On the anomalous increase of the eccentricity of the lunar orbit: search for possible explanations

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OUTLINE

UNEXPLAINED INCREASE OF THE LUNAR ECCENTRICITY

- [Anderson & Nieto 2010] mentioned an anomalous secular increase of the eccentricity e of the lunar orbit

$$\dot{e}_{☾} = (9 \pm 3) \times 10^{-12}\ \mathrm{yr}^{-1} \quad (1)$$

  based on an analysis of LLR data by [Williams & Boggs 2009] over 38.7 yr with the the DE421 ephemerides.

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CAUTIONARY REMARKS (I)

- Simple dimensional evaluations of the effect on e due to an extra-acceleration A can be made by noticing that

$$\dot{e} \approx \frac{A}{na}, \tag{3}$$

where a is the orbital semi-major axis, and $n \doteq \sqrt{\mu/a^3}$ is the Keplerian mean motion in which $\mu \doteq GM(1 + m/M)$ is the gravitational parameter of the Earth-Moon system. An extra-acceleration

$$A \approx 3 \times 10^{-16}\ \mathrm{m\ s^{-2}} = 0.3\ \mathrm{m\ yr^{-2}} \tag{4}$$

would satisfy eq. (1)

- But a mere order-of-magnitude analysis would be insufficient to draw meaningful conclusions: just finding an extra-acceleration of the right order of magnitude may be misleading [Iorio 2011a].

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CAUTIONARY REMARKS (II)

- Exact calculations of the *secular* variation of e caused by such putative promising extra-accelerations A must be performed in order to check if they, *actually*, cause an averaged *non-zero* change of the eccentricity [Iorio 2011a].
- Moreover, also in such potentially favorable cases, i.e. when a perturbative acceleration A with the right order of magnitude yields a non-vanishing $\langle \dot{e} \rangle$, caution is still in order. Indeed, it may well happen, in principle, that the resulting analytical expression for $\langle \dot{e} \rangle$ retains multiplicative factors $1/e^j, j = 1, 2, 3, ...$ or $e^j, j = 1, 2, 3...$ which would notably alter the size of the non-zero secular change of the eccentricity found with respect to the expected values according to eq. (3).

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OUTLINE

A Rindler-type acceleration

- Recently, [Grumiller 2010] has constructed a dilaton-based, effective model for gravity of a central object of mass M at large scales. Among other things, it predicts the existence of a constant and uniform Rindler-type acceleration

$$\boldsymbol{A} = A_{\mathrm{Rin}}\hat{\boldsymbol{r}} \tag{5}$$

radially directed towards M.

- Actually, eq. (5) does *not* induce any secular variation of the eccentricity. Indeed, from the standard Gauss perturbation equation for e it turns out

$$\Delta e = -\frac{A_{\mathrm{Rin}}\left(1-e^{2}\right)\left(\cos E - \cos E_{0}\right)}{n^{2}}, \tag{6}$$

where E is the eccentric anomaly, i.e. a parametrization of the polar angle in the orbital plane, so that

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A YUKAWA-TYPE EFFECT (I)

- Many theoretical paradigms allow for a Yukawa-like perturbation of the Newtonian gravitational potential

$$U_{\rm Y} = -\frac{\alpha\mu_\infty}{r}\exp\left(-\frac{r}{\lambda}\right), \tag{8}$$

where μ_∞ is the gravitational parameter evaluated at distances r much larger than the scale length λ.

- In order to compute the long-term effects of eq. (8) on the eccentricity of a test particle it is convenient to adopt the Lagrange perturbative equation for e

$$\left\langle\frac{de}{dt}\right\rangle = \frac{1}{na^2}\left(\frac{1-e^2}{e}\right)\left(\frac{1}{\sqrt{1-e^2}}\frac{\partial\mathcal{R}}{\partial\omega} - \frac{\partial\mathcal{R}}{\partial\mathcal{M}}\right), \tag{9}$$

where ω is the argument of pericenter, $\mathcal{M} = E - e\sin E$ is the mean anomaly, and $\mathcal{R}$ is the average of the perturbing potential over one orbital revolution.

