Gravitational self-force correction to the innermost stable circular orbit of a Kerr black hole

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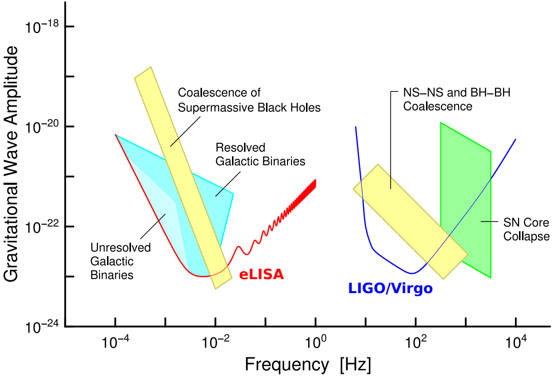
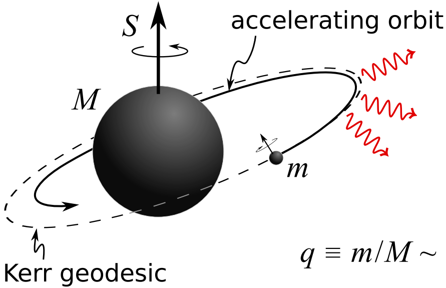
PRL **113** (2014) 161101, arXiv:1404.6133 [gr-qc]

Promising sources of gravitational waves



- Binary neutron stars ($2\times\sim 1.4M_\odot$)
- Stellar-mass black hole binaries ($2\times\sim 10M_\odot$)
- Supermassive black hole binaries ($2\times\sim 10^6M_\odot$)
- Extreme mass ratio inspirals ($\sim 10M_\odot + \sim 10^6M_\odot$)

Extreme mass ratio inspirals (EMRIs)

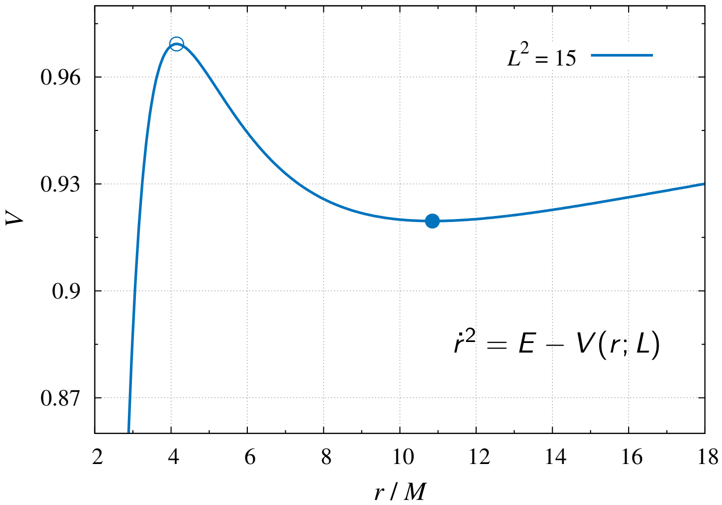


$$q \equiv m/M \sim 10^{-5}$$

Gravitational self-force (GSF)

- Dissipative component $\longleftrightarrow$ gravitational waves
- Conservative component $\longrightarrow$ some secular effects

Innermost stable circular orbit (ISCO)



