

Gravity with a Preferred Scale

Ali Mozaffari

Theoretical Physics Group, Imperial College

ali.mozaffari05@imperial.ac.uk

Recontres de Moriond
Sunday 22nd March 2015

Introduction

One of the outstanding accomplishments of 20th century physics was the formulation of Einstein's celebrated theory of General Relativity:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$
$$S = \int \left(\frac{c^4}{16\pi G} R + \mathcal{L}_m \right) \sqrt{-g} d^4x$$

This description of gravity matches a wide variety of tests -local, large scale astrophysical and cosmological. The great phenomenological success of the Λ CDM model is built upon a strong precedent that the gravity theory is a good fit to nature.

However did Einstein have the final word? Are there significant or radical departures to be made from the GR paradigm without losing the great match to observations we cherish?

Acceleration

We may imagine that gravity matches very well the local and Solar System based experiments we may perform - but there is a lower limit to all of these, they occur at $a \gtrsim 10^{-5} \text{ ms}^{-1}$. Nearby galactic accelerations are often $a \lesssim 10^{-10}$ - giving a range of **untested** parameter space.

Acceleration

We may imagine that gravity matches very well the local and Solar System based experiments we may perform - but there is a lower limit to all of these, they occur at $a \gtrsim 10^{-5} \text{ ms}^{-1}$. Nearby galactic accelerations are often $a \lesssim 10^{-10}$ - giving a range of **untested** parameter space.

Additionally we appear to find ourselves in a cosmological period of some Λ domination, what if the embedding of a scale is related to the accelerated expansion?

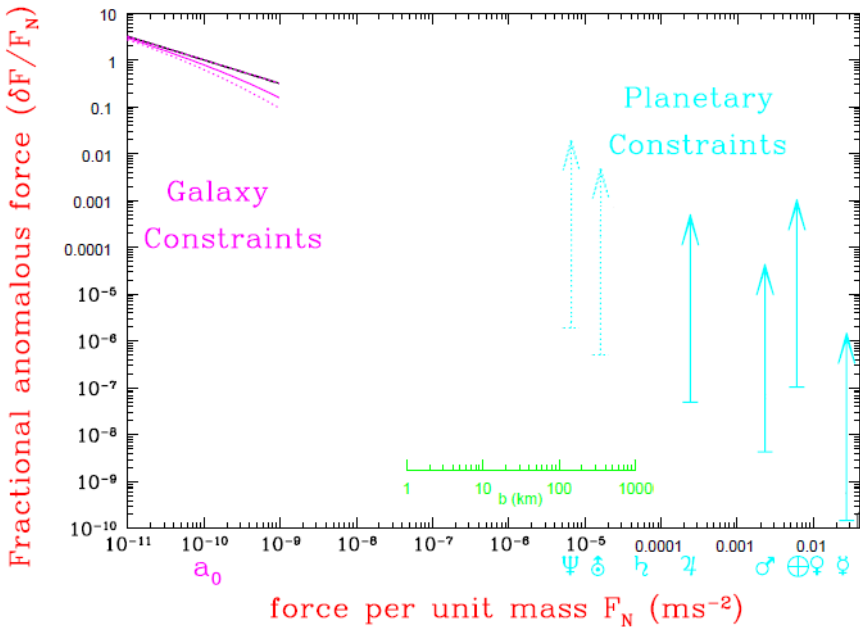
Acceleration

We may imagine that gravity matches very well the local and Solar System based experiments we may perform - but there is a lower limit to all of these, they occur at $a \gtrsim 10^{-5} \text{ ms}^{-1}$. Nearby galactic accelerations are often $a \lesssim 10^{-10}$ - giving a range of **untested** parameter space.

Additionally we appear to find ourselves in a cosmological period of some Λ domination, what if the embedding of a scale is related to the accelerated expansion?

On quantum scales, we understand that an effective field theory approach can be made with æther theories (0905.2446), adding the vector field v_{ν} to the scales in consideration.

