Practical Aspects of the EFT of LSS

- The Eastcoast Story -

Tobias Baldauf

with Lorenzo Mercolli, Mehrdad Mirbabayi, Enrico Pajer,
Emmanuel Schaan, Marko Simonovic and Matias Zaldarriaga

[1406.4135],[1504.04366],[1505.07098],[1507.02255] and [1507.02256]

School of Natural Sciences
Institute for Advanced Study
Princeton, NJ

Rencontres de Moriond, La Thuile
21.03.2016

1 Introduction

2 Lagrangian Space

3 Eulerian Space

4 Summary

1 Introduction

2 Lagrangian Space

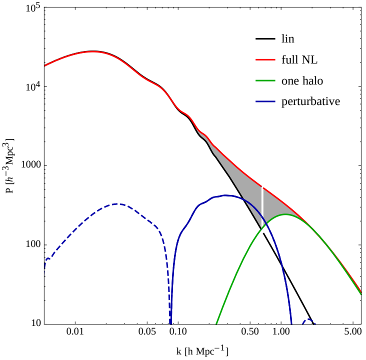
3 Eulerian Space

4 Summary

LSS Clustering Statistics

Structure of the Power Spectrum

- Linear Theory valid on large scales
- Perturbative regime
- Non-perturbative virialized regime
- bias, redshift space . . .



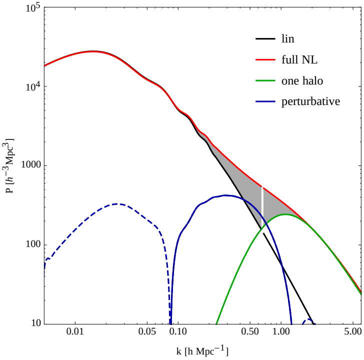
Why bother with difficulties in LSS?

- Number of modes in CMB is saturated
- Large number of 3D modes in LSS (scale as $k_{\rm max}^3$)
- Provides distinct signatures not present in CMB

LSS Clustering Statistics

Structure of the Power Spectrum

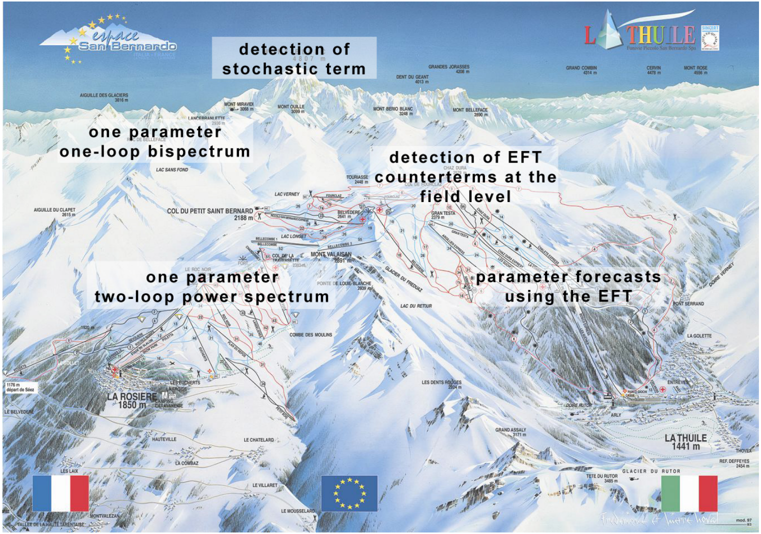
- Linear Theory valid on large scales
- Perturbative regime
- Non-perturbative virialized regime
- bias, redshift space . . .



Why bother with difficulties in LSS?

- Number of modes in CMB is saturated
- Large number of 3D modes in LSS (scale as k_{max}^3)
- Provides distinct signatures not present in CMB

Don't get lost – Take Home Messages



Credits - Most of EFT is SPT^{++}

Lagrangian Perturbation Theory and resummations

Bouchet, Colombi, Hivon, Matsubara, Juskiewicz et al.

Standard Perturbation Theory and resummations

Bernardeau, Crocce, Grinstein, Pietroni, Scoccimarro, Wise et al.

EFT

Basic Idea [Baumann, Nicolis, Senatore, Zaldarriaga 2010]

One-Loop Power Spectrum [Carrasco, Hertzberg, Senatore 2012]

Two-Loop Power Spectrum [Carrasco, Foreman, Green, Senatore 2013]

Bispectrum [Angulo, Foreman, Schmittfull, Senatore 2014]

Lagrangian Space [Porto, Senatore, Zaldarriaga 2014]

IR-resummation [Senatore and Zaldarriaga 2014]

Bias [Senatore 2014] [Mirbabay, Schmidt, Zaldarriaga 2014]

Redshift Space Distortions [Senatore and Zaldarriaga 2014]

...

Credits - Most of EFT is SPT^{++}

Lagrangian Perturbation Theory and resummations

Bouchet, Colombi, Hivon, Matsubara, Juskiewicz et al.

Standard Perturbation Theory and resummations

Bernardeau, Crocce, Grinstein, Pietroni, Scoccimarro, Wise et al.

