# String point of view on Gravity and Cosmology

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# Instead of outline

- **GR is incomplete as long as you do not like singularities**
  **The Raychaudhuri equation is one of the fastest way to show this.**
- **IR tests of GR are extremely solid**
- **We therefore need a UV completion of GR**
- **Natural expectation is that only one completion exists**
- **Strings?**

# If strings, at least up to some extent then ...

- **Strings contain gravity and reproduce GR in IR**
- **Strings are non-local objects by constructions and the final action for fields is also non-local**
- **Gravitational field should also exhibits non-local (self-)interactions**
- **The underlying construction, String Field Theory, leads to a finite unitary QFT**

# History

- **Classical gravity and GR; also Ostrogradski 1850**
- **Strings, the second half of the previous century**
- **Stelle, 1977,1978, renormalizable $R^2$ type gravity (containing ghosts)**
- **Starobinsky, 1980-s, $R^2$ inflation**
- **Witten, 1986, String Field Theory which by construction contains non-local vertexes**
- **Aref'eva, AK, 2004, models of non-local stringy inspired scalar fields coupled to gravity**
- **Biswas, Mazumdar, Siegel, 2005, first explicit non-local gravity modification**

## Exorcising ghosts

- **In some cases ghosts do not appear, like in $f(R)$ gravity for special parameters.**

  **This is because the system is constrained.**

- **There are special field theories which have higher derivatives in the Lagrangian but no more than 2 derivatives act on a field in the equations of motion. For example KGB models or galileons.**

  **The fine-tuning is required.**

- **Propagators can be modified and be non-local without changing the perturbative physics**

$$\Box - m^2 \Rightarrow \mathcal{G}(\Box) = (\Box - m^2)e^{\gamma(\Box)}$$

  $\gamma(\Box)$ **must be an entire function. This guarantees that no extra degress of freedom appear.**

# $\mathcal{G}(\Box)$ physics, SFT motivation

**Low level example action from SFT:**

$$L \sim \frac{1}{2}\phi(\Box - m^2)\phi + \frac{\lambda}{4}\left(e^{-\beta\Box}\phi\right)^4 \Rightarrow \frac{1}{2}\varphi(\Box - m^2)e^{2\beta\Box}\varphi + \frac{\lambda}{4}\varphi^4$$

**The Lagrangian to understand is**

$$S = \int d^D x \left(\frac{1}{2}\varphi\mathcal{G}(\Box)\varphi - \lambda v(\varphi) + \dots\right)$$

$\mathcal{G}(\Box) = \sum_{n\geq 0} g_n \Box^n$, **i.e. it is an analytic function.**

**Canonical physics has** $\mathcal{G}(\Box) = \Box - m^2$, **i.e.** $L = \frac{1}{2}\varphi\Box\varphi - \frac{m^2}{2}\varphi^2$

**Ghosty example** $\mathcal{G}(\Box) = \Box - m^2 + g_2\Box^2$

# SFT motivated non-local gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}R\mathcal{F}(\Box)R - \Lambda + \ldots\right), \;\; M_P^2 = \frac{1}{8\pi G_N}$$

# What is special about such a gravity

- **Ghost-free:**

$$\mathcal{F}(\Box) = \frac{e^{\gamma(\Box)} - 1}{\Box}, \;\; \gamma(\Box) \textbf{ is an entire function}$$

- **Asymptotically-free:**

$$\gamma(\Box) = -\frac{\Box}{M}, \;\; \Phi \sim -\frac{1}{r}\mathrm{erf}\left(\frac{Mr}{2}\right) \to \begin{cases} \text{const as } r \to 0 \\ \frac{1}{r} \text{ as } r \to \infty \end{cases}$$

- **Singularity-free following the Raychaudhuri equation analysis**
  **Conroy, AK, Mazumdar, PRD, 2014**

# Solutions

**Claim: any solution of the local $R^2$ gravity is a solution here upon 3 algebraic conditions on the action parameters**

**Accounting** $\Box R = r_1 R + r_2$

$$\mathcal{F}^{(1)}(r_1) = 0,\ \ \frac{r_2}{r_1} = -\frac{M_P^2 - 6\lambda\mathcal{F}(r_1)r_1}{2\lambda[\mathcal{F}(r_1) - \mathcal{F}(0)]},\ \ \Lambda = -\frac{r_2 M_P^2}{4r_1},$$

## Explicit non-singular bouncing solutions

$$a = a_0 \cosh(\sigma t)$$

**Biswas, Muzumdar, Siegel, JCAP, 2006; AK, CQG, 2013**

$$a = a_0 \sqrt{\cosh(\sigma t)} \textbf{ and also } a = a_0 e^{-\frac{\sigma}{2} t^2}$$

**AK, CQG, 2013**

## Starobinsky solution

$$a = a_0 \sqrt{t_* - t} e^{\sigma (t_* - t)^2}$$

**Craps, De Jonckheere, AK, JCAP, 2014**

**($a$ – is the scale factor of the FRW metric)**

# A wishful solution

**Apart from the local $R^2$ gravity one can arrange such a parameter range that both bounce type and inflation type solutions exist**

**We are however lack of an explicit construction of such a solution yet**

**AK, work in progress**

# Scalar reformulation of the non-local gravity

**The previous action is equivalent to the following one**

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2R}{2}(1+\psi)-\frac{M_P^4}{8\lambda}\psi\frac{1}{\mathcal{F}(\Box)}\psi+\dots\right)$$

**The conformal transform** $(1+\psi)^2\,g_{\mu\nu}=\bar{g}_{\mu\nu}$ **allows us to decouple the gravity and the scalar field**

$$S=\int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R}-\frac{M_P^2}{4}\frac{3}{(1+\psi)^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi-\frac{M_P^4}{8\lambda(1+\psi)^2}\psi\mathcal{G}(\mathcal{P})\psi\right)$$

**Here**

$$\mathcal{G}(\mathcal{P})=\frac{1}{\mathcal{F}(\mathcal{P})}\ \textbf{and}\ \mathcal{P}=(1+\psi)\,\bar{\Box}-\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

**The ghost-free condition on** $\psi$ **implies** $\mathcal{G}(\mathcal{P})=\sum\limits_{n\geq 0}g_n\mathcal{P}^n$**, i.e. it is an analytic function.**

# Limits

**The weak field limit**

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \left(\frac{3}{2} - \frac{g_1 M_P^2}{4\lambda}\right)\psi\bar{\Box}\psi - \frac{g_0 M_P^2}{4\lambda}\psi^2 - \sum_{n>1}\frac{g_n M_P^2}{4\lambda}\left(\bar{\Box}\psi + (\partial\psi)^2\right)\left(\bar{\Box} - \partial^\rho\psi\partial_\rho\right)^{n-1}\psi\right)$$

**Here we recognize the KGB models and structures similar to Galileon field theories.**

**The limit of large field**

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} - \frac{3}{2}\frac{(\partial\psi)^2}{\psi^2} - \frac{M_P^2}{4\lambda}\frac{1}{\psi}\mathcal{G}\left(\psi\bar{\Box} - \partial^\rho\psi\partial_\rho\right)\psi\right)$$

**As a special limit we restore the p-adic string theory**

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \frac{\kappa}{2}\psi e^{-\beta\bar{\Box}}\psi\right)$$

**Proposal:** **the non-local theories can be considered as *generating functionals* for other models on the market.**

# Conclusions

- **Non-local generalization of Einstein's gravity is presented**

- **There are exact analytic solutions including bounce and the Starobinsky inflation in this framework**

- **Scalar reformulation with a possible connection to other interesting models is discussed**

**Thank you for listening!**