

The $m-z$ relation
for type Ia supernovae,
locally inhomogeneous
cosmological models,
and the nature of dark matter

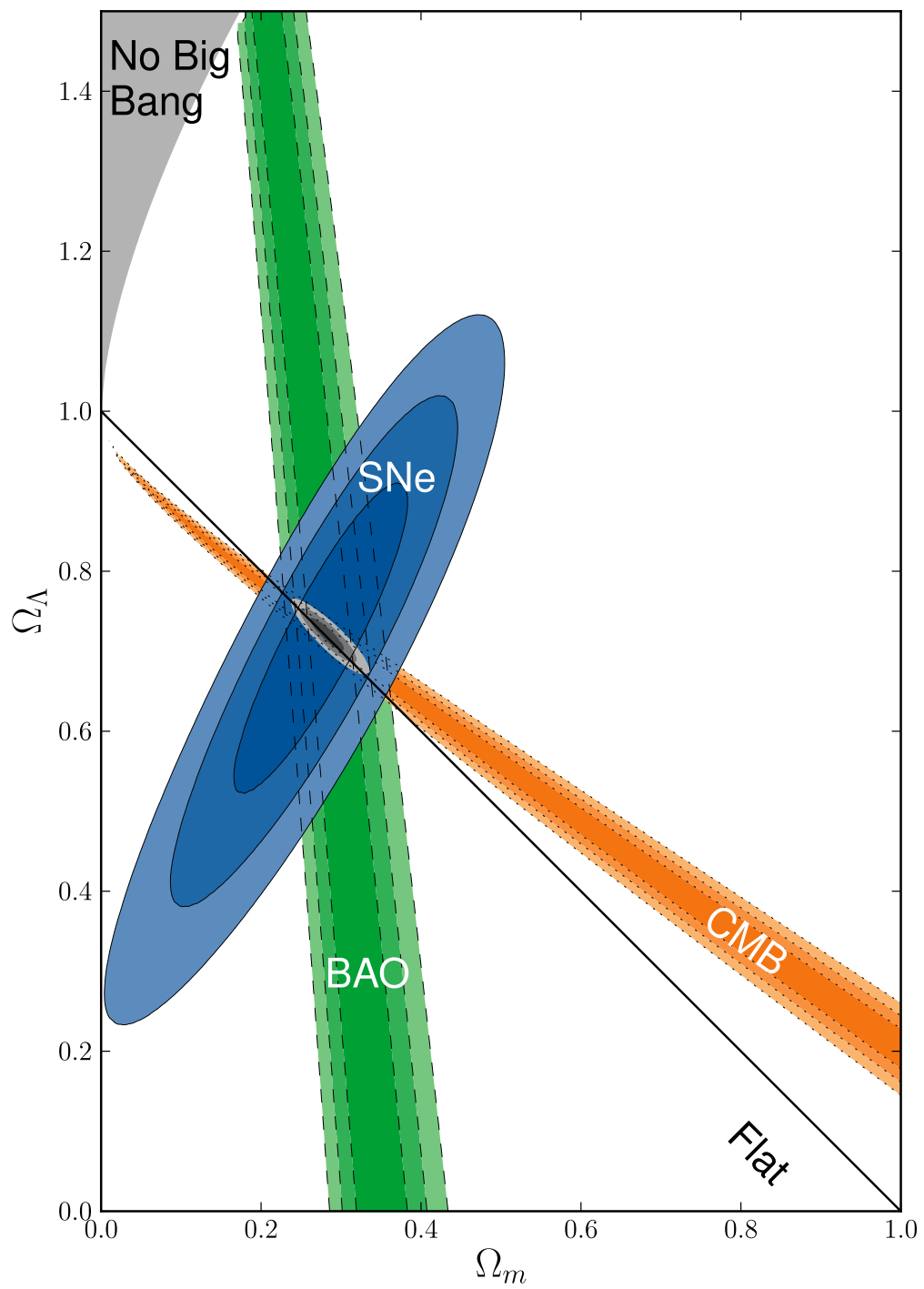
Phillip Helbig

Overview

- Introduction
- Basic theory
- History
- This work
 - calculations
 - results and discussion—and surprises
 - averaging
- Conclusions

Introduction

- The $m-z$ relation is a classic cosmological test.
- The 2011 Nobel Prize was awarded for work with type Ia supernovae.
- It is now one of many complementary cosmological tests.



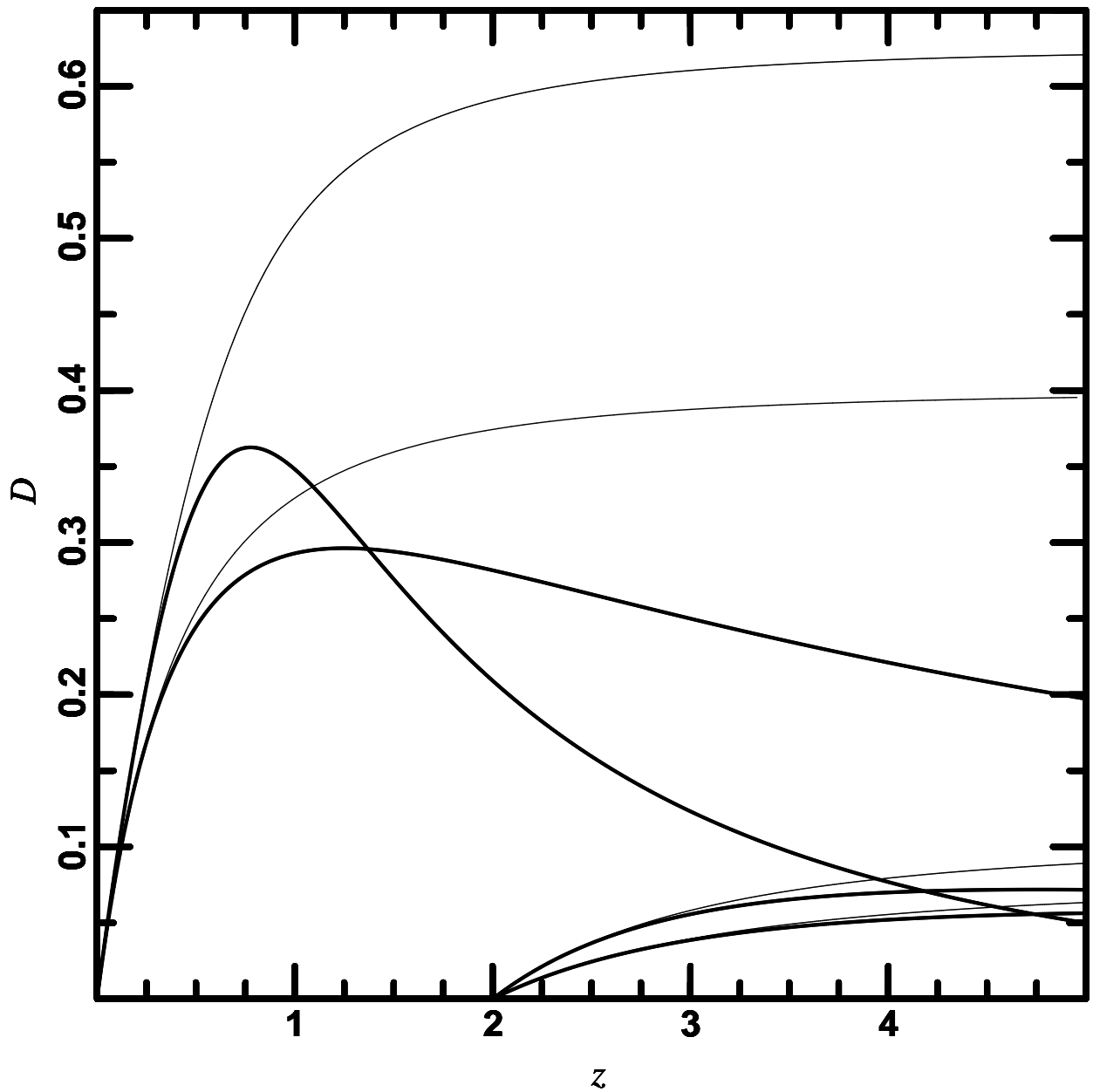
Suzuki *et al.* (2011)

Executive abstract

- The universe is obviously inhomogeneous.
- Does this matter for observational cosmology?
- It depends.
- The universe appears to be homogeneous on large scales but of course not on small scales.
- How does this affect one of the classical cosmological tests, the magnitude-redshift ($m-z$) relation, in particular for type Ia supernovae?

Basic theory

- Fraction η is distributed homogeneously.
- Fraction $1 - \eta$ is distributed clumpily.
- η depends on the relevant scale.
- If η isn't mentioned, the authors assume $\eta = 1$.
- See Kayser, Helbig & Schramm (1997) for more details and Fortran code.



