Fundamental limitations to tests of the universality of free fall by dropping atoms

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Why have torsion balances defeated Galileo-like mass dropping tests by almost 4 orders of magnitude?

Effect of initial condition errors in drop tests

$$\Delta g(t) = -2\frac{GM_\oplus}{R_\oplus^3} z(t) = \gamma \left(\frac{1}{2} g t^2 - \Delta v_\circ t - \Delta z_\circ \right)$$

$$\gamma = g \frac{2}{R_\oplus} = 3.14 \cdot 10^{-7} \; g/\mathrm{m}$$

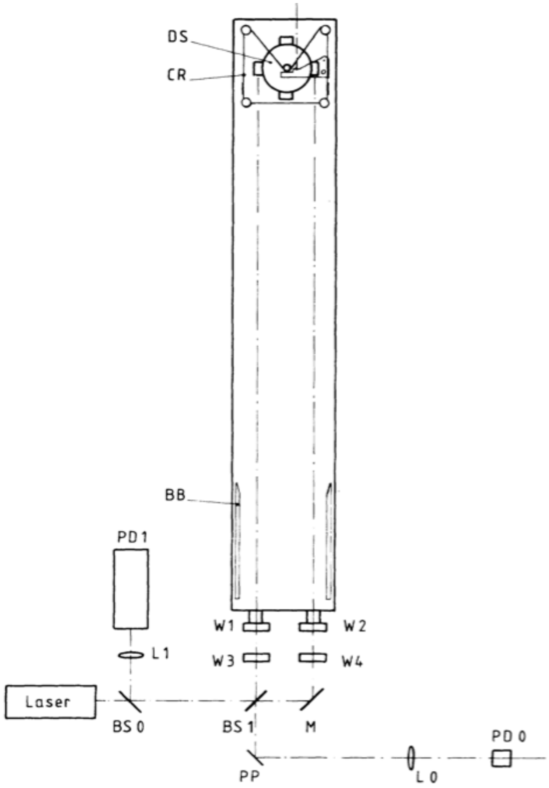
Gravity is not uniform $\Rightarrow$ gravity gradients (tides) inevitable

$\Downarrow$

any difference in position and velocity of the test masses relative to each other (in the direction to the center of mass of Earth) at initial time will mimic a violation

GAL: the most clever mass dropping test ever performed

Proposed in Pisa in the group of E. Polacco
Performed at CERN; ***Carusotto et al., PRL 1992***



- A vertical disk made of two halves of different material is dropped from a height. If UFF/WEP holds it should not rotate (Differential/null experiment: zero differential effect, zero signal).
- If the two halves are attracted differently by Earth (UFF/WEP violation) the disk rotates: the effects is measured very precisely by means of a modified Michelson interferometer in which the two arms terminate on corner-cube reflectors mounted on the rim of the disk

...but the torsion balance is far from being defeated

70 drops performed with disk made of two halves of same composition (Al) set the limiting sensitivity with which this apparatus can test UFF/WEP:

$$\frac{\Delta g}{g} = (3.2 \pm 9.5) \cdot 10^{-10}$$

Al-Cu disk, 63+65 drops (disk reversed):

$$\left(\frac{\Delta g}{g}\right)_{Al-Cu} = (2.9 \pm 7.2) \cdot 10^{-10}$$

Carusotto et al, PRL 1992
Read-out noise is not an issue
Spurious disk rotations at release are the limiting factor

At the time of publication this was the best result in the field of Earth (remember that Earth was the relevant source body for ***Fischbach et al., PRL 1986*** *claim on 5th force), but slowly rotating torsion balances soon proved no violation from Earth to* 10^{-12} ***(Su et al., PRD 1994)*** *and later to* 10^{-13} ***(Shlamminger et al., PRL 2008)***

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Dropping free masses inside a spacecraft

Initial condition (release) errors cannot be separated from violation and would not even allow to match the best torsion balance tests.

Free test masses inside a spacecraft cannot in any way compete with weakly suspended/coupled tests masses (whatever the type of suspension/coupling: mechanical, electrostatic, by superconducting coils..)

Blaser, CQG 2001

Free masses in space: UFF/WEP tests by LLR & SLR

Nobili et al., GRG 2008

- LLR, Earth/Moon in the field of the Sun:

$$\eta = 3\frac{\Delta a_{meas}}{d_{\oplus\odot}} \simeq 3\frac{10^{-2}\,\mathrm{m}}{\mathbf{1.5\cdot 10^{11}\,m}} \simeq 2\cdot 10^{-13} \rightarrow 2\cdot 10^{-14} \text{ if LLR to } 1\,\mathrm{mm}$$

Not related to being little affected by non-gravitational forces, but only to the large distance from source body (Sun) (*Blaser, CQG 2001 was wrong on that..*)

- Limitations much more stringent for SLR, Earth/LAGEOS (testing Yukawa-like deviation does not help):

$$\eta = 3\frac{\Delta a_{meas}}{d_{\oplus LAGEOS}} \simeq \frac{10^{-2}\,\mathrm{m}}{\mathbf{1.23\cdot 10^{7}\,m}} \simeq 2.4\cdot 10^{-9}$$

Absolute distance measurements not needed for UFF/WEP tests (only relative displacements). LLR/SLR tests cannot compete with in-situ measurements of relative displacements inside a spacecraft (which can easily sense 1 pm*)*

What's magic about the torsion balance?

