Standard Electroweak Interactions in Gravitational Theory with Chameleon Field and Torsion

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Content

Properties of Torsion Tensor Field $\mathcal{T}^{\alpha}{}_{\mu\nu}$

Torsion tensor $\mathcal{T}^{\alpha}{}_{\mu\nu}$,
introduced as an antisymmetric part of
the affine connection $\Gamma^{\alpha}{}_{\mu\nu}$

$$\mathcal{T}^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\nu\mu} - \Gamma^{\alpha}{}_{\mu\nu} = -\mathcal{T}^{\alpha}{}_{\nu\mu},$$

has 24 independent components,
which can be given in the following general form

$$\mathcal{T}_{\alpha\mu\nu} = \frac{1}{3}(g_{\alpha\mu}\mathcal{E}_{\nu} - g_{\alpha\nu}\mathcal{E}_{\mu}) - \frac{1}{2}\,\varepsilon_{\alpha\mu\nu\beta}\mathcal{B}^{\beta} + \mathcal{M}_{\alpha\mu\nu},$$

where 4–vector $\mathcal{E}_{\nu}$ and 4–axial vector $\mathcal{B}^{\alpha}$ are defined by

$$\mathcal{E}_{\nu} = g^{\alpha\mu}\mathcal{T}_{\alpha\mu\nu} \quad , \quad \mathcal{B}^{\alpha} = \frac{1}{2}\,\varepsilon^{\alpha\beta\mu\nu}\mathcal{T}_{\beta\mu\nu}$$

and tensor $\mathcal{M}_{\alpha\mu\nu} = -\mathcal{M}_{\alpha\nu\mu}$ with 16 independent components obeys the constraints

$$g^{\alpha\mu}\mathcal{M}_{\alpha\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}\mathcal{M}_{\beta\mu\nu} = 0$$

Dirac Action and Minimal Torsion-Fermion Coupling

For metric spacetime

$$\tilde{g}_{\mu\nu;\beta} = \tilde{g}_{\mu\nu,\beta} - \tilde{\Gamma}^{\lambda}{}_{\mu\beta}\,\tilde{g}_{\lambda\nu} - \tilde{\Gamma}^{\lambda}{}_{\nu\beta}\,\tilde{g}_{\mu\lambda} = 0$$

the affine connection $\tilde{\Gamma}^{\alpha}{}_{\mu\nu}$ with torsion $\tilde{\mathcal{T}}^{\alpha}{}_{\mu\nu}$ is given by

$$\tilde{\Gamma}^{\alpha}{}_{\mu\nu} = \widetilde{\{^{\alpha}{}_{\mu\nu}\}} - \frac{1}{2}(\tilde{\mathcal{T}}^{\alpha}{}_{\mu\nu} - \tilde{\mathcal{T}}_{\mu}{}^{\alpha}{}_{\nu} - \tilde{\mathcal{T}}_{\nu}{}^{\alpha}{}_{\mu})$$

Action for Dirac fermions (neutrons) coupled to the chameleon field in the curved spacetime with torsion

$$S_{\psi} = \int d^4x\,\sqrt{-\tilde{g}}\,\Big(i\,\frac{1}{2}\,\bar{\psi}(x)\tilde{\gamma}^{\mu}(x)\overleftrightarrow{D}_{\mu}\,\psi(x) - m\bar{\psi}(x)\psi(x)\Big)$$

Covariant derivative of a fermion (neutron) field

$$D_{\mu} = \partial_{\mu} - \frac{i}{4}\,\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}\sigma^{\hat{\alpha}\hat{\beta}}$$

Spin connection $\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}$

$$\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x) = -\eta_{\hat{\alpha}\hat{\varphi}}\Big(\partial_{\mu}\tilde{e}^{\hat{\varphi}}_{\nu}(x) - \tilde{\Gamma}^{\alpha}{}_{\mu\nu}(x)\tilde{e}^{\hat{\varphi}}_{\alpha}(x)\Big)\tilde{e}^{\nu}_{\hat{\beta}}(x)$$