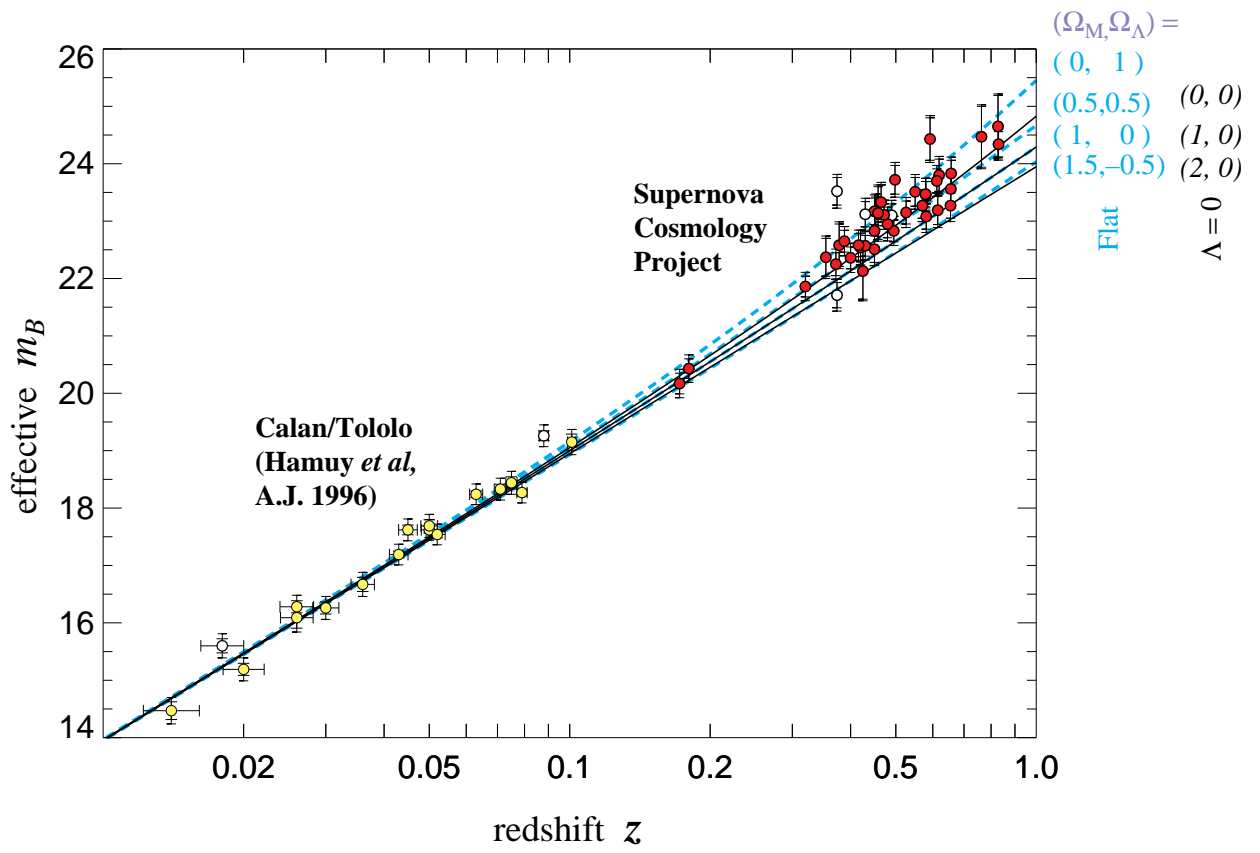
Thin: $\eta = 0$, thick: $\eta = 1$. The upper curves near $z = 0$ ($z = 2$ at lower right) are for $\lambda_0 = 2$, the lower for $\lambda_0 = 0$. $\Omega_0 = 1$ for all curves.

Previous work

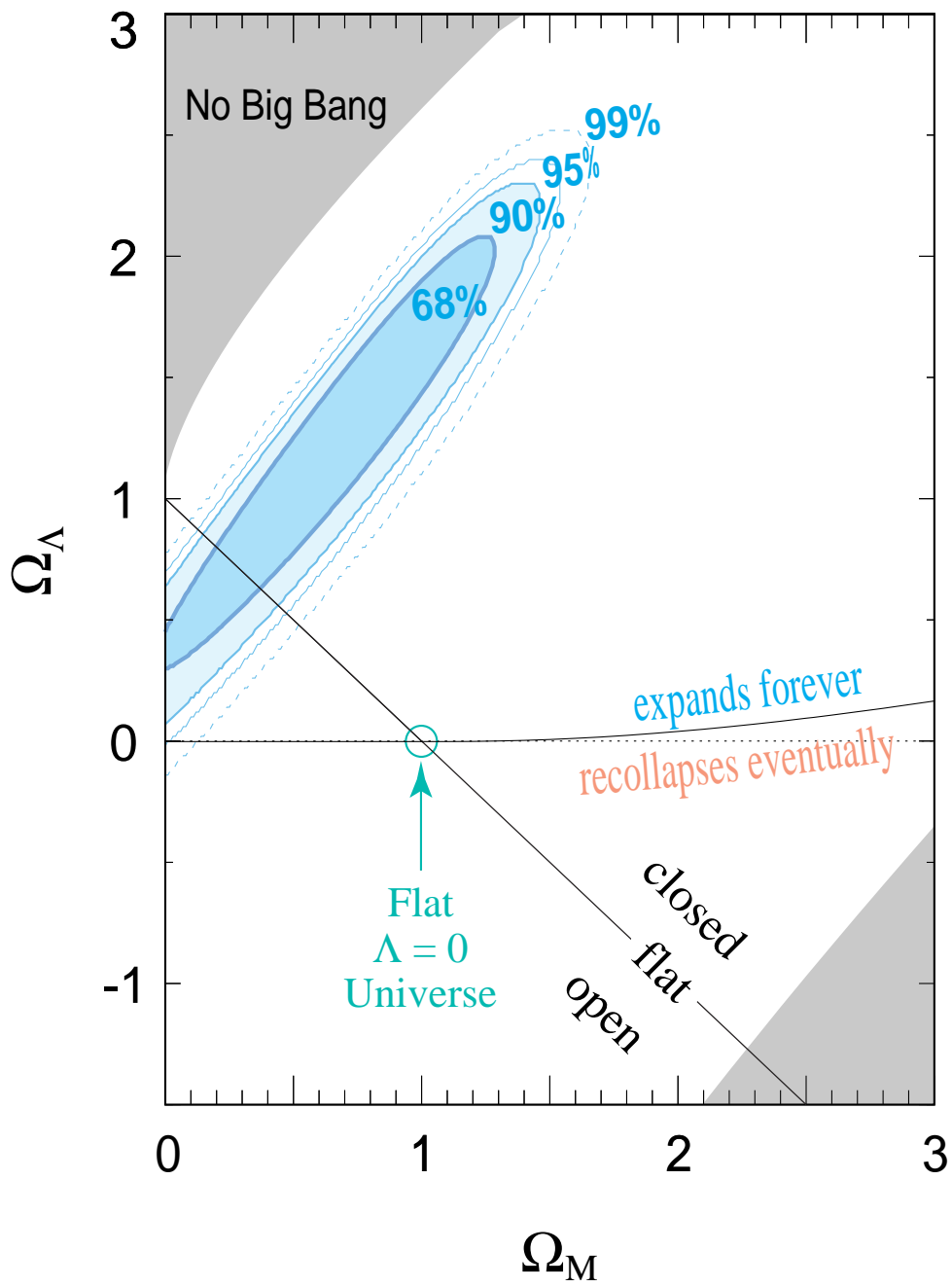
- Light propagation in locally inhomogeneous cosmological models was first investigated in detail by Zel'dovich (1964), Dashevskii & Zel'dovich (1965), Dashevskii & Slysh (1966)
- This topic entered 'mainstream cosmology' with papers by Dyer and Roeder (several papers, alone and together).
- The importance of η in the for the $m-z$ relation has been emphasized by Kantowski.

Previous work

- Is it a good approximation?
 - Mörtzell (2002)
 - Jönsson, Kronborg, Mörtzell & Sollerman (2008)
 - Bergström, Goliath, Goobar & Mörtzell (2000)
- There is a simple analysis in the most important SCP paper.



Perlmutter *et al.* (1999)

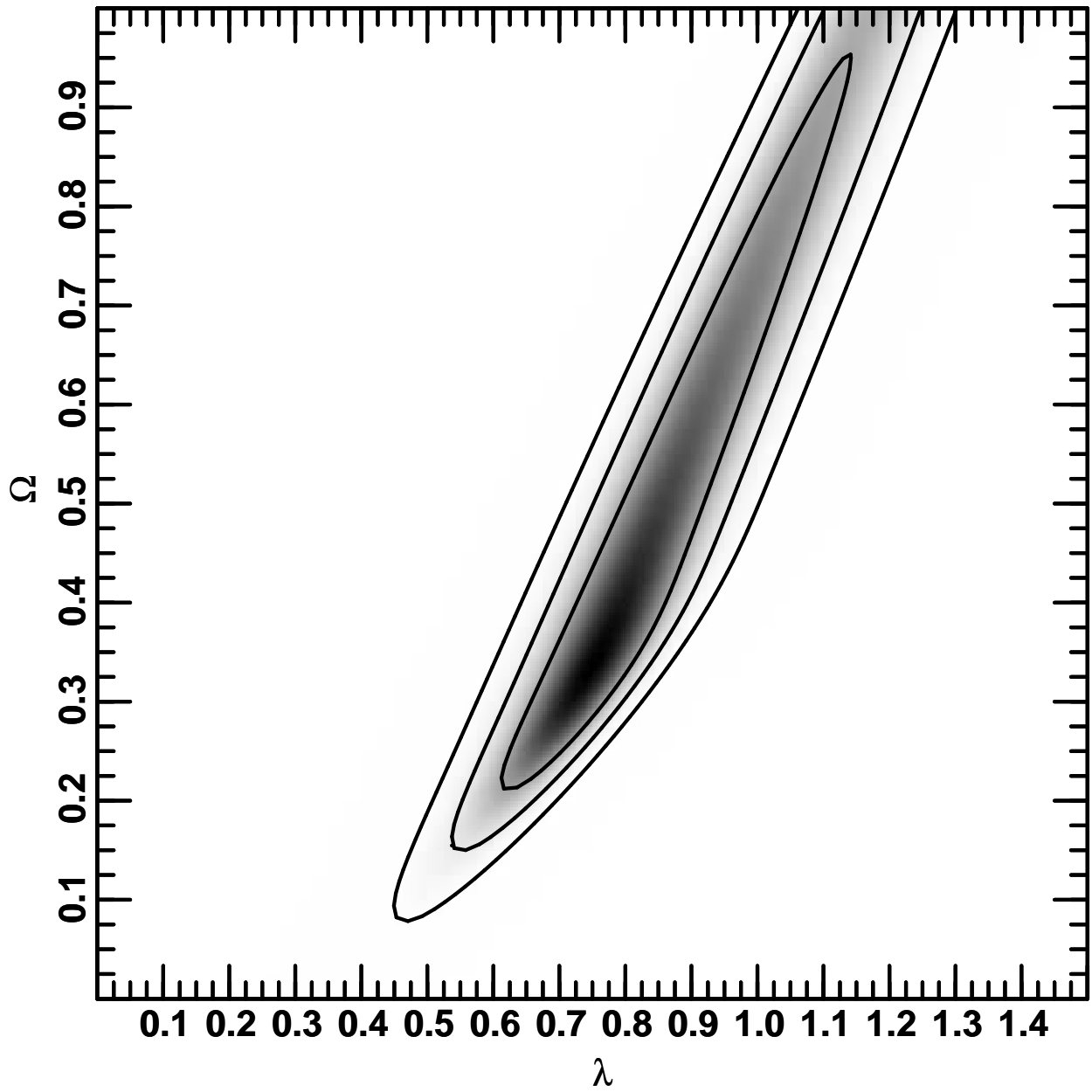


Perlmutter *et al.* (1999)

