Dark energy from backreaction

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Backreaction

- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**
- The Einstein equation is non-linear in the metric:
  $\langle \mathbf{G}_{\mu\nu}(\mathbf{g}_{\alpha\beta}) \rangle \neq \mathbf{G}_{\mu\nu}(\langle \mathbf{g}_{\alpha\beta} \rangle) \Leftrightarrow$ average quantities ($\langle\rho\rangle$, $\langle\theta\rangle$, …)
  do not satisfy the Einstein equation.
- This is **the fitting problem** (Ellis 1984): when we are fitting FRW model parameters to observations, are we fitting the right model?
- The full problem of smoothing an inhomogeneous spacetime to obtain the relation between the average sources and the average geometry has not been solved.
- A more modest approach: take an already smooth spacetime and study the effect of perturbations on the background.

- Let us consider the Einstein-de Sitter universe ($\boldsymbol{\Omega_m = 1}$) with a scale-invariant spectrum of adiabatic perturbations.
- The metric is

$$\mathbf{d}s^2 = -(1+2\Phi)\,\mathbf{d}t^2 + (1-2\Phi)\,a^2\,\delta_{ij}\,\mathbf{d}x^i\mathbf{d}x^j$$

$$= -\,\mathbf{d}\tau^2 + 2g^{(\tau)}{}_{0i}\,\mathbf{d}\tau\mathbf{d}y^i + g^{(\tau)}{}_{ij}\,\mathbf{d}y^i\mathbf{d}y^j$$

- We are interested in the expansion rate $\theta = u^{\mu}{}_{;\mu}$
- Naive expectation:

$$<\theta>_\tau \simeq 3H_\tau\,(\,1 + \alpha_1 < \Phi > + \alpha_2\,(< \Phi >)^2 + \alpha_3 < \Phi^2 >\,)$$

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- The metric is

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- We are interested in the expansion rate $\theta = u^{\mu}{}_{;\mu}$
- Actual calculation (for Φ independent of t):

$$<\theta>_\tau \simeq 3H_\tau \left(1 + \frac{\beta_1}{(aH)^2} <\nabla^2\Phi> + \frac{\beta_2}{(aH)^4} (<\nabla^2\Phi>)^2 + \frac{\beta_3}{(aH)^2} <\partial_i\Phi\,\partial_i\Phi> + \frac{\beta_4}{(aH)^2} <\partial_i(\Phi\,\partial_i\Phi)> + \frac{\beta_5}{(aH)^4} <\partial_i(\nabla^2\Phi\,\partial_i\Phi)> \right)$$
$$= 3\,H_\tau\,(1 + \lambda_1 a + \lambda_2 a^2)$$

Backreaction as 'dark energy'

- The average expansion rate is
  $(\tfrac{1}{3} < \theta >_\tau)^2$
  $\simeq H_\tau^2 \,(1 + \lambda_1 a + \lambda_2 a^2)^2$
  $\propto a^{-3} + 2\,\lambda_1 a^{-2} + (\lambda_1^2 + 2\,\lambda_2) a^{-1} + 2\,\lambda_1\lambda_2 + \lambda_2^2 a$
- Fitting to the FRW expansion law, we have
  $(\tfrac{1}{3}\,\theta)^2 = \rho/(3M_{Pl}^2)$
  $= (\rho_m + \rho_{de})/(3M_{Pl}^2)$
- From $\rho_{de} \propto a^{-3(1+w)}$ we find the equations of state
  w = –1/3, –2/3, –1 and **–4/3.**
- What are the magnitudes of λ_1 and λ_2?

- The contributions from linear perturbations are

  $\lambda_1 \sim \langle \partial_i\Phi\, \partial_i\Phi \rangle / H_0^2 \sim 10^{-5}$

  $\lambda_2 \sim \langle \partial_i(\nabla^2\Phi\, \partial_i\Phi) \rangle / H_0^4 \sim \langle \delta^2 \rangle / a^2 \sim 1$

- However, λ_2 is a boundary term which is exactly zero because of Fourier decomposition: it would have to be evaluated with realistic boundary conditions.
- Summary: the quadratic backreaction on $\langle \theta \rangle$ from the linear regime, for periodic boundary conditions, is negligible.
- Backreaction from higher order terms or the non-linear regime can be sizeable. For example,

$$\langle (\tfrac{1}{3}\,\theta)^2 \rangle_\tau \simeq H_\tau^2 \left(1 + 4/81 \langle \nabla^2\Phi\, \nabla^2\Phi \rangle / (aH)^4 \right)$$

$$= H_\tau^2 \left(1 + 1/9 \langle \delta^2 \rangle \right)$$

Conclusion

- We know that backreaction
  - is non-zero
  - is boosted by powers of $\boldsymbol{k^2/(aH)^2}$
  - naturally involves $\mathbf{w < 0}$
  - must be taken into account in interpreting observations
- Possibly backreaction
  - from higher order terms could be of order one (for $< \theta >$)
  - from non-linear perturbations could give 'dark energy' at the right time to solve the coincidence problem