Dirac Equation and Dirac Hamilton Operator for Minimal Torsion–Fermion Coupling

Dirac equation for fermions with mass m
in curved spacetime with chameleon and torsion:
V. A. Kostelecky, PRD69, 105009 (2004)

$$\Big(i\,\tilde{e}^{\mu}_{\hat{\lambda}}(x)\gamma^{\hat{\lambda}}D_{\mu} - \frac{1}{2}\,i\,\tilde{\mathcal{T}}^{\alpha}{}_{\alpha\mu}(x)\tilde{e}^{\mu}_{\hat{\lambda}}(x)\gamma^{\hat{\lambda}}$$

$$-\frac{1}{2}\,i\,\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x)\tilde{e}^{\mu}_{\hat{\lambda}}(x)\Big(\eta^{\hat{\lambda}\hat{\beta}}\gamma^{\hat{\alpha}} + \frac{1}{4}\,i\,[\sigma^{\hat{\alpha}\hat{\beta}},\gamma^{\hat{\lambda}}]\Big) - m\Big)\,\psi(x) = 0$$

Dirac Hamilton operator: $\mathrm{H} = \mathrm{H}_0 + \delta\mathrm{H} = \gamma^{\hat{0}}m - i\gamma^{\hat{0}}\vec{\hat{\gamma}}\cdot\vec{\nabla} + \delta\mathrm{H}$

$$\delta\mathrm{H} =$$

$$= (\tilde{e}^{\hat{0}}_{0}(x) - 1)\gamma^{\hat{0}}m - i\,(\tilde{e}^{\hat{0}}_{0}(x) - 1)\gamma^{\hat{0}}\gamma^{\hat{j}}\delta^{j}_{\hat{j}}\frac{\partial}{\partial x^{j}} - i\tilde{e}^{\hat{0}}_{0}(x)(\tilde{e}^{j}_{\hat{j}}(x) - \delta^{j}_{\hat{j}})\gamma^{\hat{0}}\gamma^{\hat{j}}\frac{\partial}{\partial x^{j}}$$

$$+\frac{1}{2}\,i\,\tilde{\mathcal{T}}^{\alpha}{}_{\alpha\mu}(x)\tilde{e}^{\mu}_{\hat{\lambda}}(x)\gamma^{\hat{0}}\gamma^{\hat{\lambda}} + \frac{1}{2}\,i\,\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x)\tilde{e}^{\hat{0}}_{0}(x)\tilde{e}^{\mu}_{\hat{\lambda}}(x)\gamma^{\hat{0}}\Big(\eta^{\hat{\lambda}\hat{\beta}}\gamma^{\hat{\alpha}} + \frac{1}{4}\,i\,\{\sigma^{\hat{\alpha}\hat{\beta}},\gamma^{\hat{\lambda}}\}\Big)$$

Schwarzschild metric $\tilde{g}_{\mu\nu}(x)$
in the weak gravitational field of the Earth approximation
modified by the chameleon field (Jordan–frame metric):

$$ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = (1 + 2U_{\rm E})\, e^{2\beta\phi/M_{\rm Pl}} (dt)^2 - (1 - 2U_{\rm E})\, e^{2\beta\phi/M_{\rm Pl}} (d\vec{r}\,)^2$$

Schrödinger–Pauli equation:
(Foldy & Wouthuysen, 1950, Fischbach *et al.*, 1981)

$$i\frac{\partial\Psi(\vec{r},t)}{\partial t} = \Big(-\frac{1}{2m}\Delta + mU_{\rm E} + \Phi_{\rm gr-ch} + \Phi_{\rm tors}\Big)\Psi(\vec{r},t)$$