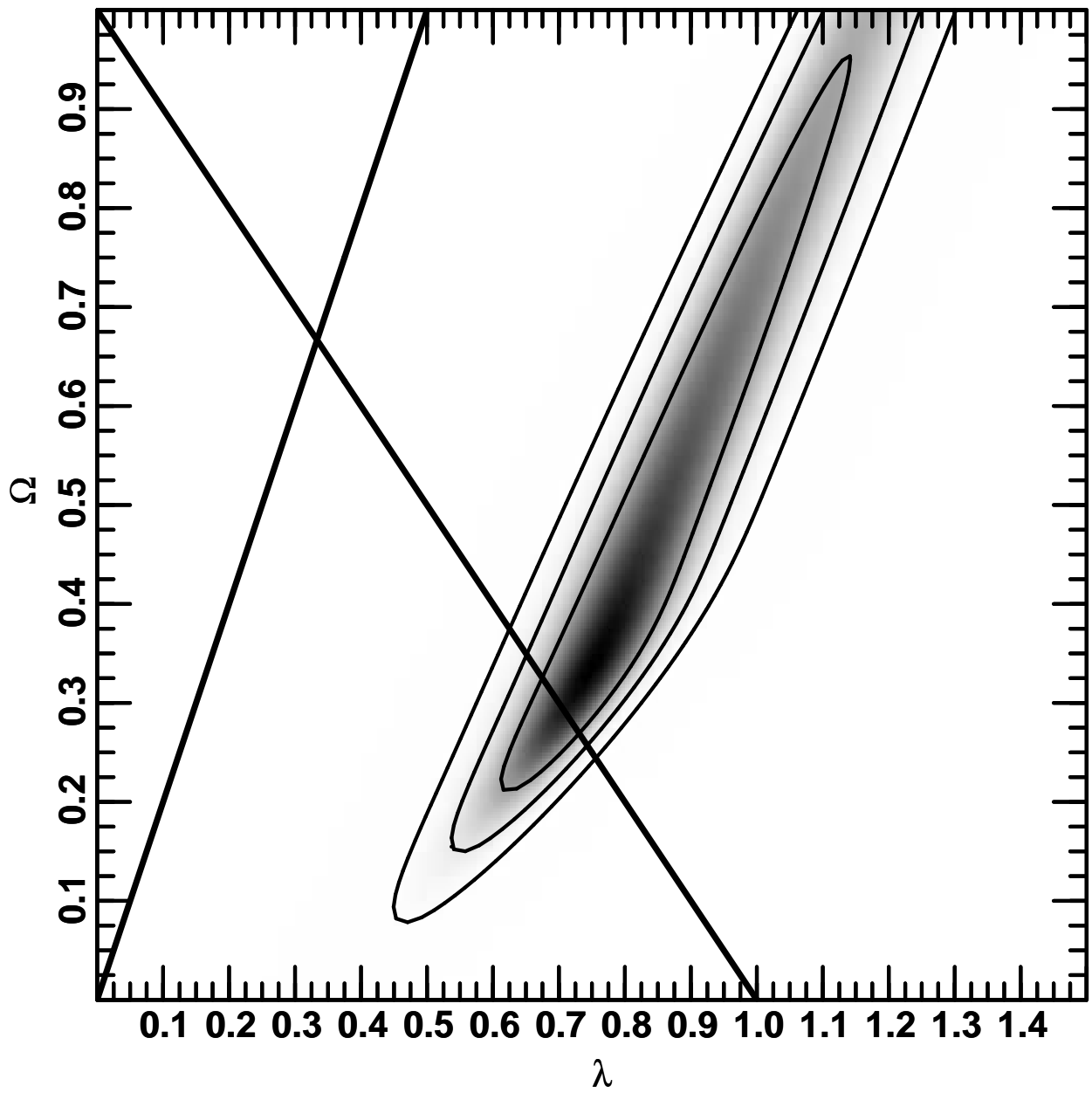
What have I done?

- Calculations of χ^2 and the corresponding probability on two three-dimensional grids:
 - large, low-resolution grid
 - * $-5 < \lambda_0 < 5$, $\Delta\lambda_0 = 0.02$ (500 points)
 - * $0 < \Omega_0 < 10$, $\Delta\Omega_0 = 0.02$ (500 points)
 - * $0 < \eta < 1$, $\Delta\eta = 0.01$ (100 points)
 - small, high-resolution grid
 - * $0 < \lambda_0 < 1.5$, $\Delta\lambda_0 = 0.003125$ (480)
 - * $0 < \Omega_0 < 1$, $\Delta\Omega_0 = 0.003125$ (320)
 - * $0 < \eta < 1$, $\Delta\eta = 0.01$ (100)
- Various two-dimensional visualizations

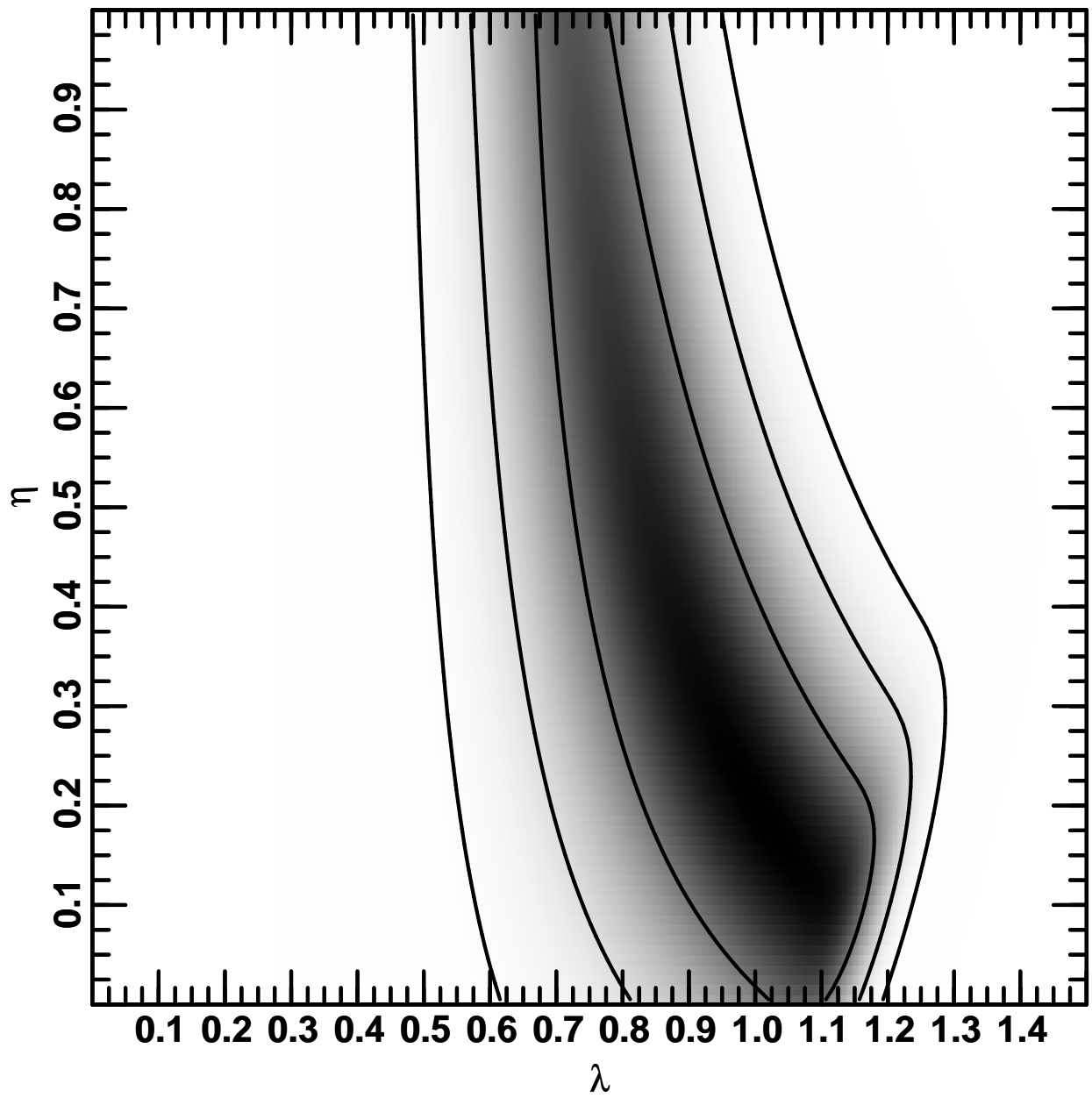
Calculate contours of
relative probability in
three-dimensional space;
marginalize over 1 dimension