Have we seen this before? - MOND



A Minimal Model

It would be great to modify GR using just some combination of the space time metric $g_{\mu\nu}$, making objects like

$$R_{\beta\mu\nu}^{\alpha}(a_{pref})$$

A Minimal Model

It would be great to modify GR using just some combination of the space time metric $g_{\mu\nu}$, making objects like

$$R_{\beta\mu\nu}^{\alpha}(a_{\text{pref}})$$

But a no-go theorem of Woodard and Soussa tells us if we make an action for a MONDian metric theory, the trace of the EE in the weak field limit may be written in a conformally invariant way - classical EM is conformally invariant!

$$S_{EM} = -\frac{1}{4} \int F_{\mu\nu} F_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta} \sqrt{-g} d^4x$$
$$F_{\mu\nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu}$$

To explain additional light bending - need additional dof external from $g_{\mu\nu}$!

Where would the scale arise?

Consider a four velocity field \mathbf{v}^μ , by constructing a projector

$$h_{\mu\nu} = g_{\mu\nu} - \xi_\mu \xi_\nu$$

to define a spatial velocity vector $\mathbf{U}^\mu = h^\mu{}_\nu \mathbf{v}^\nu$. Then through $\xi^\alpha \nabla_\alpha \mathbf{U}^\beta$ we may assign a norm $a_{norm} = |\xi^\alpha \nabla_\alpha \mathbf{U}^\beta|$, $a_{pref} \sim a_{norm}$. **Local object!**

Where would the scale arise?

Consider a four velocity field \mathbf{v}^μ , by constructing a projector

$$h_{\mu\nu} = g_{\mu\nu} - \xi_\mu \xi_\nu$$

to define a spatial velocity vector $\mathbf{U}^\mu = h^\mu{}_\nu \mathbf{v}^\nu$. Then through $\xi^\alpha \nabla_\alpha \mathbf{U}^\beta$ we may assign a norm $a_{norm} = |\xi^\alpha \nabla_\alpha \mathbf{U}^\beta|$, $a_{pref} \sim a_{norm}$. **Local object!**

Generically such a mechanism may be introduced through a variational principle, e.g.

$$\begin{aligned} S_\lambda &= \int \lambda \left(\mathcal{X}^\alpha \mathcal{X}^\beta g_{\mu\nu} - a_{pref}^2 \right) \sqrt{-g} d^4x \\ \mathcal{X}^\beta &= \xi^\alpha \nabla_\alpha \mathbf{U}^\beta \end{aligned}$$

alongside the usual canonical kinetic terms for \mathbf{U}^α .

A little precedent

An acceleration scale appears quite generically in relativistic theories with extra degrees of freedom, examples can include:

- Einstein-Æther \iff Hořava-Gravity

A little precedent

An acceleration scale appears quite generically in relativistic theories with extra degrees of freedom, examples can include:

- Einstein-Æther \iff Hořava-Gravity

$$L = \left(\frac{M_{pl}^2}{2} [R + \alpha^2 \mathcal{K}] + \lambda (A^\mu A^\nu g_{\mu\nu} + 1) \right)$$

$$\mathcal{K} = \alpha^{-2} \mathcal{K}_{\mu\nu}{}^{\alpha\beta} \nabla_\alpha A^\mu \nabla_\beta A^\nu$$

$$\mathcal{K}_{\mu\nu}{}^{\alpha\beta} = c_1 g_{\mu\nu} g^{\alpha\beta} + c_2 \delta_\mu^\alpha \delta_\nu^\beta + c_3 \delta_\mu^\beta \delta_\nu^\alpha + c_4 g_{\mu\nu} A^\alpha A^\beta$$

and Hořava's theory maybe implemented with a Hypersurface Orthogonal solutions

$$A^\alpha = \frac{g^{\alpha\beta} \nabla_\beta Y}{(g^{\mu\nu} \nabla_\mu Y \nabla_\nu Y)^{1/2}}$$

- Chameleon Screening Mechanism - these will arise later

A Toy Model

Consider a scalar field non-minimally coupled to gravity

$$\begin{aligned} S &= \int \left(\frac{M_{pl}^2}{2} R + \frac{a_1}{\alpha} X - V(\phi) - F(a_1) \right) \sqrt{-g} d^4x \\ &+ \int \mathcal{L}_M \sqrt{-\tilde{g}} d^4x \\ \tilde{g}_{\mu\nu} &= f(\phi) g_{\mu\nu} \end{aligned}$$

A Toy Model

Consider a scalar field non-minimally coupled to gravity

$$\begin{aligned} S &= \int \left(\frac{M_{pl}^2}{2} R + \frac{a_1}{\alpha} X - V(\phi) - F(a_1) \right) \sqrt{-g} d^4x \\ &+ \int \mathcal{L}_M \sqrt{-\tilde{g}} d^4x \\ \tilde{g}_{\mu\nu} &= f(\phi) g_{\mu\nu} \end{aligned}$$

EoM's fix relations between F , a_1 and X , but form F , a_1 left as a free function.

