NONLOCAL COSMOLOGY

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SFT (string field theory) nonlocal models of the cosmological dark energy providing $w < -1$ are briefly surveyed. We summarize recent developments and open problems, as well as point out some theoretical issues related with others applications of the SFT nonlocal models in cosmology, in particular, in inflation and cosmological singularity.

1 Introduction

The origin of the dark energy (DE) is still a fascinating puzzle. Present cosmological observations do not exclude an evolving DE state parameter w and do not exclude $w < -1$. More precisely, recent data do not have enough accuracy to exclude neither of these two possibilities (see talks presented on section "Dark Energy: ongoing and future surveys" at this meeting concerning future projected aimed to answer to questions about a dynamical DE and $w < -1$).

One might wonder whether there is a room for $w < -1$ in a theory. Dark energy models with the state parameter $w < -1$ violate the null energy condition (NEC). All local models realizing the NEC violation are unstable and violate usual physical requirements.

One can get a viable theory with $w < -1$ as an effective theory which describes a decay to a stable vacuum of some fundamental theory. As a fundamental theory one can have in mind a string theory. A particular realization of this scenario is related with consideration of our Universe as a brane embedded in D-dimensional space-time. Dynamics of the brane is described by the string field theory [1]. String theory has at least two minima, one of which is a perturbative vacuum. A transition from a perturbative vacuum to a non-perturbative one is interpreted as D-brane decay.

In some approximation the dynamics of D-brane can be described by a string tachyon. In this model the role of the dark energy plays the string tachyon and string theory dictates the self interaction of the tachyon as well as interactions with other modes. The tachyon has an effective potential with at least two minima and we can consider the evolution of our Universe as a decay of an unstable vacuum to a stable one. Dynamics in this model is nonlocal. This nonlocality is originated from a nonlocal character of string interaction in SFT and a typical parameter of this nonlocality is the string scale α'.

More precise, we are going to consider SFT nonlocal model. In this model our Universe is considered as a D3 non-BPS brane embedded in the 10 dimensional space-time. The role of the dark energy plays the string tachyon leaving in GSO$-$ sector. The tachyon action is dictated by the cubic fermionic SFT [2,3,4].

It happens that SFT nonlocal model under some conditions displays a phantom behaviour [5]. Note that unlike phenomenological phantom models here phantom appears in an effective theory. Since SFT is a consistent theory this approach does not suffer from usual problems which are inevitable for phenomenological phantom models. The UV completion is supposed to be solved by extending the one mode (tachyon) approximation. The back reaction can be taken into account in an approximation of the dilaton field [6](see also the talk by A.Koshelev)

SFT in the flat background dictates a particular value of the D-brane tension. It can be found from the requirement that the total energy of the system in the true non-perturbative vacuum is zero. In cosmology this total energy of the system in the true non-perturbative vacuum can be interpreted as the cosmological constant. It has been conjectured that an existence of a rolling solution describing a smooth transition to the true vacuum defines the value of the cosmological constant [1]. We cannot prove this conjecture but arguments to it favor can be given using the local approximation [9]. A recent breakthrough in solving numerically the full nonlinear and nonlocal system of equations [8] also supports this expectation.

2 Model

Our model is given by the following action [1]

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2}+\frac{1}{\lambda_4^2}\left(-\frac{\xi^2\alpha'}{2}g^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x)+\frac{1}{2}\phi^2(x)-\frac{1}{4}\Phi^4(x)-T'\right)\right)$$

Here $g_{\mu\nu}$, κ and λ_4 are the four-dimensional metric, gravitational coupling constant and scalar field coupling constant, respectively; $\frac{1}{\lambda_4^2}=\frac{v_6M_s^4}{g_o}(\frac{M_s}{M_c})^6$, g_o is the open string dimensionless coupling constant, M_s is the string scale $M_s=1/\sqrt{\alpha'}$ and M_c is a scale of the compactification, v_6 is a number related with a volume of the 6-dimensional compact space. T' is a constant related with the difference of the values of potential in the extremal points (1/4 in our case) and a dimensionless cosmological constant Λ'

$$T'=1/4+\Lambda'.$$

ϕ is a tachyon field and Φ is related with ϕ by the following relation

$$\Phi=e^{\frac{\alpha'}{8}\Box_g}\phi,\quad\text{where}\quad\Box_g=\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu.$$

This form of the nonlocal interaction is defined by the cubic fermionic SFT (CFSFT) [3]. More precisely, the CFSFT brings a more complicated form of the interaction, but by an analogy with the flat case [7,5,8] we believe that this approximation catches essential physical properties of the model and $\xi^2\approx 0.9556$ is a constant defined by the CFSFT.