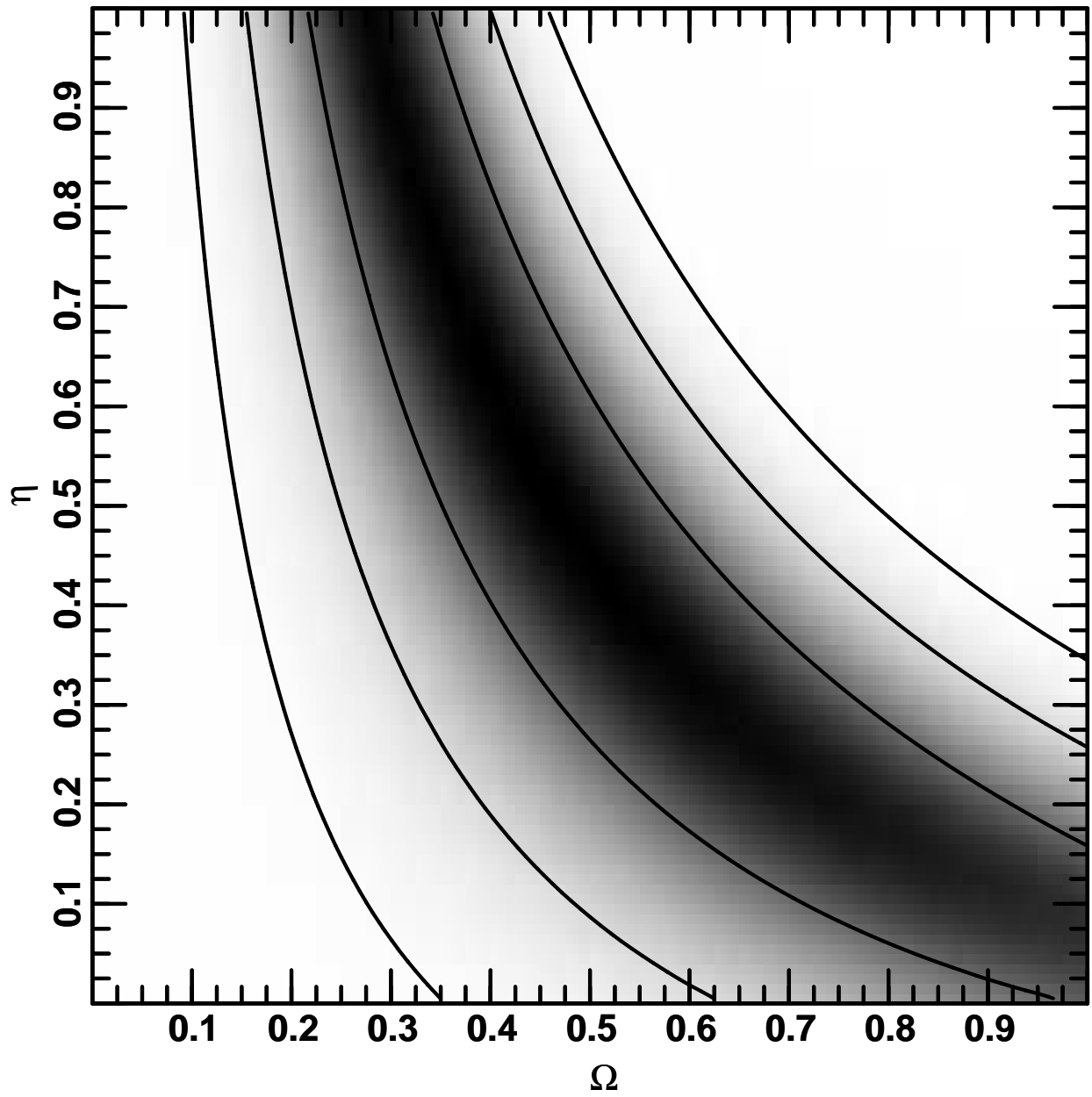
small, high-resolution grid
marginalized over η



small, high-resolution grid
marginalized over η

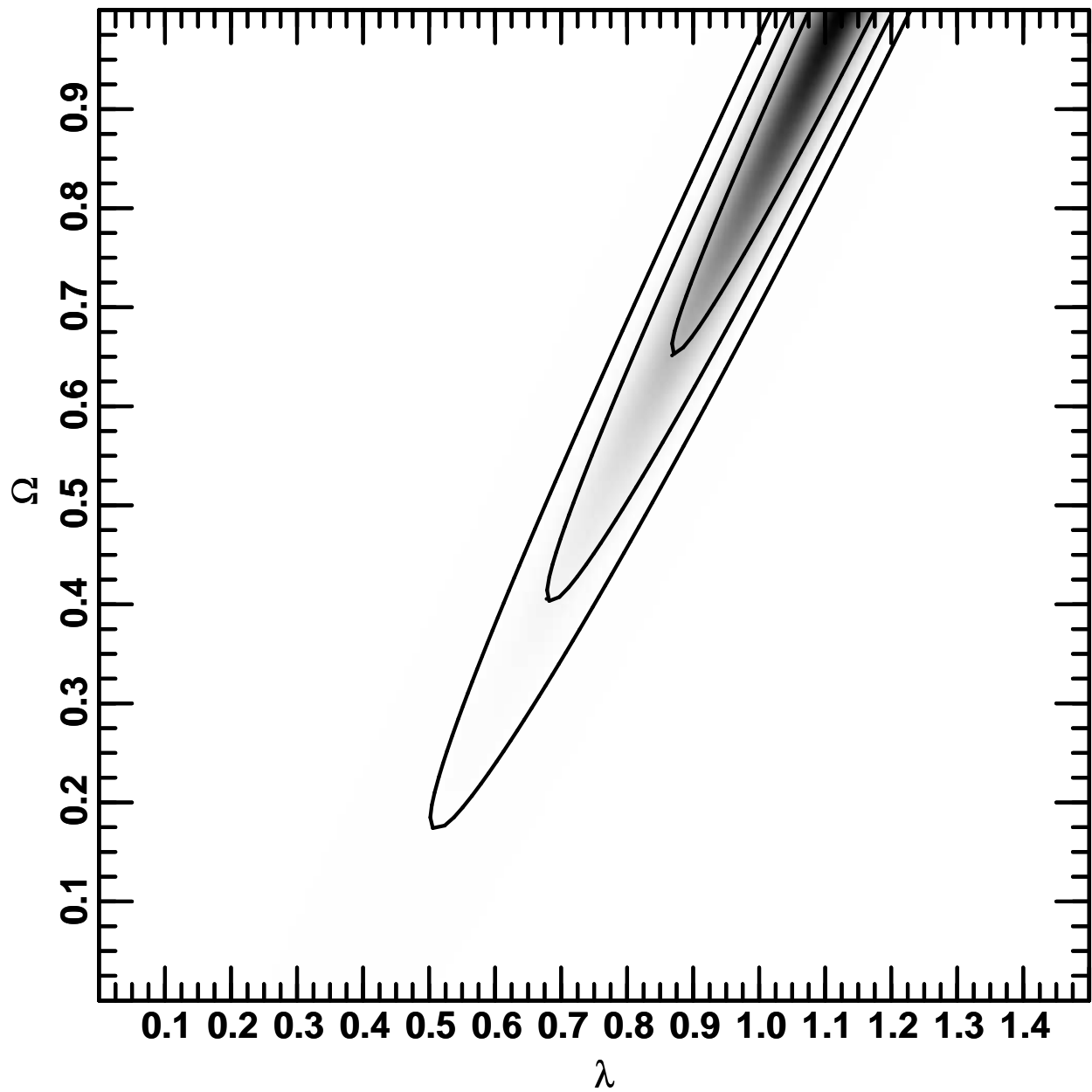


small, high-resolution grid
marginalized over Ω_0



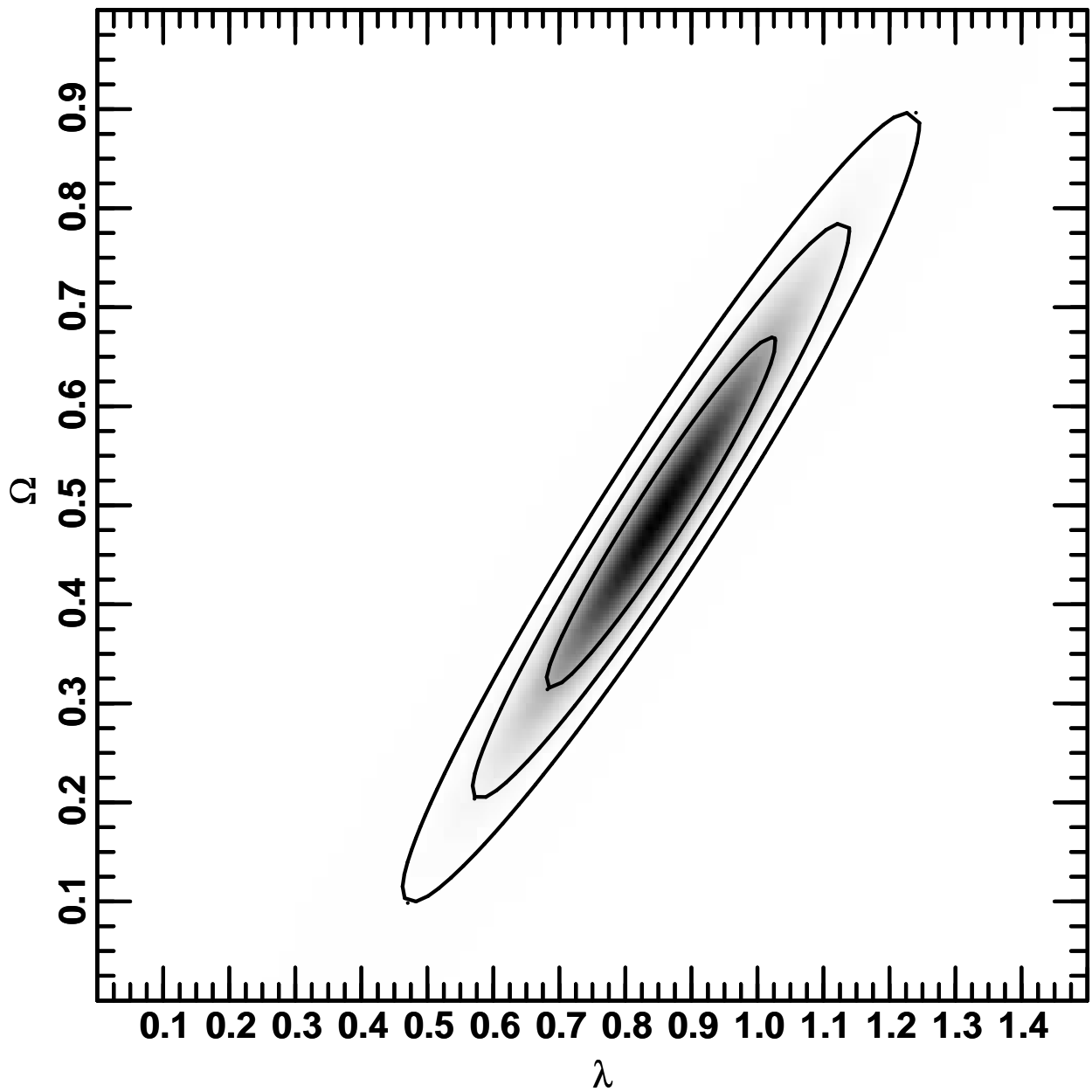
small, high-resolution grid
marginalized over λ_0

Calculate contours of
relative probability in
two-dimensional space;
fix value of the 3rd dimension
(δ -function prior)