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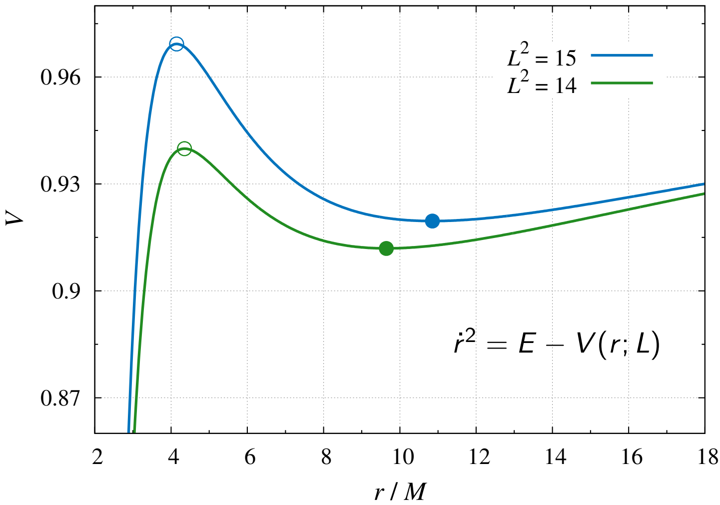
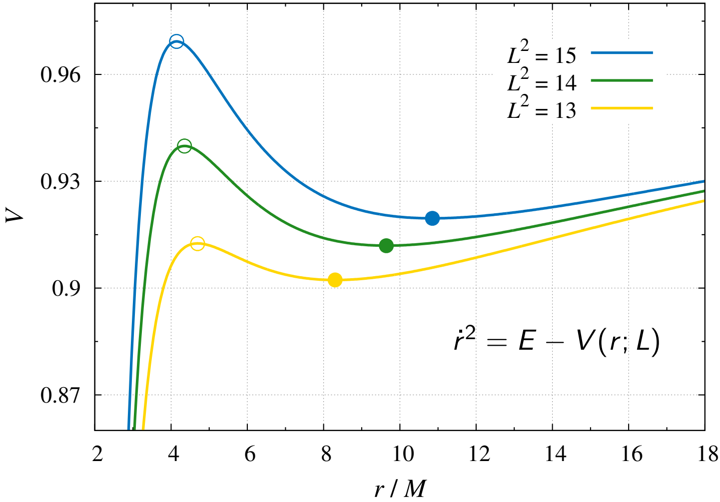


Innermost stable circular orbit (ISCO)



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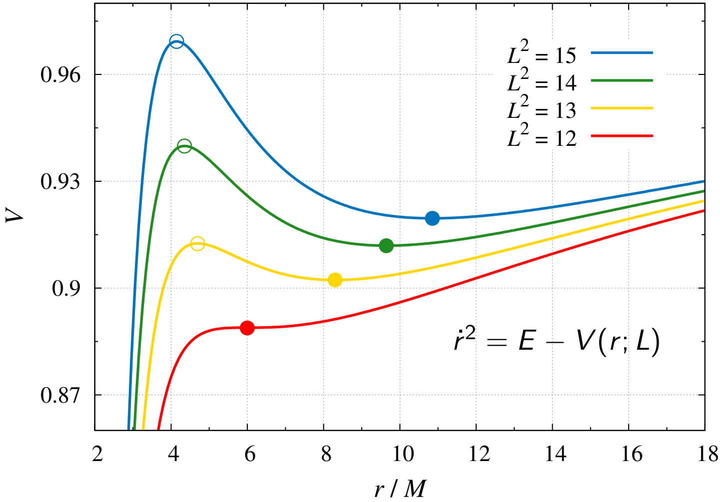
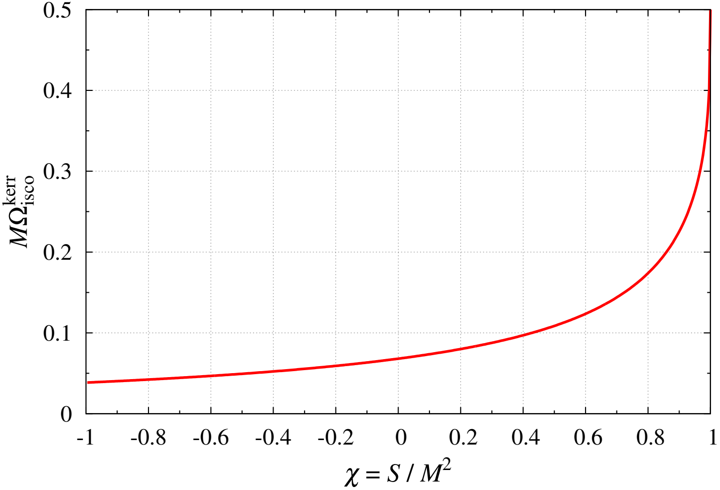


Kerr ISCO frequency *vs* black hole spin

[Bardeen *et al.*, ApJ 1972]