Effective low–energy potentials $\Phi_{\rm gr-ch}$ and $\Phi_{\rm tors}$:

$$\Phi_{\rm gr-ch} = m(U_+ - U_{\rm E}) - \frac{1}{2m}\vec{\nabla}(U_+ + 2U_-)\cdot\vec{\nabla} - \frac{1}{2m}(U_+ + 2U_-)\,\Delta - \frac{1}{8m}\Delta(U_+ + 2U_-) - \frac{i}{4m}\vec{\sigma}\cdot\Big(\vec{\nabla}(U_+ + 2U_-)\times\vec{\nabla}\Big)$$

$$\Phi_{\rm tors} = -\frac{1}{4}\vec{\sigma}\cdot\vec{\mathcal{B}} + \frac{i}{4m}\mathcal{B}_0\,\vec{\sigma}\cdot\vec{\nabla} + \frac{i}{8m}\vec{\sigma}\cdot\vec{\nabla}\mathcal{B}_0$$

Constraints on the Axial 4–Vector Torsion Field Value

Upper bound on $\mathcal{B}_0$ and $\vec{\mathcal{B}}$:
(V. A. Kostelecky *et al.*, PRL100, 111102 (2008))

$$|\mathcal{B}_0| < 8.7 \times 10^{-18}\,\mathrm{eV}$$

$$|\vec{\mathcal{B}}| < 3.0 \times 10^{-20}\,\mathrm{eV}$$

These upper bounds are
in agreement with the estimates obtained by
C. Lämmerzahl, PLA228, 223 (1997)
and Yu. N. Obukhov *et al.*, PRD90, 124068 (2014)

- The upper bounds on the torsion–fermion axial 4–vector couplings were obtained from “null–result” in measurements of Lorentz invariance violation and Zeeman transition frequencies between neighbouring atomic levels.

What will happen if we set $\mathcal{B}_0 = \vec{\mathcal{B}} = 0$?

Spin–Chameleon–Fermion Interaction

Spin–chameleon–fermion potential:
A. N. Ivanov & M. Pitschmann, PRD90, 045040 (2014)

$$\Phi_{\rm spin-ch} = \frac{1}{4m}\,\frac{\beta}{M_{\rm Pl}}\,i\,\vec{\sigma}\cdot\Big(\vec{\nabla}\phi \times \vec{\nabla}\Big)$$

Spin–chameleon–fermion potential
corresponds to the phenomenological Lagrangian of a torsion–fermion interaction:
V. A. Kostelecky *et al.*, PRL100, 111102 (2008)

$$\delta\mathcal{L}_{\mathcal{T}}(x) = \frac{i}{2}\,g_{\mathcal{T}}\,\mathcal{E}_{\mu}(x)\bar{\psi}(x)\,\sigma^{\mu\nu}\overleftrightarrow{\partial_{\nu}}\,\psi(x)$$

From comparison we obtain:
A. N. Ivanov & M. Pitschmann, PRD90, 045040 (2014)

$$g_{\mathcal{T}}\,\vec{\mathcal{E}} = \frac{1}{4m}\,\frac{\beta}{M_{\rm Pl}}\,\vec{\nabla}\phi$$

Covariant form
of the torsion tensor field in terms of the chameleon field

$$\mathcal{T}_{\alpha\mu\nu} = \frac{\beta}{M_{\rm Pl}}\Big(g_{\alpha\nu}\,\partial_\mu\phi - g_{\alpha\mu}\,\partial_\nu\phi\Big)$$

How can such a torsion tensor field be introduced ?:
S. Hojman *et al.*, PRD17, 3141 (1976)

- Abstract: A formalism is given which makes it possible for a modified form of local gauge invariance and minimal coupling to be compatible with torsion . . . The Lagrangian for interacting electromagnetic, gravitational, torsion, and complex scalar fields is presented . . .

Extension to Standard Electroweak Model:
A. N. Ivanov & M. Wellenzohn (submitted to PRD)

- The Lagrangian for interacting leptons, baryons, photons, W–, Z– and Higgs bosons with gravitational, chameleon and torsion fields is presented

Standard Electroweak Interactions in Gravitational Theory with Torsion as Gradient of Chameleon