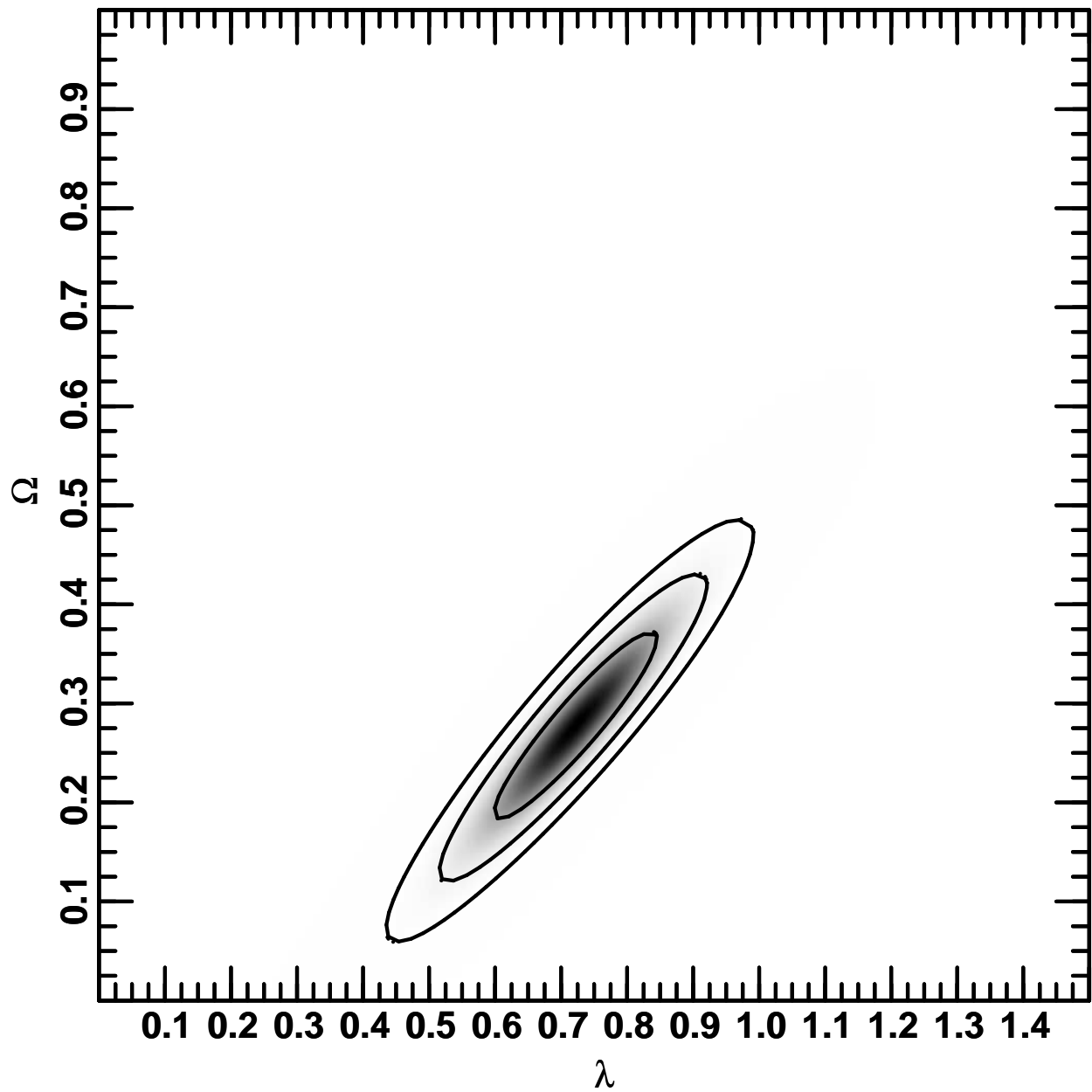
small, high-resolution grid

prior: $\eta = 0$ (completely empty beam)



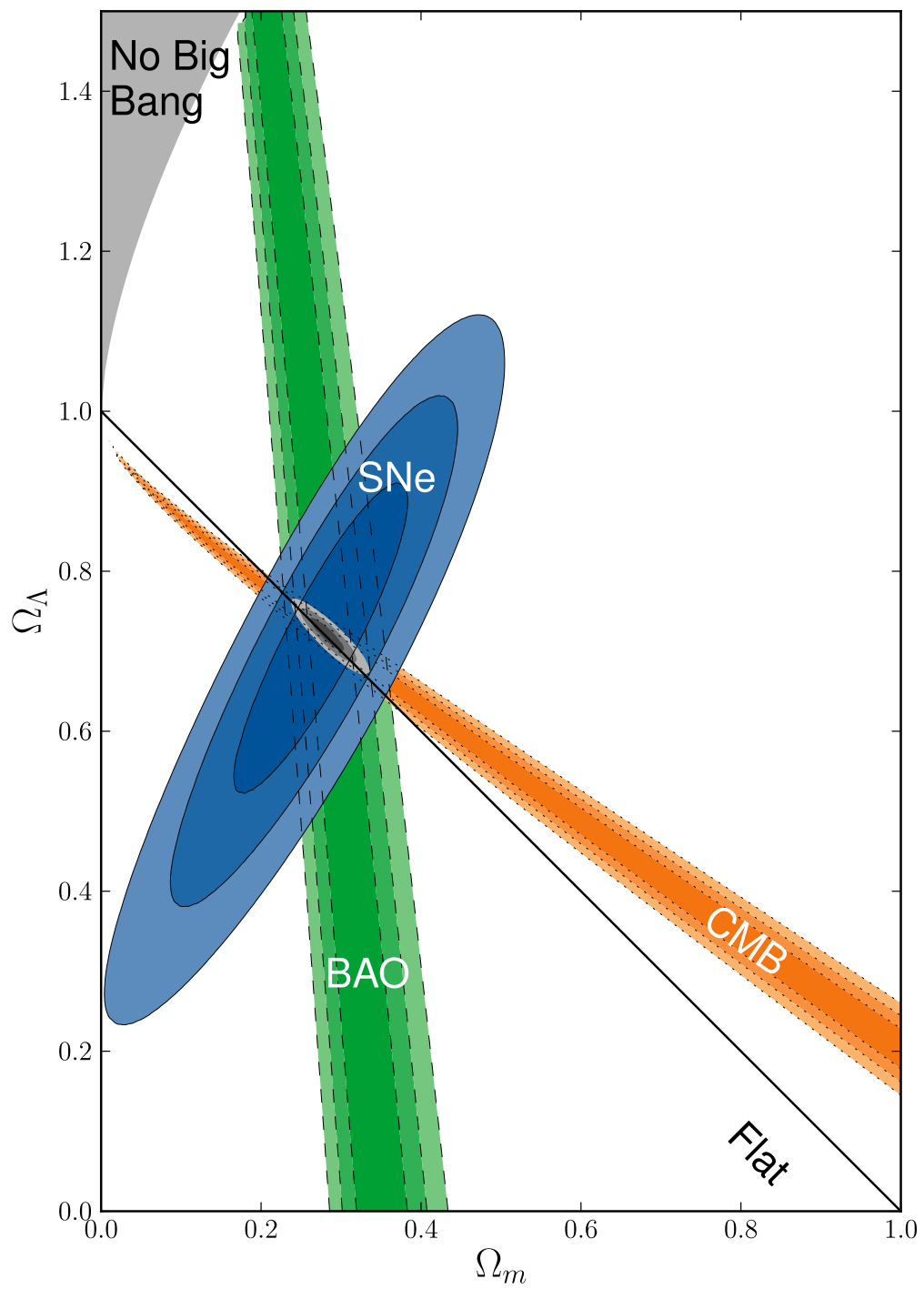
small, high-resolution grid

prior: $\eta = 0.455$ (value of η at the overall maximum)



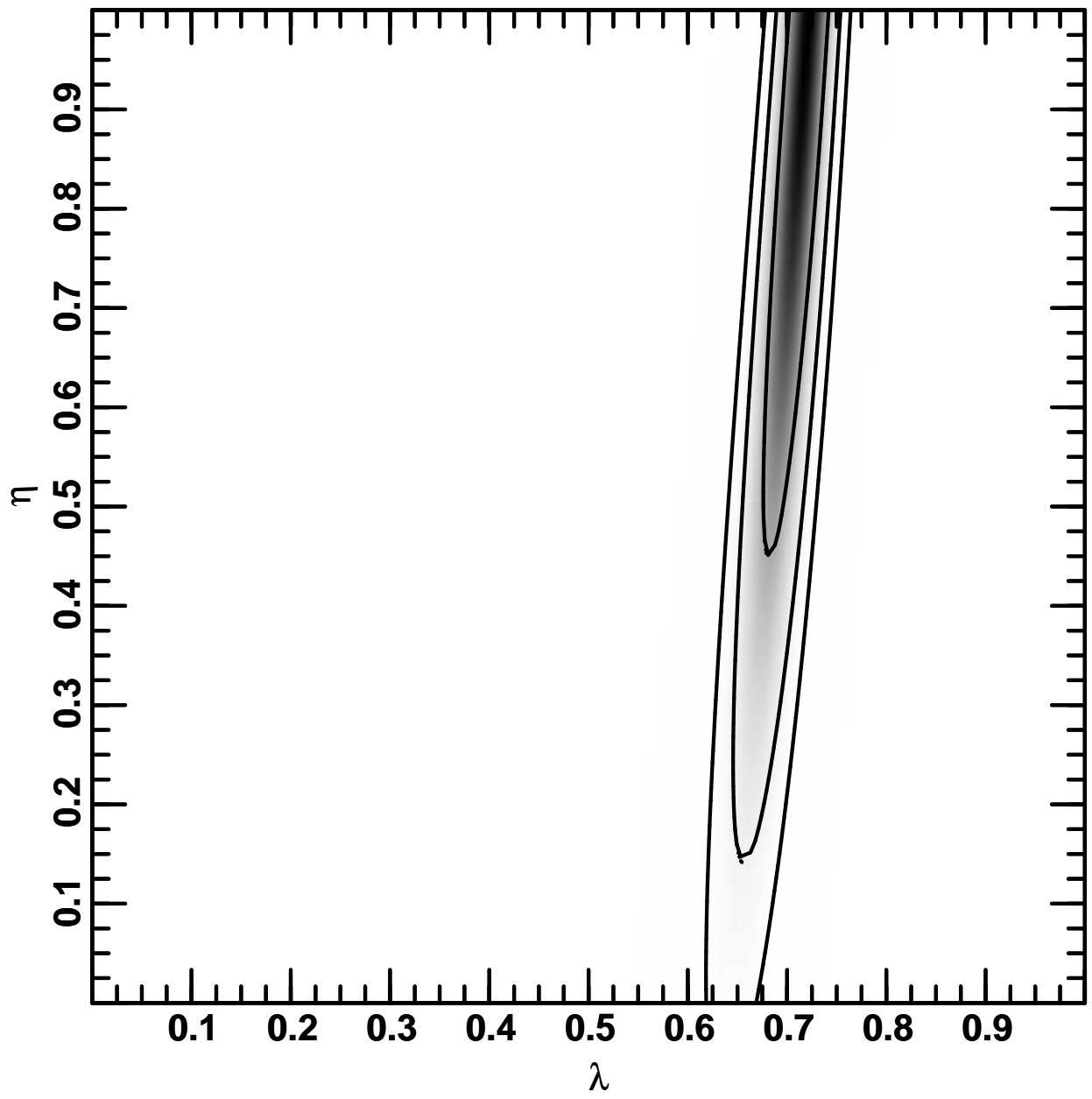
small, high-resolution grid

prior: $\eta = 1$ (value which is usually implicitly assumed)