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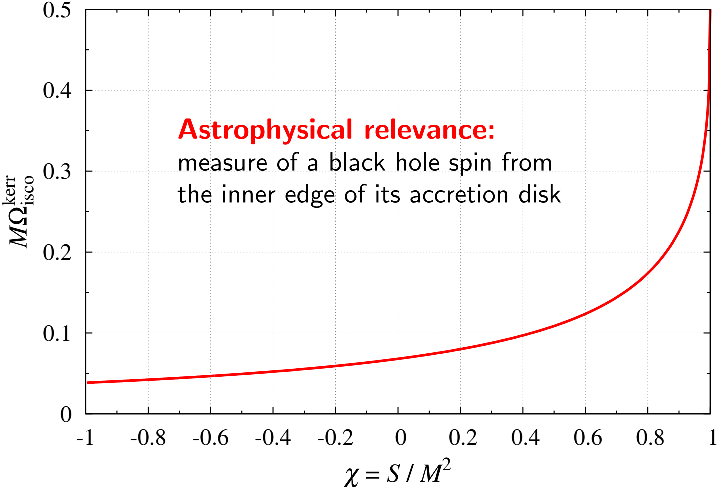
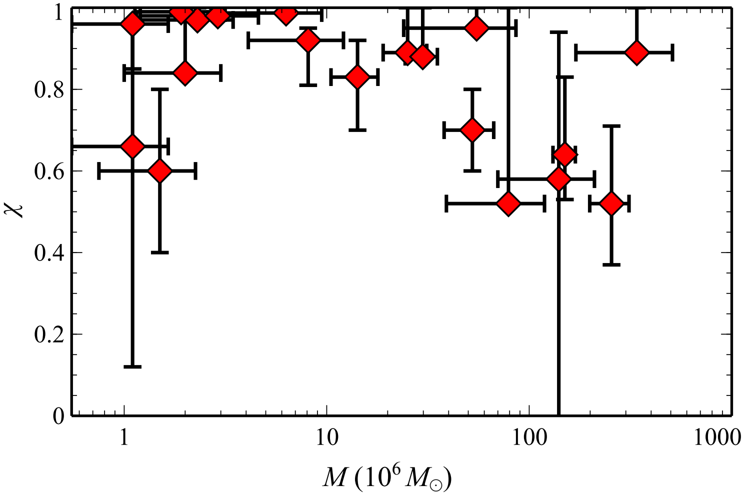


Spins of supermassive black holes

[Reynolds, CQG 2013]



Geodesic motion of a test mass in Kerr

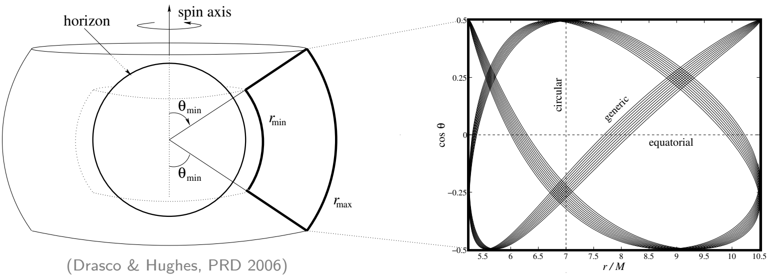
Hamiltonian formulation

Hamiltonian of a *test mass* m in the Kerr geometry g_{ab}:

$$H(x,p) = \frac{1}{2m}\, g^{ab}(x)\, p_a p_b$$

Constants of the motion

- Rest mass m
- Energy $E = -t^a p_a$
- Ang. momen. $L = \phi^a p_a$
- Carter constant $Q = K^{ab} p_a p_b$



(Drasco & Hughes, PRD 2006)

Hamiltonian first law of mechanics

[Le Tiec, CQG 2014]

- The Hamilton-Jacobi equation is completely separable
- Perform a canonical transformation $(x^a, p_a) \to (q^\alpha, J_\alpha)$ to action-angle variables:

$$\frac{\mathrm{d}q^\alpha}{\mathrm{d}t} = \frac{\partial H}{\partial J_\alpha} \equiv \Omega_\alpha \,, \quad \frac{\mathrm{d}J_\alpha}{\mathrm{d}t} = -\frac{\partial H}{\partial q^\alpha} = 0$$