Action for particles coupled to gravitational, chameleon and torsion fields:

$$S_{\rm g,ch,EW} = \int d^4x\,\sqrt{-g}\,\Big(\frac{1}{2}\,M^2_{\rm Pl}\,R + \frac{1}{2}\,\phi_{,\mu}\phi^{,\mu} - V(\phi)\Big)$$

$$-\frac{1}{4}\int d^4x\,,\sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}\mathcal{F}_{\alpha\beta}\mathcal{F}_{\mu\nu} + \int d^4x\,\sqrt{-\tilde{g}}\,\mathcal{L}_m[\tilde{g}_{\mu\nu}] =$$

$$= \int d^4x\,\sqrt{-g}\,\Big(\frac{1}{2}\,M^2_{\rm Pl}\,R + \frac{1}{2}\,\phi_{,\mu}\phi^{,\mu} - V(\phi)\Big)$$

$$-\frac{1}{4}\int d^4x\,\sqrt{-g}\,g^{\alpha\mu}g^{\beta\nu}\mathcal{F}_{\alpha\beta}\mathcal{F}_{\mu\nu} + \int d^4x\,\sqrt{-\tilde{g}}\,\mathcal{L}_m[\tilde{g}_{\mu\nu}]$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}f^2\;,\;\tilde{g}^{\mu\nu} = g^{\mu\nu}\,f^{-2}\;,\;\sqrt{-\tilde{g}} = f^4\,\sqrt{-g}$$

Photon–Torsion (Chameleon) Interactions

Lagrangian
of photon–torsion (chameleon) interactions

$$\frac{\mathcal{L}_{\rm em-tors}}{\sqrt{-g}} = -\frac{1}{2}\,\frac{\beta}{M_{\rm Pl}}\,F^{\mu\nu}(A_{\mu}\phi_{,\nu} - A_{\nu}\phi_{,\mu})$$

$$-\frac{1}{4}\,\frac{\beta^2}{M^2_{\rm Pl}}\,(A^{\mu}\phi^{,\nu} - A^{\nu}\phi^{,\mu})(A_{\mu}\phi_{,\nu} - A_{\nu}\phi_{,\mu})\Big) - \frac{1}{2\xi}\Big(A^{\mu}{}_{;\mu} - 4\,\frac{\beta}{M_{\rm Pl}}\,\phi_{,\nu}A^{\nu}\Big)^2,$$

where the last term fixes gauge and ξ is a gauge parameter

Torsion (Chameleon) Two–Photon Decay

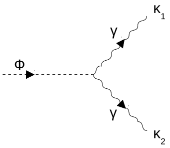

Figure: Feynman diagram for the $\phi \to \gamma + \gamma$ decay

Amplitude of torsion (chameleon) two–photon decay

$$M(\phi \to \gamma\,\gamma) = -\,2\,\frac{\beta}{M_{\rm Pl}}\,((\varepsilon^*_1\cdot\varepsilon^*_2)(k_1\cdot k_2) - (\varepsilon^*_1\cdot k_2)(\varepsilon^*_2\cdot k_1))$$

Partial width of torsion (chameleon) two–photon decay

$$\Gamma(\phi \to \gamma\,\gamma) = \frac{\beta^2}{M^2_{\rm Pl}}\,\frac{m^3_{\phi}}{8\pi}\quad,\quad m^2_{\phi} = \frac{\partial^2 V(\phi)}{\partial\phi^2}\Big|_{\phi=\phi_m}$$

m_{ϕ} **is a chameleon mass:**

J. Khoury & A. Weltman, PRD69, 044026 (2004)

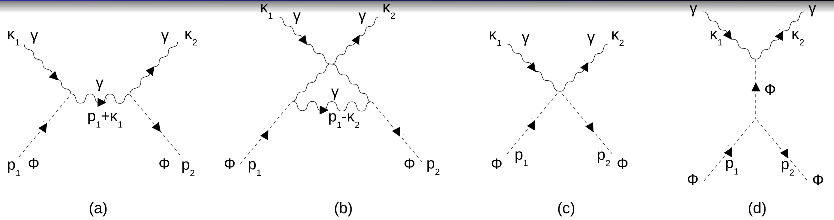

Figure: Feynman diagrams for torsion(chameleon)-photon scattering

Gauge invariance

$$\tilde{M}(\gamma\,\phi \to \phi\,\gamma)\Big|_{\varepsilon_1 \to k_1} = \sum_{j=a,b,c} M^{(j)}(\gamma\,\phi \to \phi\,\gamma)\Big|_{\varepsilon_1 \to k_1} = 0$$