Suzuki *et al.* (2011)

Calculate contours of
relative probability in
two-dimensional space
assuming $k = 0$



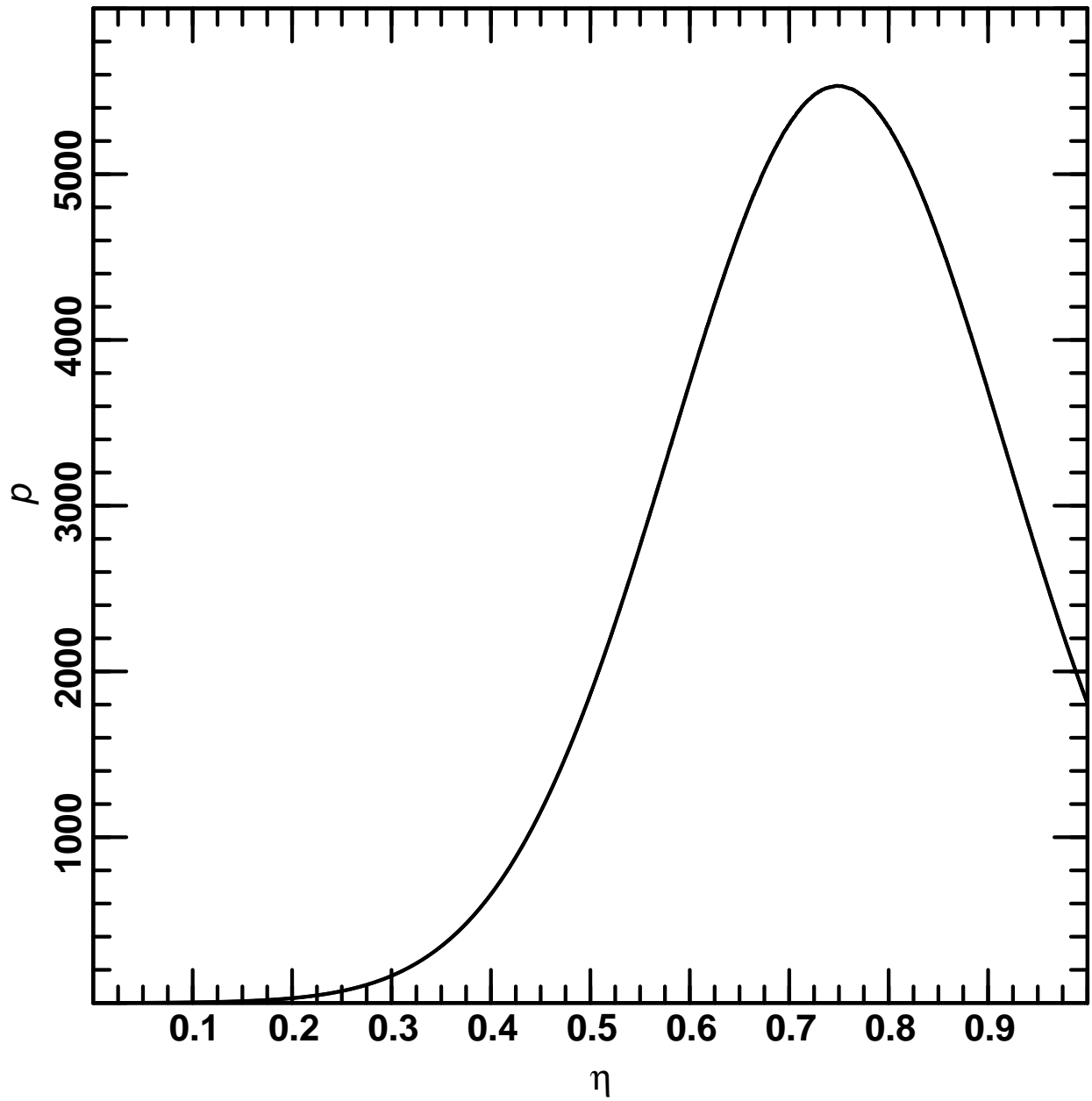
small, high-resolution grid

prior: $k = 0$

Calculate relative probability

in one-dimensional space

fix $\lambda_0 = 0.7$ and $\Omega_0 = 0.3$



small, high-resolution grid
 $p(\eta)$ for $\lambda_0 = 0.7$ and $\Omega_0 = 0.3$

What do we find: expected

- Constraints on λ_0 and Ω_0 are weaker if η is not constrained.
- The concordance model is reasonably probable.
- There is a degeneracy between η and the amount of spatial curvature ($\lambda_0 + \Omega_0$).
- λ_0 is constrained best, then Ω_0 , then η .

What about dark matter?

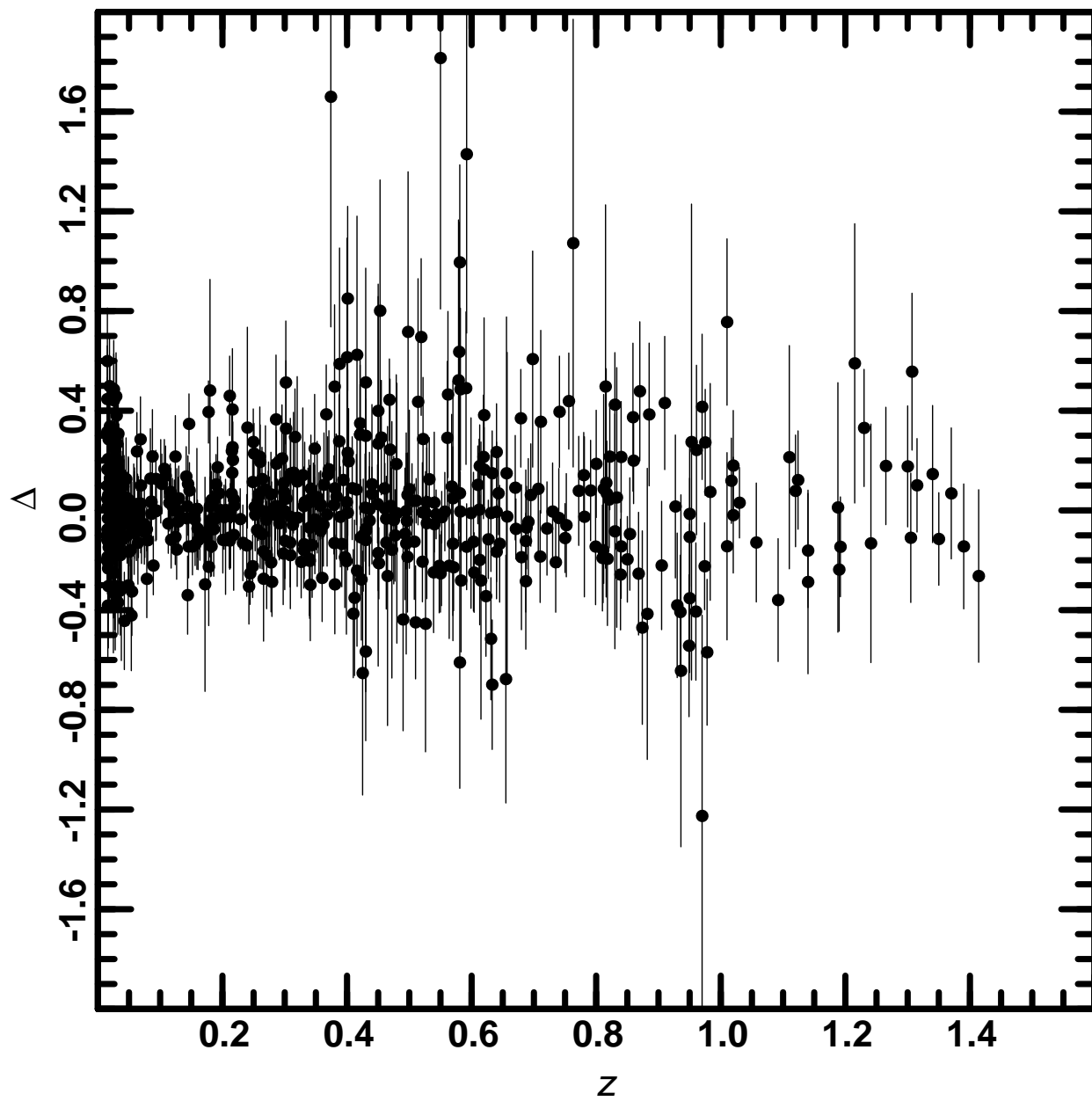
- Assuming the concordance model, the best-fit value of η is approximately 0.75:
 - $0.60 < \eta < 0.90$ (68.3%)
 - $0.46 < \eta < 1.00$ (95.4%)
 - $0.28 < \eta < 1.00$ (99.7%)
- Question: can we take such constraints on η seriously, if we don't believe the overall best fit?
- Answer: The best fit is really not statistically significantly better than many other points, but some values of η can be ruled out at high confidence levels.

What do we find: surprising?