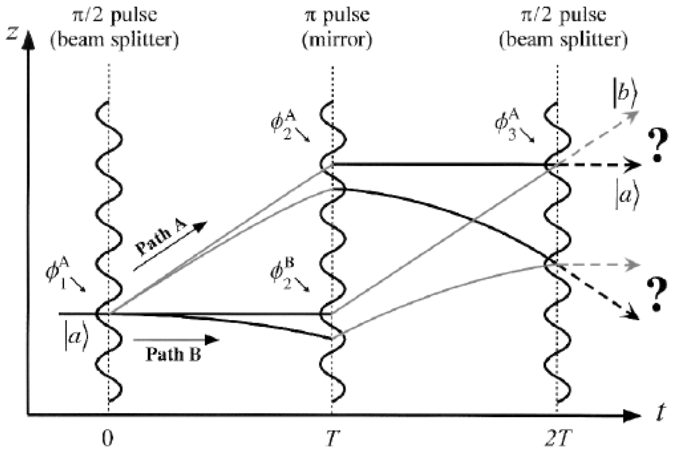
What's magic about the torsion balance

- Differential/null experiment (no differential effect $\Rightarrow$ no signal)
- Motion of test masses around position of relative equilibrium $\Rightarrow$ **it does not depend on initial conditions at all**
- Signal up-conversion to higher frequency provided by rotation
- If fiber is thin, it has very low natural frequency (Eöt-Wash balance: 798 s period). TMs very weakly coupled $\Rightarrow$ highly sensitive to differential effects
- **On ground (not in space!)** suspension fiber aligns itself with local gravity (towards the center of mass of the Earth) $\Rightarrow$ common mode forces are rejected (almost) perfectly by physics (no torque from common mode forces, no deflection of the wire, no residual differential signal.. (almost))

Only GReAT mass dropping proposed test shares some of the properties of the torsion balance, but has not been performed yet (Iafolla et al., RSI 1998)

Initial condition errors in measurements of g by dropping atoms in a light pulse atom interferometer (I)

$$\Delta g_{AI} = \gamma \left(\left(\frac{1}{2} + \mathbf{\frac{1}{12}} \right) gT^2 - \Delta v_\circ T - \Delta z_\circ \right) \qquad \gamma = g\frac{2}{R_\oplus} = 3.14 \cdot 10^{-7}\ g/\mathrm{m}$$



- The excess error ***(quadratic in time!!!)***

$$\Delta g_{ExcessError} = \mathbf{\frac{1}{12}}\gamma \mathbf{g T^2}$$

is due to the approximation made by the light pulse atom interferometer in measuring the acceleration of the falling atoms (measured as a second difference of their positions at the times 0, T and $2T$ when, during their ballistic flight, they are subjected to light pulses – instead of second time derivative)

Peters, PhD thesis, Stanford U., 1998

Wolf & Turrenc, PLA 1999

Initial condition errors in measurements of g by dropping atoms in a light pulse atom interferometer (II)

$$\Delta g_{ExcessError} = \frac{1}{12}\gamma g T^2 \simeq 6.5 \cdot 10^{-9}\, g \qquad (T = 160\,\mathrm{ms})$$

$$\Delta g_{AI} = \gamma\left(\left(\frac{1}{2} + \frac{1}{12}\right) g T^2 - \Delta v_\circ T - \Delta z_\circ\right) \simeq 31 \cdot 10^{-9}\, g$$

Checks of systematic errors with many runs $\Rightarrow$

$$\left(\frac{\Delta g}{g}\right)_{AI} \simeq 3 \cdot 10^{-9}$$

Peters, Chung & Chu, Nature 1999

With free falling corner cube reflector & laser interferometry:

$\Delta g/g \simeq 1.1 \cdot 10^{-9}$ *by* ***Niebauer et al., Metrologia 1995***

Initial condition errors in UFF/WEP tests by dropping atoms in a dual isotope/atom interferometer

The free fall acceleration is measured simultaneously and independently for each of two overlapped clouds $\Rightarrow \Delta g = \eta g(h)$ *(UFF/WEP test to* $\eta = \frac{\Delta g}{g(h)}$*)*

Gravity gradient $\gamma \simeq 3 \cdot 10^{-7}\, g(h)/\mathrm{m}$ & initial condition errors $\Rightarrow$

- For a single atom: $\Delta g_{_{ICE-singleatom}} = \gamma\,(\Delta z_\circ + \Delta v_\circ t)$
- If N atoms are released together in a single cloud: $\Delta g_{_{ICE-singlecloud}} = \gamma \left(\frac{\Delta z_\circ}{\sqrt{N}} + \frac{\Delta v_\circ}{\sqrt{N}} t \right)$ x $\sqrt{2}$ gives the relative random error between two clouds
- If the two clouds have non zero position and velocity offsets at initial time: $\Delta g_{_{ICE-offsets}} = \gamma \Big(\Delta z_{\circ-rel} + \Delta v_{\circ-rel} t \Big)$ **systematic**