$$\tilde{M}(\gamma\,\phi \to \phi\,\gamma)\Big|_{\varepsilon_2^* \to k_2} = \sum_{j=a,b,c} M^{(j)}(\gamma\,\phi \to \phi\,\gamma)\Big|_{\varepsilon_2^* \to k_2} = 0$$

Cross–section: $\sigma_{\gamma\phi\to\phi\gamma}(\omega) = \sigma_0\, f(\omega)$

$$\sigma_0 = \frac{1}{16\pi}\frac{\beta^4}{M_{\rm Pl}^4} < 2.5 \times 10^{-50}\,{\rm barn/MeV^2} \quad , \quad \beta < 5.8 \times 10^8$$

Chameleon–Photon Coupling Constant $g_{\rm eff} = \beta_\gamma / M_{\rm Pl}$

Since in our approach $\beta_\gamma = \beta$, one can estimate the constraints on astrophysical sources of the chameleon field using the following experimental data on β

$$g_{\rm eff} < \begin{cases} 7.8 \times 10^{-12}\,{\rm GeV}^{-1} & , \quad \beta < 1.9 \times 10^{7} \quad n = 1 \\ 2.0 \times 10^{-11}\,{\rm GeV}^{-1} & , \quad \beta < 5.8 \times 10^{7} \quad n = 2 \\ 8.2 \times 10^{-11}\,{\rm GeV}^{-1} & , \quad \beta < 2.0 \times 10^{8} \quad n = 3 \\ 2.0 \times 10^{-10}\,{\rm GeV}^{-1} & , \quad \beta < 4.8 \times 10^{8} \quad n = 4 \end{cases}$$

- H. Lemmel *et al.*, PLB743, 310 (2015) “Neutron interferometry constrains dark energy chameleon fields” $g_{\rm eff} < 2.4 \times 10^{-10}\,{\rm GeV}^{-1}\ ,\ \beta < 5.8 \times 10^{8}\ ,\ 1 \le n \le 10$
- T. Jenke *et al.*, PRL112, 115105 (2014) “Gravity Resonance Spectroscopy Constrains Dark Energy and Dark Matter Scenarios ”
- A.-C. Davis, C. A. O. Schelpe,& D. J. Shaw, PRD80, 064016 (2009) “Effect of a chameleon scalar field on the cosmic microwave background” and references therein

Neutron and Proton Charge Radii

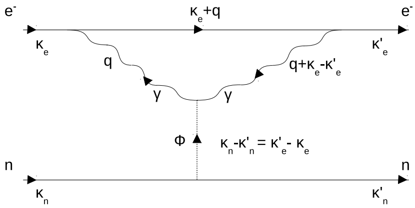

Figure: Feynman diagram for the contribution of the torsion(chameleon) to charge radii of the neutron and the proton

$$r_n^2 = -\frac{9}{4\pi^2}\frac{\beta^2}{m_\phi^2}\frac{m_e m_n}{M_{\rm Pl}^2}\ell n\left(\frac{M_{\rm Pl}}{m_e}\right) \quad , \quad (r^2)_{\rm exp} = -0.1161(22)\,{\rm fm}^2$$

$$\delta r_p^2 = -\frac{9}{4\pi^2}\frac{\beta^2}{m_\phi^2}\frac{m_\mu m_p}{M_{\rm Pl}^2}\ell n\left(\frac{M_{\rm Pl}}{m_\mu}\right) , \; \delta E_{2s\to 2p} = -5.180\,\delta r_p^2 = 0.311\,{\rm meV}$$

- K. A. Olive *et al.*, Chin. Phys. C **38**, 1 (2014)
- R. Pohl, Hyperfine Interactions **227**, 23 (2014)