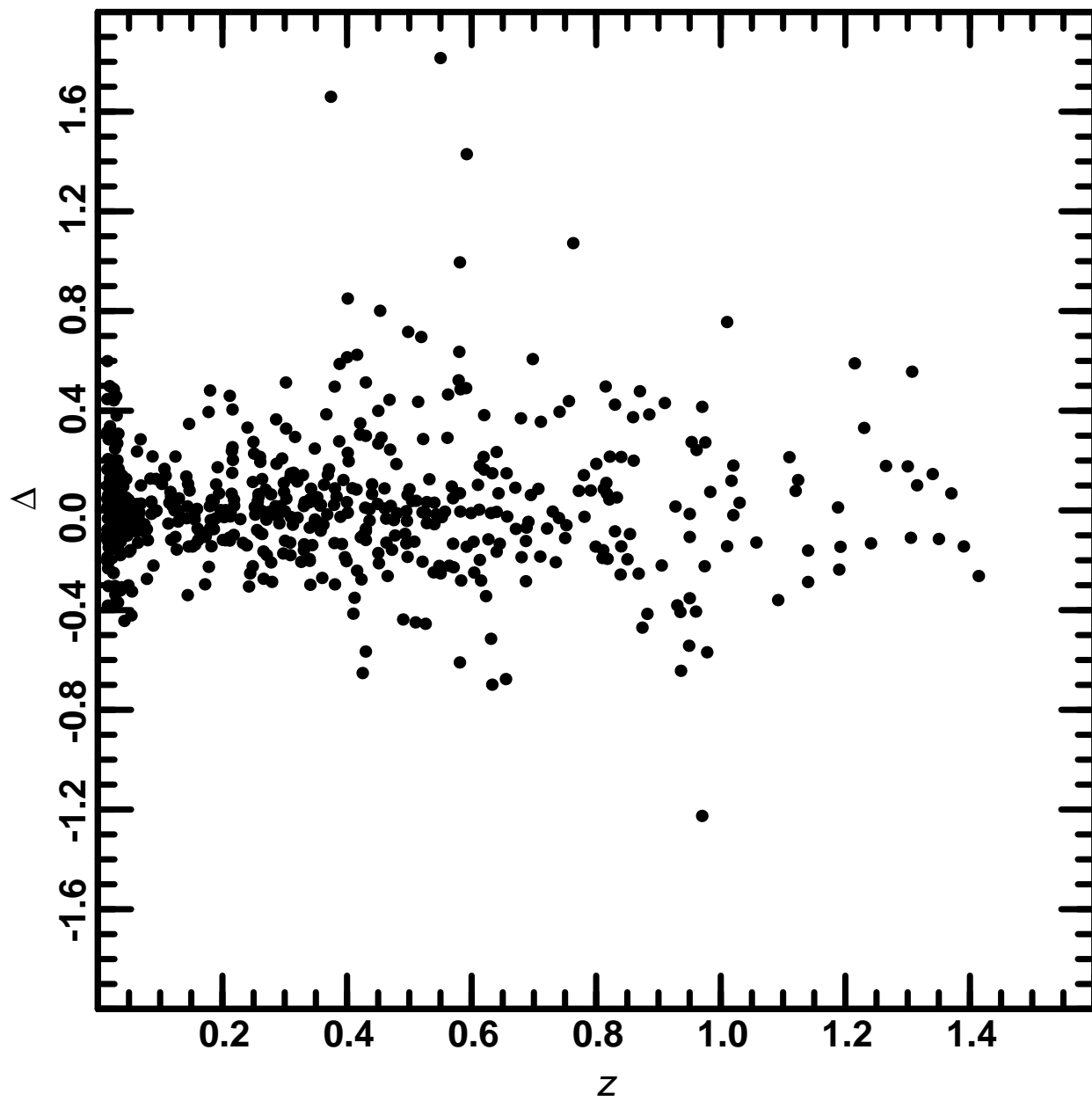
- Overall best fit is ruled out by other tests (overfitting?).
- If we *assume* $k = 0$ then the best fit is the concordance model *and* has $\eta = 1$.
- If we *assume* $\eta = 1$, then the best-fit is very close to the concordance model.
- If we assume the concordance model, then can probably rule out low values of η , even though the relevant scale is extremely small.
- We cannot rule out $\eta = 1$, and there is some tentative evidence for it.

Averaging: Safety in numbers, or safely without worry?

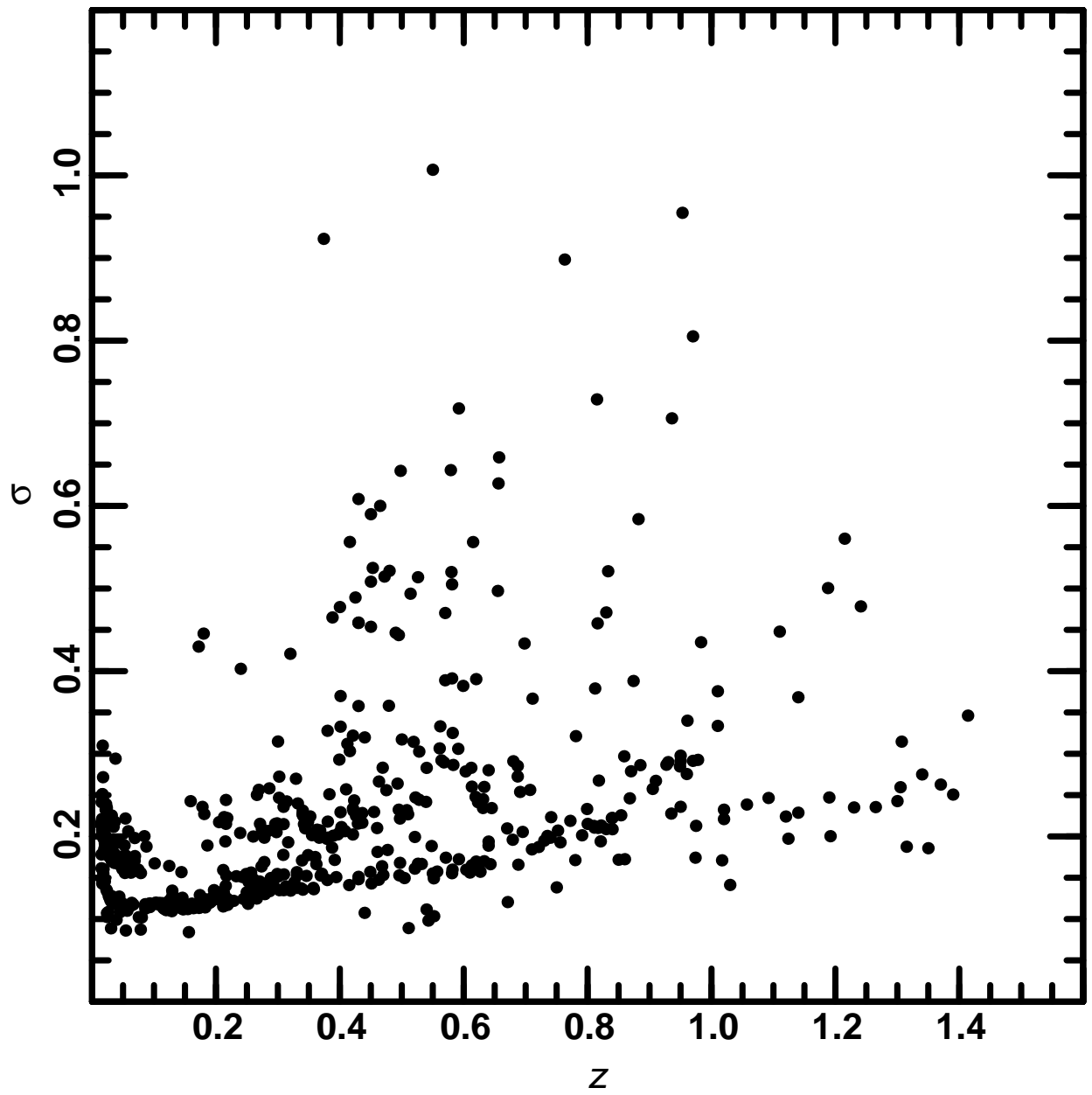
- Two possibilities to have $\eta \approx 1$:
- Safety in numbers: Some lines of sight are overdense, some underdense (implies more general definition of η), and $\eta \approx 1$ results from appropriate 'averaging'.
- Safely without worry: Each line of sight has $\eta \approx 1$, so the classical distance formula (i.e. $\eta = 1$) can be used safely without worry.
- Idea: compare residuals with observational uncertainties.



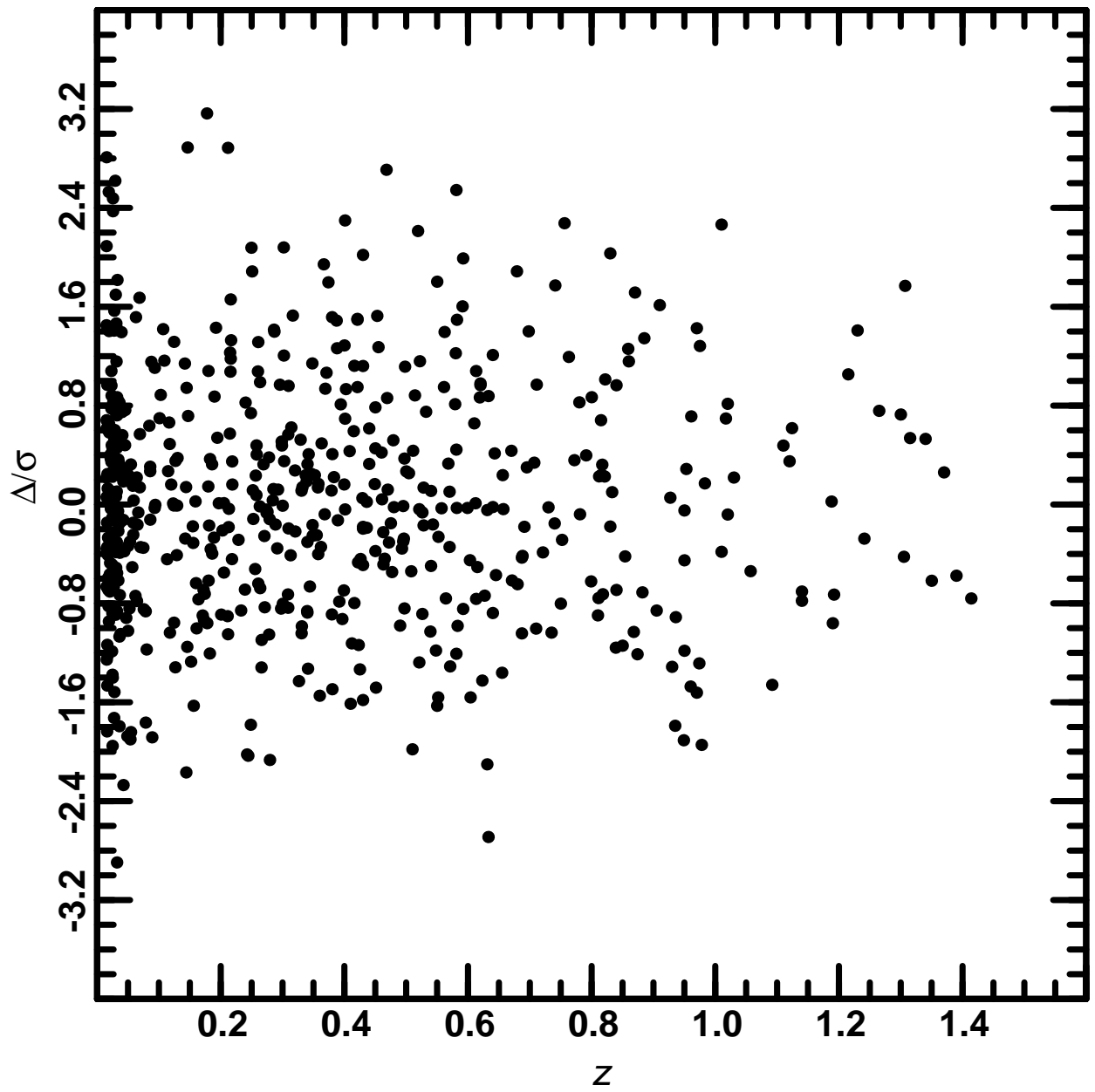
Residuals (points) with observational uncertainties (error bars)