Experimental results

- ^{85}Rb,^{87}Rb $\quad \eta \simeq 10^{-7}$ ***Bonnin et al., PRA 2013***

  They report a rejection of common mode effects by 550 and estimate: $\Delta z_{\circ-rel} = \pm 2\,\text{mm}$, $\Delta v_{\circ-rel} \leq 6\,\text{mm/s}$

- ^{87}Rb,^{39}K $\quad \eta \simeq 10^{-7}$ ***Schlippert et al., PRL 2014***

  With $\gamma \simeq 3 \cdot 10^{-7}\, g/\text{m}$ initial condition errors are not an issue at this level of UFF/WEP test

  (140 times worse than best drop test; 6 orders of magnitude worse then best torsion balance test)

- In a 10-m tall vacuum chamber at Stanford, by imaging $N = 4{\cdot}10^6$ ^{87}Rb atoms, 200 μm initial radius, 2 mm/s initial velocity spread (50 nK) & $t = 1.15$ s free fall time $\Rightarrow$

  $\Delta g_{ICE-singleatom} \simeq 8 \cdot 10^{-10}\, g$

  - then reduced to $\Delta g \simeq 6.7 \cdot 10^{-12}\, g$ (comparable to single shot noise)

  - i.e. statistical reduction by $\sqrt{N}$=2000 not fully measured (seismic noise dominant)

  Dickerson et al., PRL 2013

Proposed tests of UFF/WEP to 10^{-15}–10^{-17} ***(Dimopoulos et al., PRL 2007)*** *far away*

Limitations from Heisenberg's Principle (HP)

Masses (especially if very small...) cannot be confined at will in position and velocity space

- Single atom: $\Delta p_\circ \cdot \Delta z_\circ \geq \hbar/2$

$$(\Delta v_\circ \cdot \Delta z_\circ)_{HP-atom} \geq \frac{\hbar}{2}\frac{1}{m_{Rb}} \geq \frac{\hbar}{2}\frac{1}{85.468 \cdot 10^{-3}} \cdot N_A \ \mathrm{m^2/s} \geq 3.7 \cdot 10^{-10} \ \mathrm{m^2/s}$$

- Cloud of $N = 10^6$ atoms released together: $(\sqrt{N}\Delta z_\circ)(m_{Rb}\sqrt{N}\Delta v_\circ) \geq \hbar/2$

$$(\Delta v_\circ \cdot \Delta z_\circ)_{HP-freemass} \geq \frac{\hbar}{2}\frac{1}{85.468 \cdot 10^{-3}} \cdot \frac{N_A}{N} \ \mathrm{m^2/s} \geq 3.7 \cdot 10^{-16} \ \mathrm{m^2/s}$$

Limitations to STE-QUEST proposed space test of UFF/WEP to $2 \cdot 10^{-15}$ (I)

Aguilera et al., CQG 2014

Dual isotope ^{85}Rb,^{87}Rb interferometer, $h = 700\,\mathrm{km}$ altitude, two clouds of $N = 10^6$ atoms each, $300\,\mu$m initial radius, $82\,\mu$m/s initial velocity spread & $t = 5\,$s free fall time

- Each atom & each cloud are HP safe by a factor 66
- Initial condition relative random error **per drop/shot**:
  $\Delta g_{ICE-random} = \gamma \left(\frac{\sqrt{2}\Delta z_\circ}{\sqrt{N}} + \frac{\sqrt{2}\Delta v_\circ}{\sqrt{N}} t \right) \simeq 2.8 \cdot 10^{-13}\, g(h)$ (comparable to single shot noise)

  $n = 1.48 \cdot 10^5$ drops/shots needed to reduce below target (assuming they are uncorrelated, that experimental conditions are unchanged, that there is no relative bias..)

  $\Rightarrow$ 3 years of mission time required for 1 single measurement

  - Microscope: 1 measurement to 10^{-15} in 1.4 d
  - GG 1 measurement to 10^{-17} in a few hours

  Both at room temperature, thermal noise limited, different signal up-conversion frequencies
  (Nobili et al., PRD 2014; Pegna et al., PRL 2011)

Limitations to STE-QUEST proposed space test of UFF/WEP to $2 \cdot 10^{-15}$ (II)

- **Required** initial offset errors between centres of mass position & velocity of the two clouds ***(give systematic effect and are never measured concurrently with the drop)***:

  $\Delta z_{\circ-rel} = 1.1\,\mathrm{nm} \quad \Delta v_{\circ-rel} = 0.31\,\mathrm{nm/s}$

  The centre of mass of a free cloud of $N = 10^6$ Rb atoms cannot be confined in position and velocity with these errors because it would violate HP by 1000*.*
  Confinement must be provided the hard way (at release) and is not measured...
  Values estimated in experiment ***(Bonnin et al., PRA 2013)*** are 2 to 20 million times larger than required.
  Check of systematics (to the target sensitivity) ruled out by the very long time required for each measurement.

  (Note: null check test with same isotopes/atoms not straightforward because identical atoms are hard to distinguish..)