- Varying $H(J_\alpha)$ and using Hamilton's equations yields a *first law of mechanics*:

$$\delta E = \Omega_\varphi \, \delta L + \Omega_r \, \delta J_r + \Omega_\theta \, \delta J_\theta + \langle z \rangle \, \delta m$$

Inclusion of the conservative self-force

[Isoyama *et al.*, in preparation]

- Geodesic motion of a *self-gravitating mass m* in perturbed geometry $g_{ab} + h_{ab}^{\text{reg}}$ derives from perturbed Hamiltonian

$$\mathcal{H}[x, p; \gamma] = H(x, p) + H_{\text{int}}[x, p; \gamma]$$

- The *first law of mechanics* can be extended up to $\mathcal{O}(q)$:

$$\delta\mathcal{E} = \Omega_\varphi \, \delta\mathcal{L} + \Omega_r \, \delta\mathcal{J}_r + \Omega_\theta \, \delta\mathcal{J}_\theta + \langle z \rangle \, \delta m$$

- The actions $\mathcal{J}_\alpha$, frequencies Ω_α, and average redshift $\langle z \rangle$ include conservative self-force corrections from H_{int}

Minimum energy circular orbit (MECO)

- For *circular equatorial* orbits, the first law reduces to

$$\delta\mathcal{E} = \Omega\,\delta\mathcal{L} + z\,\delta m$$

- The MECO is the circular orbit whose frequency obeys

$$\mathcal{E}'(\Omega_{\text{meco}}) = 0 \quad \Longleftrightarrow \quad \tilde{z}''(\Omega_{\text{meco}}) = 0$$

- Since $\Omega_{\text{meco}} = \Omega_{\text{isco}}$ for Hamiltonian systems such as ours, the ISCO frequency obeys

$$\tilde{z}''(\Omega_{\text{isco}}) = 0\,, \quad \text{where} \quad \tilde{z} \equiv z_{\text{kerr}} + \frac{q}{2}\,z_{\text{gsf}}$$

Frequency shift of the Kerr ISCO

[Isoyama *et al.*, PRL 2014]

- The orbital frequency of the Kerr ISCO is shifted under the effect of the conservative self-force:

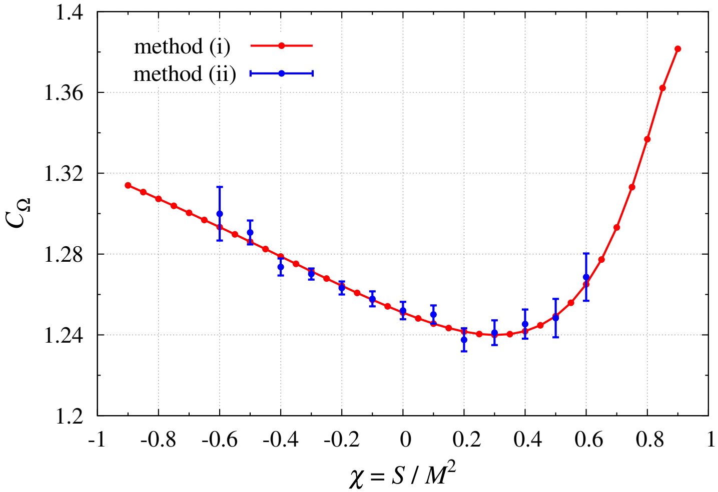
$$\Omega_{\text{isco}} = \underbrace{\Omega_{\text{isco}}^{\text{kerr}}(\chi)}_{\substack{\text{test mass}\\ \text{result}}} \left\{ 1 + \underbrace{q\, C_\Omega(\chi)}_{\substack{\text{self-force}\\ \text{correction}}} + \mathcal{O}(q^2) \right\}$$

- From the condition $\tilde{z}''(\Omega_{\text{isco}}) = 0$, the frequency shift reads

$$C_\Omega = \frac{1}{2} \frac{z''_{\text{gsf}}(\Omega_{\text{isco}}^{\text{kerr}})}{E''(\Omega_{\text{isco}}^{\text{kerr}})}$$