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- In the case of a Yukawa-type perturbation, eq. (8) yields

$$\langle U_{\mathrm{Y}} \rangle = -\frac{\alpha\mu_{\infty} \exp\left(-\frac{a}{\lambda}\right)}{a} I_0\left(\frac{ae}{\lambda}\right), \tag{10}$$

  where $I_0(x)$ is the modified Bessel function of the first kind $I_q(x)$ for $q = 0$.

- An inspection of eq. (9) and eq. (10) immediately tells us that, since eq. (10) does contain neither ω nor $\mathcal{M}$, there is *no* secular variation of e caused by an anomalous Yukawa-type perturbation.
- A Yukawa-like modification of gravity *cannot* explain the measured variation of the lunar eccentricity of eq. (1).

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OTHER LONG-RANGE EXOTIC MODELS OF GRAVITY

- The previous analysis has the merit of elucidating certain *general* features pertaining to a vast category of long-range modified models of gravity Indeed, eq. (9) tells us that a long-term change of e occurs only if the averaged extra-potential considered *explicitly* depends on ω and on time through $\mathcal{M}$ or, equivalently, E. Actually, the anomalous potentials arising in the majority of long-range modified models of gravity are *time-independent* and *spherically symmetric*
- Anomalous accelerations $\boldsymbol{A}$ exhibiting a dependence on the test particle's velocity $\boldsymbol{v}$ were also proposed in different frameworks. It was straightforward to perturbatively infer that *no* long-term variations of the eccentricity arose at all.

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GENERAL RELATIVITY: THE LENSE-THIRRING EFFECT

- The magnitude of the general relativistic [Lense & Thirring 1918] acceleration of the Moon due to the Earth's angular momentum $S = 5.86 \times 10^{33}$ kg m^2 s^{-1} is just

$$A_{\rm LT} \approx \frac{2vGS}{c^2 a^3} = 1.6 \times 10^{-16}\ {\rm m\ s^{-2}} = 0.16\ {\rm m\ yr^{-2}}, \quad (11)$$

  i.e. it is close to eq. (4).

- The Lense-Thirring effect does *not* cause long-term variations of the eccentricity since the integrated shift of e from an initial epoch corresponding to f_0 to a generic time corresponding to f, where f is the true anomaly, is [Soffel 1989]

$$\Delta e = -\frac{2GS \cos I\,(\cos f - \cos f_0)}{c^2 n a^3 \sqrt{1-e^2}}. \quad (12)$$

  Thus, after one orbital revolution, i.e. for $f \rightarrow f_0 + 2\pi$, the gravitomagnetic shift of e vanishes

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OUTLINE

A *potentially* viable candidate: Planet X (I)

- A promising candidate for explaining the anomalous increase of the lunar eccentricity may be, *at least in principle*, a trans-Plutonian massive body of planetary size located in the remote peripheries of the solar system: Planet X/Nemesis/Tyche?
- The perturbing potential felt by a test particle orbiting a central body due to a very distant, pointlike mass can be cast into the following quadrupolar form [Hogg et al. 1991]

$$U_{\rm X} = \frac{\mathcal{K}_{\rm X}}{2}\left[r^2 - 3\left(\boldsymbol{r}\cdot\boldsymbol{\hat{l}}\right)^2\right], \quad (13)$$

where $\mathcal{K}_{\rm X} \doteq Gm_{\rm X}/d_{\rm X}^3$ is the tidal parameter of X, and $\boldsymbol{\hat{l}} = \{l_x, l_y, l_z\}$ is a unit vector directed towards X determining its position in the sky. In eq. (13) $\boldsymbol{r} = \{x, y, z\}$ is the geocentric position vector of the perturbed particle, which, in the present case, is the Moon

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A *potentially* viable candidate: Planet X (II)

- The average of eq. (13) over one orbital revolution of the particle is [Iorio 2011b]

$$\langle U_{\rm X}\rangle = \frac{\mathcal{K}_{\rm X} a^2}{32}\mathcal{U}\left(I, \Omega, \omega; \hat{\boldsymbol{l}}\right), \tag{14}$$

where $\mathcal{U}\left(I, \Omega, \omega; \hat{\boldsymbol{l}}\right)$ is a quite involved function of the inclination I, the node Ω and the perigee ω of the Moon, and of the position of X in the sky.