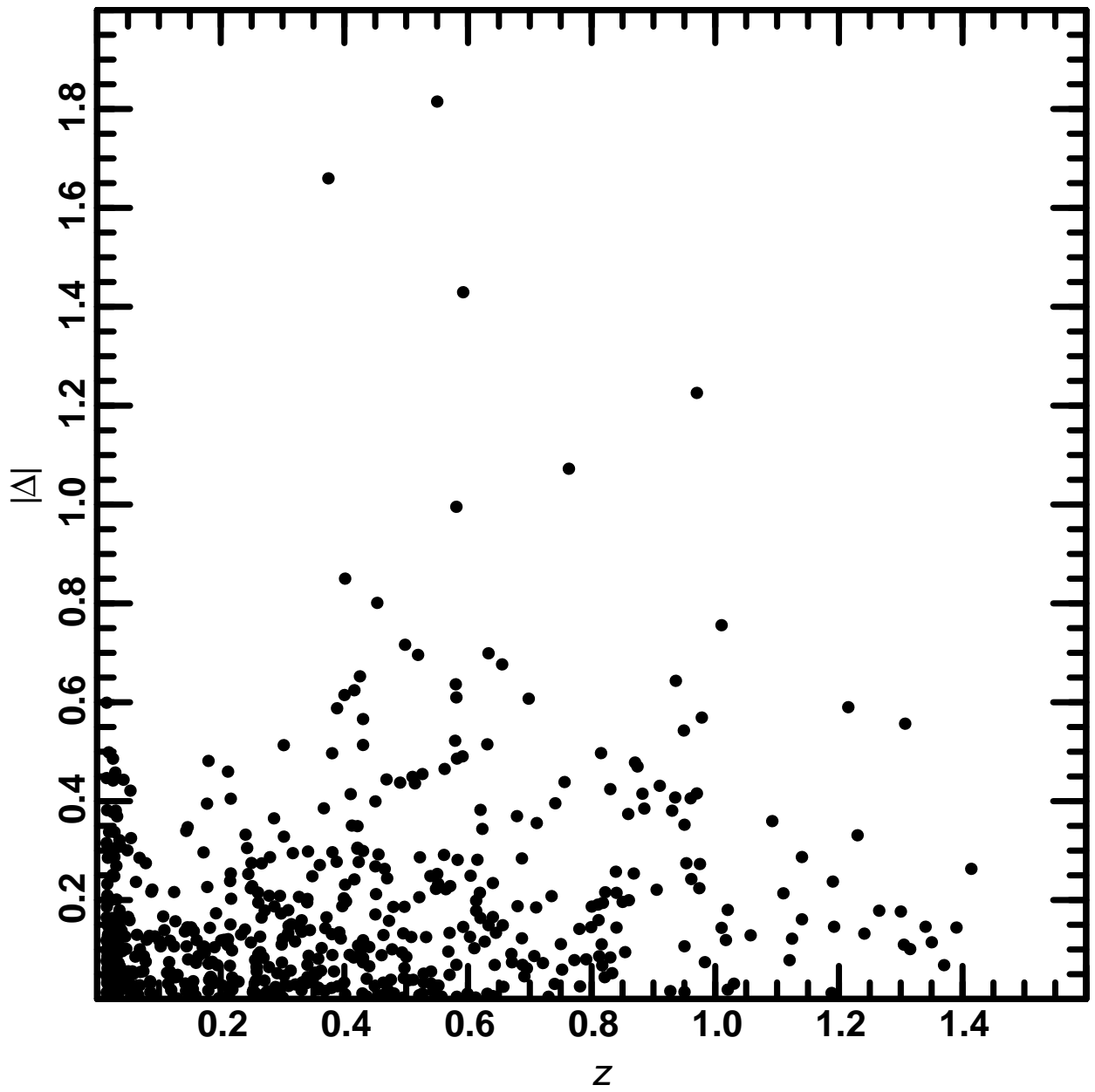
Residuals



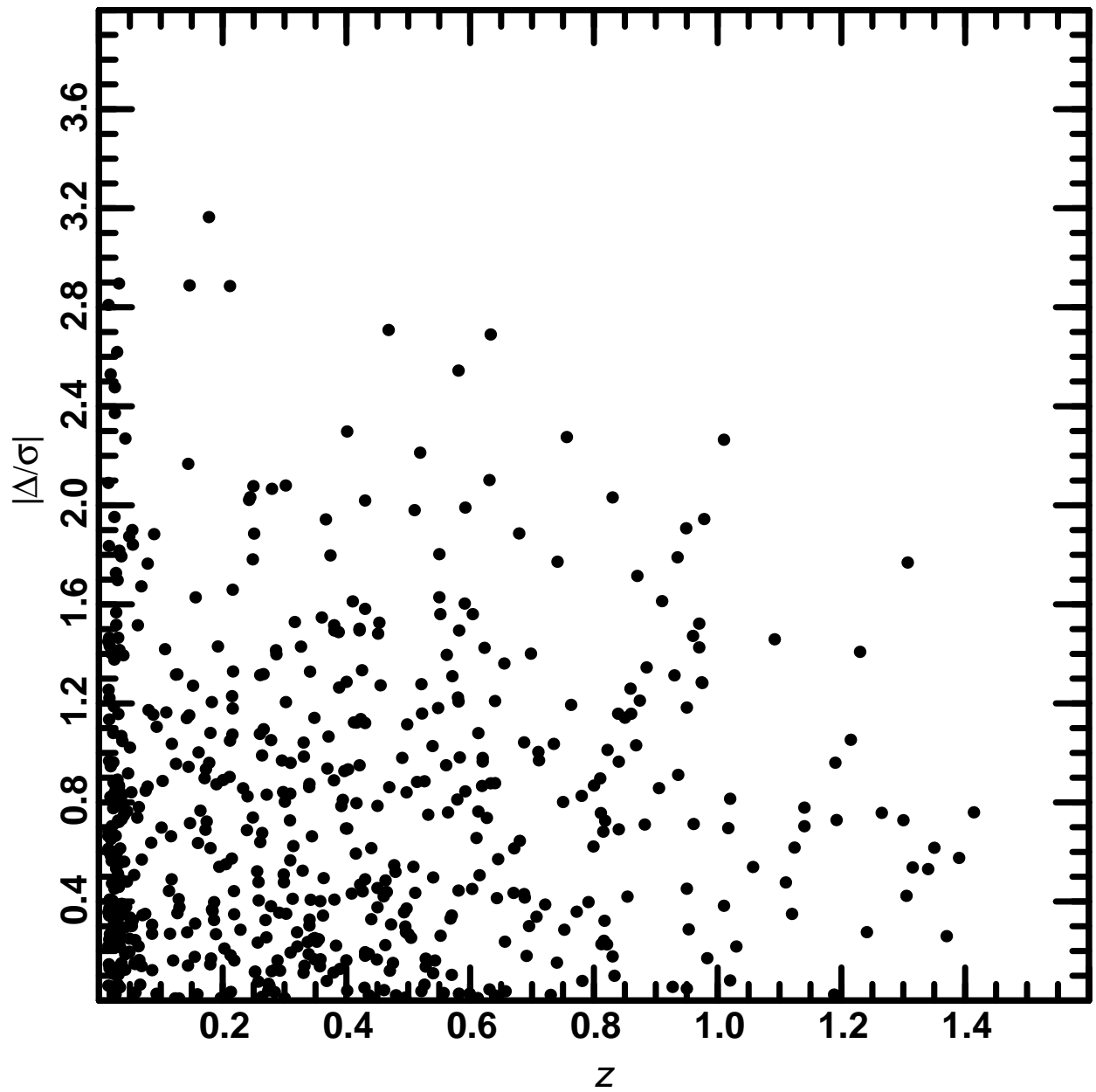
Observational uncertainties



Residuals scaled with observational uncertainties



Absolute value of residuals



Absolute value of residuals scaled with observational uncertainties

Conclusions

- Allowing η as a free parameter significantly alters both the best fit in the λ_0 - Ω_0 plane and the allowed region of this plane.
- The concordance model is still allowed.
- There are non-significant hints that $\eta \approx 1$.
- Low values of η can probably be ruled out, which is not obvious considering the very small scales involved.
- Each line of sight is probably a fair sample of the universe; we don't have to average over many to justify $\eta \approx 1$.

Further reading

more general η concept:

Lima, Busti, & Santos: arXiv:1301.5360;
PRD, **89**, 6, 067301

η and the $m-z$ relation:

P. Helbig: arXiv:1505.02917;
MNRAS, **451**, 2, 2097

residuals and uncertainties:

P. Helbig: arXiv:1508.05544;
MNRAS, **453**, 4, 3975

recent paper with all the gory details:

Kaiser & Peacock: arXiv:1503.08506
MNRAS **455**, 4, 4518