In the spatially flat FRW metric with a scale factor $a(t)$ the equation for a space homogenous tachyon field Φ and the Friedmann equations have the form [1]

$$\left(\xi^2\mathcal{D}+1\right)e^{-\frac{1}{4}\mathcal{D}}\Phi = \Phi^3,\qquad \mathcal{D}=-\partial_t^2-3H(t)\partial_t,\quad H=\partial_t a/a \tag{1}$$

$$3H^2 = \frac{\kappa^2}{\lambda_4^2}\left(\frac{\xi^2}{2}\partial_t\phi^2-\frac{1}{2}\phi^2+\frac{1}{4}\Phi^4+\mathcal{E}_1+\mathcal{E}_2+T'\right), \tag{2}$$

$$\mathcal{E}_1 = -\frac{1}{8}\int_0^1 ds\left((\xi^2\mathcal{D}+1)\ e^{\frac{s-2}{8}\mathcal{D}}\Phi\right)\cdot\left(\mathcal{D}\ e^{-\frac{1}{8}s\mathcal{D}}\Phi\right), \tag{3}$$

$$\mathcal{E}_2 = -\frac{1}{8}\int_0^1 ds\left(\partial_t(\xi^2\mathcal{D}+1)\,e^{\frac{s-2}{8}\mathcal{D}}\Phi\right)\cdot\left(\partial_t e^{-\frac{1}{8}s\mathcal{D}}\Phi\right). \tag{4}$$

The non-local energy $\mathcal{E}_1$ plays the role of an extra potential term and $\mathcal{E}_2$ the role of the kinetic term. Note that here we use a dimensionless time $t\to t\sqrt{\alpha'}$.

3 How we study our model and what we get

Equations (1) and (2) form a rather complicated system of nonlinear nonlocal equations for functions Φ and $H(t)$ because of the presence of an infinite number of derivatives and a non-flat metric. Before to discuss the methods of study this model let us mention the known methods of study equation (1) in the flat background, $H = 0$,

$$\left(-\xi^2\partial_t^2 + 1\right) e^{\frac{1}{4}\partial_t^2}\Phi(t) = \Phi(t)^3. \tag{5}$$

Equation (2) in the flat case describes the energy conservation [5].

A boundary problem $\Phi(\pm\infty) = \pm 1$ for (5) has been studied using:

- a numerical method[7] based on an integral representation of (5); it is related with a diffusion equation method [10] which uses an auxiliary function of two variables $\Psi(r,t)$ that is the subject of a linear equation and $\Psi(\frac{1}{4},t) = \Phi(t)$;
- a decomposition on local fields [11,13,14,12]; this method works well for linear equations and has been used to study solutions to (5) near vacuum ± 1;
- existence theorems [16,17,18];
- almost exact solutions methods [19,20]; the approach [19] uses a diffusion equation method.

The following two characteristic properties of (5) have been obtained

- an existence of a critical point $\xi^2_{\text{cr}} \approx 1.38$ such that for $\xi^2 < \xi^2_{\text{cr}}$ eq. (5) has a rolling solution [7] interpolating between ± 1;
- an existence of a dominance of an extra non-local kinetic term $\mathcal{E}_2$ over the local kinetic one [5] and as a result, an appearance of a phantom behavior providing $w < -1$.

These result have been obtained using numerical calculations. It is very interesting to study the problem analytically and also try to find approximate models admitting explicit solutions and having above mentioned properties. They could be two or more components local models.

An investigation of non-flat eqs. (1) and (2) is essentially more complicated. The following methods are used:

- A decomposition on local fields and a modification of the potential have been used in [9,20,22,14]. A simplest one phantom mode approximation with an explicit form of the solution $\phi(t) = \tanh(t)$ is realized for a six-order potential [9] and gives $H_0 = 1/3m_p^2$. Assuming that $M_c \sim M_p$ and $M_s \sim 10^{-6.6}M_p$ we get

$$H_0 \sim 10^{-60}M_p.$$

- an analytic approach that is closely related with the diffusion equation method [23].
- A numerical study has been performed in [8], where the diffusion equation method has been used to define $\exp\mathcal{D}$ and a double-step iteration procedure has been proposed.

The following physical effects are found in [8]

- For $\xi^2 < \xi^2_{\text{cr}} \approx 1.18$ and $\Lambda = \Lambda(\xi)$ the system (1), (2) has a rolling solution.

- For $\xi^2 < \xi^2_{\rm shape}$ and $t > 0$ the Hubble function $H(t)$ is a function which has small fluctuations about a monotonic function $H_l(t)$ with an asymptotic H_0;
  for $\xi^2_{\rm shape} < \xi^2 < \xi^2_{\rm cr}$ $H(t)$ describes fluctuations about a function $H_l(t)$ that has two maximum. To realize this approximated shape $H_l(t)$ by local fields one needs at least two fields [22].

As in the flat case it would be very interesting to find approximate analytical solutions which exhibit these properties. Note that two maximum shape regime for $H(t)$ is interesting in a context of building an unified cosmological evolution. Let us also note that there are applications of non-local SFT models to inflation [24] and cosmological singularity ([14,15] and refs therein).

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