Cosmological Limit

In the cosmological regime, this action clearly fits just a class of k-essence theories

$$S = \int \left(\frac{M_{pl}^2}{2} R + P(X, \phi) \right) \sqrt{-g} d^4x$$

$$P = \frac{a_1}{\alpha} X - F\left(\frac{X}{\alpha}\right) - V(\phi)$$

$$\rho = \frac{a_1}{\alpha} X + F\left(\frac{X}{\alpha}\right) + V(\phi)$$

Find a perfectly viable (tuned) inflationary theory - $n_s \simeq 1$, f_{NL} small and etc ...

Interestingly the reheating regime appears to be the same, upto a field redefinition - hiding some order higher effects?

Weak Field Limit

Expanding around a flat background, using a $1/c$ style expansion

$$\begin{aligned}g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{c^2} h_{\mu\nu} \\ \partial_0 &\equiv \frac{1}{c} \frac{\partial}{\partial t} \\ \phi &\equiv \phi_0(t) + \frac{1}{c^2} \phi_1(t, \mathbf{x})\end{aligned}\tag{1}$$

Weak Field Limit

Expanding around a flat background, using a $1/c$ style expansion

$$\begin{aligned}g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{c^2} h_{\mu\nu} \\ \partial_0 &\equiv \frac{1}{c} \frac{\partial}{\partial t} \\ \phi &\equiv \phi_0(t) + \frac{1}{c^2} \phi_1(t, \mathbf{x})\end{aligned}\tag{1}$$

At c^{-2} order, with a quasi-static approximation $|\dot{\phi}| \ll 1$ and bosonic SET $\tilde{T}^0_0 = \bar{\rho}$,

$$\begin{aligned}\nabla^2 \Phi_N &= 4\pi G \bar{\rho} \\ \nabla \cdot (a_1 \nabla \phi) &= \alpha \left(V_{,\phi} + \frac{1}{4} f_{,\phi}^2 \bar{\rho} \right)\end{aligned}$$

Weak Field Limit

Expanding around a flat background, using a $1/c$ style expansion

$$\begin{aligned}g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{c^2} h_{\mu\nu} \\ \partial_0 &\equiv \frac{1}{c} \frac{\partial}{\partial t} \\ \phi &\equiv \phi_0(t) + \frac{1}{c^2} \phi_1(t, \mathbf{x})\end{aligned}\tag{1}$$

At c^{-2} order, with a quasi-static approximation $|\dot{\phi}| \ll 1$ and bosonic SET $\tilde{T}^0_0 = \bar{\rho}$,

$$\begin{aligned}\nabla^2 \Phi_N &= 4\pi G \bar{\rho} \\ \nabla \cdot (a_1 \nabla \phi) &= \alpha \left(V_{,\phi} + \frac{1}{4} f_{,\phi}^2 \bar{\rho} \right)\end{aligned}$$

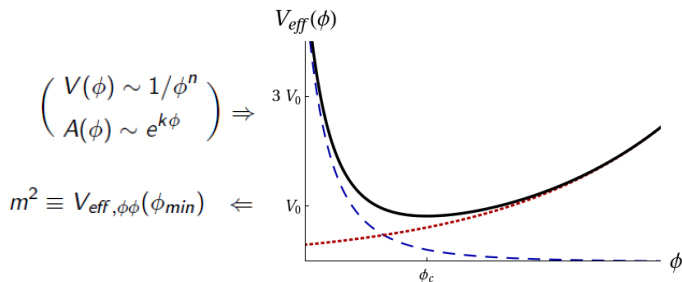
as well as physical metric perturbation

$$\Phi = f(\phi_0) \Phi_N + \phi$$

Chameleon

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{M^2}{2} R + X - V(\phi) \right) + \mathcal{S}_m (A^2(\phi) g_{\mu\nu}, \Psi_i)$$

$$\square\phi = V_{,\phi} + A_{,\phi} \hat{\rho} \implies V_{\text{eff}} = V(\phi) + \hat{\rho} A(\phi)$$



$$\nabla \cdot (a_1 \nabla \phi) = \alpha \rho \iff \nabla^2 \phi = F(\phi, X, \dots) \rho + G(\phi, X, \dots)$$