Neutron and Proton Charge Radii

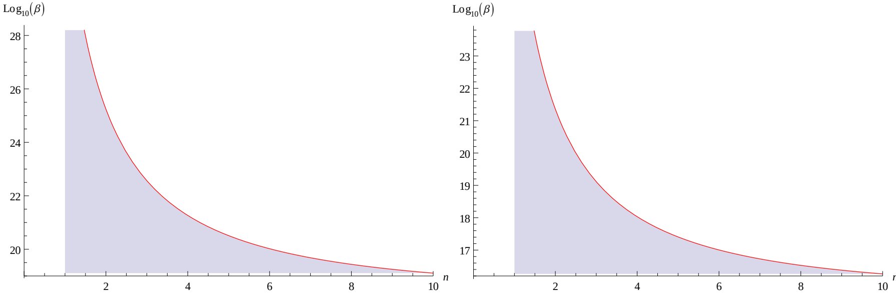

Figure: The lower bounds of the chameleon–matter coupling constant β from the experiment data on the electric charge radii of the neutron (left) and the proton (right), respectively. The shaded area is excluded: $\beta < 10^{19}$ (left) and $\beta < 10^{17}$ (right)

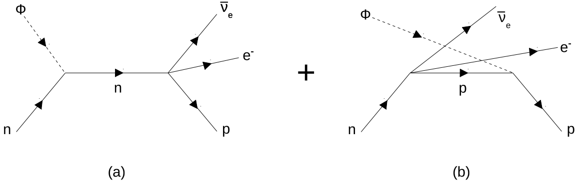

Figure: Feynman diagrams for the reaction $\phi + n \to p + e^- + \bar{\nu}_e$

Decay rate of $\phi + n \to p + e^- + \bar{\nu}_e$

$$\lambda_{\phi n} = \int_0^{\infty} dE_{\phi}\, \Phi_{\rm ch}(E_{\phi})\, \sigma_{\phi\, n \to p\, e^-\, \bar{\nu}_e}(E_{\phi})$$

$$\sigma_{\phi\, n \to p\, e^-\, \bar{\nu}_e}(E_{\phi}) = (1 + 3\lambda^2)\, \frac{G_F^2 |V_{ud}|^2}{2\pi^3}\, \beta^2\, \frac{(m_n + m_p)^2}{M_{\rm Pl}^2}\, \frac{(E_0 + E_{\phi})^5}{120 E_{\phi}^3}$$

Half–life: $T_{1/2} = \ell n\, 2/\lambda_{\phi n} > 3 \times 10^{33}\,{\rm yr}$

Proton half–life $T_{1/2} > 5.9 \times 10^{33}$ **(Super–Kamiokande)**

- $\Phi_{\rm ch}(E_{\phi})$: Ph. Brax *et al.*, PRD85, 043014 (2012)

Summary

- Gauge invariance of the chameleon–photon interactions might be interpreted as unrenormalisability of the chameleon–matter coupling β by any interactions. This may imply that a screening, caused by the Vainstein mechanism, should not be valid in such an approach.
- The relation $\beta_\gamma = \beta$ gives stronger constraints on astrophysical sources of chameleons, coming from direct photon–chameleon transitions in magnetic fields.
- For the chameleon – induced β^-–decay one may propose the reaction:

$$\phi + {}^{112}_{48}\mathrm{Cd} \to {}^{112}_{49}\mathrm{In} + e^- + \bar{\nu}_e \quad , \quad E_\phi > 0.7\,\mathrm{MeV},$$

where ${}^{112}_{48}\mathrm{Cd}$ is stable and ${}^{112}_{49}\mathrm{In}$ is unstable under EC (56%) and β^-(44%) decays with $T_{1/2} = 14.97(10)\,\mathrm{m}$. In principle the chameleon – induced electron spectrum can be distinguished from the ${}^{112}_{49}\mathrm{In}$ – decay electron spectrum. For comparison the half–life of the proton for specific modes is $T_{1/2} > 5.9 \times 10^{33}\,\mathrm{yr}$ (Super–Kamiokande).

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Thank You for Attention