ISCO frequency shift *vs* black hole spin

[Isoyama *et al.*, PRL 2014]



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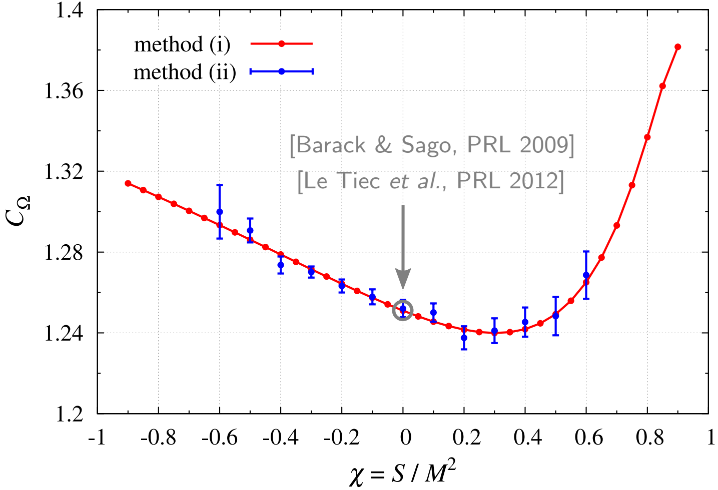

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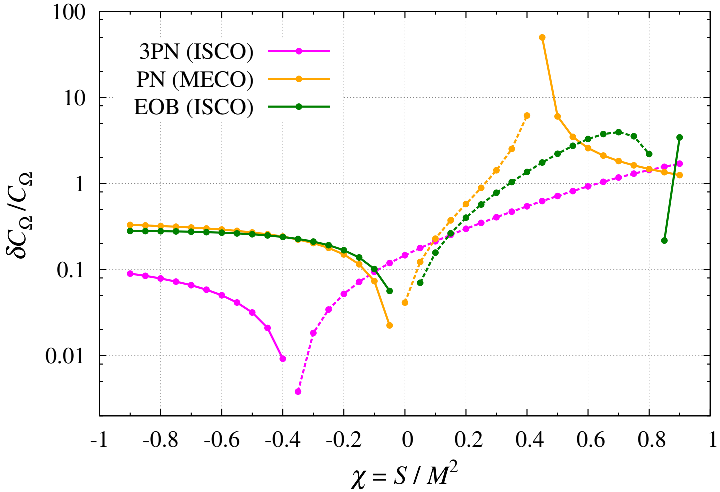

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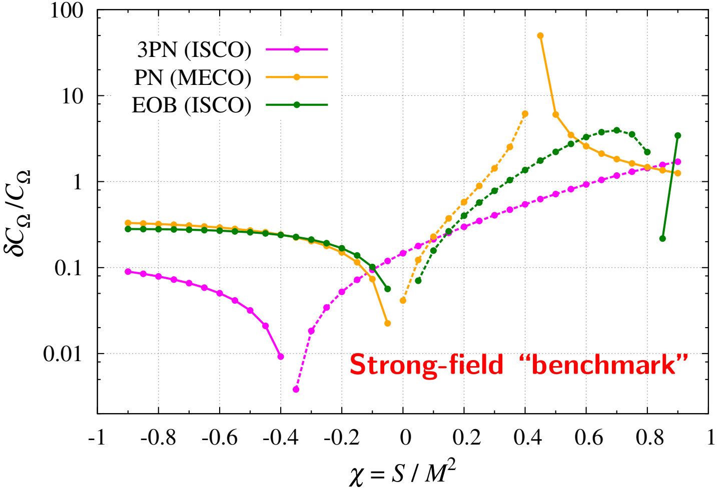

Summary and prospects

- EMRIs are prime targets for the planned eLISA observatory
- Highly accurate template waveforms are a prerequisite for doing science with GW observations
- We computed the shift in the Kerr ISCO frequency induced by the conservative piece of the GSF
- This result provides an accurate strong-field “benchmark” for comparison with other methods (PN, EOB)
- Future work: beyond circular equatorial orbits