Beyond Æther Theories

Making a Generalised AE theory

$$L = \left(\frac{M_{pl}^2}{2} [R + \alpha^2 \mathcal{F}(\mathcal{K})] + \lambda (A^\mu A^\nu g_{\mu\nu} + 1) \right)$$

$$\mathcal{K} = \alpha^{-2} \mathcal{K}_{\mu\nu}{}^{\alpha\beta} \nabla_\alpha A^\mu \nabla_\beta A^\nu$$

$$\mathcal{K}_{\mu\nu}{}^{\alpha\beta} = c_1 g_{\mu\nu} g^{\alpha\beta} + c_2 \delta_\mu^\alpha \delta_\nu^\beta + c_3 \delta_\mu^\beta \delta_\nu^\alpha + c_4 g_{\mu\nu} A^\alpha A^\beta$$

Beyond Æther Theories

Making a Generalised AE theory

$$\begin{aligned}L &= \left(\frac{M_{pl}^2}{2} [R + \alpha^2 \mathcal{F}(\mathcal{K})] + \lambda (A^\mu A^\nu g_{\mu\nu} + 1) \right) \\ \mathcal{K} &= \alpha^{-2} \mathcal{K}_{\mu\nu}{}^{\alpha\beta} \nabla_\alpha A^\mu \nabla_\beta A^\nu \\ \mathcal{K}_{\mu\nu}{}^{\alpha\beta} &= c_1 g_{\mu\nu} g^{\alpha\beta} + c_2 \delta_\mu^\alpha \delta_\nu^\beta + c_3 \delta_\mu^\beta \delta_\nu^\alpha + c_4 g_{\mu\nu} A^\alpha A^\beta \\ &+ \left[(c_5 g^{\alpha\beta} + c_6 A^\alpha A^\beta) g_{\gamma\mu} g_{\sigma\nu} A^\sigma \right. \\ &\left. + (c_7 \delta_\mu^\beta g_{\gamma\nu} + c_8 \delta_\nu^\beta g_{\gamma\mu}) A^\alpha \right] A^\gamma\end{aligned}$$

The original EA theories picked $\mathcal{F}(\mathcal{K}) \rightarrow \mathcal{K}$, $c_i = 0, i \geq 4$ or likewise from a perturbative action

This may also be written as a generalised Hořava theory - (messy!)

A Geometric Perspective

Another way in which to view these modified theories may be examined is moving just in GR with a modified spacetime, Generalised Einstein Æther theories take the form

$$ds^2 = -f(r)dt^2 + \frac{dL^2}{f(r)}$$

$$f(r) = 1 + \frac{c}{3} \ln\left(\frac{r}{r_0}\right)$$

c is a dimensionless parameter implementing the a_{pref} and r_0 is an integration constant. Our toy model will take the form

$$ds^2 = -\left(\frac{r}{r_0}\right)^{\frac{2c}{1-c}} dt^2 + \frac{dr^2}{(1-c)^2} + r^2 d\Omega^2$$

Barriola-Wilenkin Global Monopole - $c > 1$ unphysical

Possible Future Constraints

Ultra-low acceleration saddle points (SP) in the Solar System, such as Earth-Sun, Jupiter-Sun, etc . . .

- Tidal Stresses - A movement away from GR predicts generically different tidal stresses around SP, possible to be tests using LISA Pathfinder
- Light Delays - Additional time delays are expected in a region of modified weak-field potentials, c.f. Lunar Laser Ranging, Long Baseline Interferometry
- Strong-Field Tests - As these theories are **acceleration** dependent, it is possible to engineer tests in different regimes of curvature
- Dynamical Dark Energy / Screening Mechanism tests

Conclusions

Whilst we have the main ingredients in our gravitational theories, it could be possible to include a modification at (to be chosen) acceleration scales.

Often some more familiar theories appear to have an a_{pref} hidden in their structure.

The possibility to make a modification for both weak-field limits and also with some inflationary limit is viable - more work needs to be done to write properly consistent action.

Connection with geometry and also massive gravity could be made - these need more investigation. (Possibility of using PPN?)