}. \end{cases}$$

The Hubble rate is characterized by the scale

$$\mathbf{H_0 t_1} = \frac{\mathbf{2}}{\lambda^{\mathbf{2}}} + \frac{\mathbf{2}(\lambda - \sigma)^{\mathbf{2}}}{\lambda^{\mathbf{2}} \rho^{\mathbf{2}}}$$

Inflation is an epoch of the Universe's evolution during which the comoving Hubble length is decreasing, namely,

$$\frac{\rho(\mathbf{H^{-1} a})}{\mathbf{dt}} < \mathbf{0}$$

Although this is satisfied by all $k = 0, \pm 1$ cosmologies, in the $k = +1$ case, the null energy condition is generally violated.

Summary

- An eternal and/or transient accelerations of the universe are generic for M theory flux compactifications on hyperbolic spaces

- It is not impossible to use M-theory motivated potential for the observed value of dark energy.

- If the spatial curvature of our universe is (perfectly) flat, then the number of accelerating phases would be just one and this is always transient!

- For the M-theory flux compactification on hyperbolic spaces with two or more scalar fields, and $k < 0$, we can find many interesting and new features, like two periods of cosmic accelerations, although for a classical compactification of supergravities the e-folding number $\mathcal{N}_e < 20$