- Thus, the Lagrange planetary equation (9) yields [Iorio 2011b]

$$\langle \dot{e}\rangle = \frac{15\mathcal{K}_{\rm X} e\sqrt{1-e^2}}{16n}\mathcal{E}\left(I, \Omega, \omega; \hat{\boldsymbol{l}}\right) \neq 0, \tag{15}$$

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- Actually, the expectations concerning X are doomed to fade away.
- Indeed, a long-term harmonic modulation is introduced in $\langle \dot{e} \rangle$ by the presence of the time-varying ω and Ω in eq. (15), contrary to the linearly increasing trend actually measured in eq. (1)
- Moreover, it turns out [Iorio 2011a] that

$$\mathcal{K}_{\rm X} = 4.46 \times 10^{-24}\ {\rm s}^{-2} \tag{16}$$

would agree with eq. (1), as far as the order of magnitude is concerned. But, eq. (16) is totally unacceptable since it corresponds to distances of X as absurdly small as $d_{\rm X} = 30$ au for a terrestrial body, and $d_{\rm X} = 200$ au for a Jovian mass.

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A LOCAL COSMOLOGICAL EFFECT? (I)

- Let us assume that there is a small radial extra-acceleration of the form

$$\boldsymbol{A} = kH_0 v_r \hat{\boldsymbol{r}}. \quad (17)$$

In it k is a positive numerical parameter of the order of unity to be determined from the observations,

$$H_0 = (71.0 \pm 2.5)\ \mathrm{km\ s^{-1}\ Mpc^{-1}} = (7.3 \pm 0.2) \times 10^{-11}\ \mathrm{yr^{-1}} \quad (18)$$

is the Hubble parameter at the present epoch, and v_r is the radial component of the velocity $\boldsymbol{v}$ of the test particle's proper motion

- It turns out that both the semi-major axis a and the eccentricity e of the test particle's orbit secularly increase according to

$$\langle \dot{a} \rangle = 2kaH_0 \left(1 - \sqrt{1 - e^2}\right),\ \langle \dot{e} \rangle = kH_0 \frac{\left(1 - e^2\right)\left(1 - \sqrt{1 - e^2}\right)}{e}. \quad (19)$$

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A LOCAL COSMOLOGICAL EFFECT? (II)

- Since $e_{☾} = 0.0647$, it turns out that eq. (19) is able to reproduce the measured anomalous increase of the lunar orbit for

$$2.5 \lesssim k \lesssim 5. \tag{20}$$

  If we assume the terrestrial semi-major axis $a_{\oplus} = 1.5 \times 10^{13}$ cm as an *approximate* measure of the astronomical unit and consider that $e_{\oplus} = 0.0167$, eq. (19) and the previous values of k yield a secular increase of just a few cm yr^{-1}, in agreement with the latest determinations of the anomalous increase of the astronomical unit [Anderson & Nieto 2010].

- It is just an *empirical* result, without any defined theoretical scenario behind it. Actually, known [Cooperstock et al. 1998, Mashhoon & Singh 2007] local cosmological effects are of tidal origin, and are much smaller, being of the order of H_O^2.

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Concluding remarks

- *All* the recently proposed long-range modified models of gravity *fail* to explain the anomalous increase of the lunar eccentricity
- General relativistic gravitomagnetism has the right order of magnitude, but it does *not* cause a long-term variation of e.
- A putative, still unseen trans-Plutonian planetary body does induce a long-term variation of e, but it should be *unrealistically close to us* to yield the right order of magnitude for $\dot{e}_{☾}$
- A *purely empirical* extra-acceleration proportional to the radial velocity of the test particle through a coefficient with the same magnitude of the Hubble parameter is qualitatively able to explain both the anomalous increase of the eccentricity of the Moon and of the astronomical unit. Also the orders of magnitude are reproduced rather well.

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