EFT

Basic Idea [Baumann, Nicolis, Senatore, Zaldarriaga 2010]
One-Loop Power Spectrum [Carrasco, Hertzberg, Senatore 2012]
Two-Loop Power Spectrum [Carrasco, Foreman, Green, Senatore 2013]
Bispectrum [Angulo, Foreman, Schmittfull, Senatore 2014]
Lagrangian Space [Porto, Senatore, Zaldarriaga 2014]
IR-resummation [Senatore and Zaldarriaga 2014]
Bias [Senatore 2014] [Mirbabay, Schmidt, Zaldarriaga 2014]
Redshift Space Distortions [Senatore and Zaldarriaga 2014]
...

EoM from the Vlaslov Equation

Vlaslov - Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma}\frac{\partial f}{\partial \mathbf{x}} - ma\boldsymbol{\nabla}\phi\frac{\partial f}{\partial \mathbf{p}} = 0$$

Definition of Density and Momentum

$$\rho = ma^{-3}\int \mathrm{d}^3p\; f(\mathbf{x},\mathbf{p},\tau) \quad \rho v_i = \int \mathrm{d}^3p\; \frac{p_i}{ma} f(\mathbf{x},\mathbf{p},\tau) / \int \mathrm{d}^3p\; f(\mathbf{x},\mathbf{p},\tau)$$

Fluid Equations

$$\delta' + \partial_j\left[(1+\delta)v_j\right] = 0$$

$$v_i' + \mathcal{H}v_i + \partial_i\phi + v_j\partial_j v_i = -\frac{1}{1+\delta}\partial_j[(1+\delta)\sigma_{ij}]$$

EoM from the Vlaslov Equation

Vlaslov - Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma}\frac{\partial f}{\partial \mathbf{x}} - ma\boldsymbol{\nabla}\phi\frac{\partial f}{\partial \mathbf{p}} = 0$$

Definition of Density and Momentum

$$\rho = ma^{-3}\int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau) \quad \rho v_i = \int \mathrm{d}^3p\ \frac{p_i}{ma} f(\mathbf{x},\mathbf{p},\tau)/\int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau)$$

Fluid Equations

$$\delta' + \partial_j\left[(1+\delta)v_j\right] = 0$$

$$v_i' + \mathcal{H}v_i + \partial_i\phi + v_j\partial_j v_i = -\frac{1}{1+\delta}\partial_j[(1+\delta)\sigma_{ij}]$$

EoM from the Vlaslov Equation

Vlaslov - Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma}\frac{\partial f}{\partial \mathbf{x}} - ma\boldsymbol{\nabla}\phi\frac{\partial f}{\partial \mathbf{p}} = 0$$

Definition of Density and Momentum

$$\rho = ma^{-3}\int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau) \quad \rho v_i = \int \mathrm{d}^3p\ \frac{p_i}{ma} f(\mathbf{x},\mathbf{p},\tau) / \int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau)$$

Fluid Equations

$$\delta' + \partial_j\left[(1+\delta)v_j\right] = 0$$

$$v_i' + \mathcal{H}v_i + \partial_i\phi + v_j\partial_j v_i = -\frac{1}{1+\delta}\partial_j[(1+\delta)\sigma_{ij}]$$

EoM from the Vlaslov Equation

Vlaslov - Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma}\frac{\partial f}{\partial \mathbf{x}} - ma\boldsymbol{\nabla}\phi\frac{\partial f}{\partial \mathbf{p}} = 0$$

Definition of Density and Momentum

$$\rho = ma^{-3}\int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau) \quad \rho v_i = \int \mathrm{d}^3p\ \frac{p_i}{ma} f(\mathbf{x},\mathbf{p},\tau) / \int \mathrm{d}^3p\ f(\mathbf{x},\mathbf{p},\tau)$$

Pressureless Perfect Fluid Equations

$$\begin{aligned} \delta' + \partial_j\left[(1+\delta)v_j\right] =& 0 \\ v_i' + \mathcal{H}v_i + \partial_i\phi + v_j\partial_j v_i =& 0 \end{aligned}$$

Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \boldsymbol{\nabla} \cdot \mathbf{v}$)

$$\delta' + \theta = -\,\alpha[\theta \star \delta]$$

$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\,\beta[\theta \star \theta]$$

Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \boldsymbol{\nabla} \cdot \mathbf{v}$)

$$\delta' + \theta = -\,\alpha[\theta \star \delta]$$
$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\,\beta[\theta \star \theta]$$

Perturbative Ansatz

$$\delta(\mathbf{k}) = \sum_n F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)\delta^{(1)}(\mathbf{q}_1)\ldots\delta^{(1)}(\mathbf{q}_n)$$

Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \boldsymbol{\nabla} \cdot \mathbf{v}$)

$$\begin{aligned} \delta' + \theta &= 0 \\ \theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta &= 0 \end{aligned}$$

Matter Power Spectrum

$$P_{\rm mm}(k, t) = D^2(t) P_{\rm lin}(k)$$

Perturbative Solution of EoM

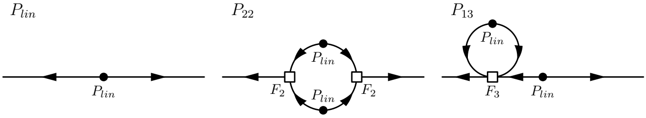
Equations of Motion for Pressureless Perfect Fluid ($\theta = \nabla \cdot \mathbf{v}$)

$$\delta' + \theta = -\alpha[\theta \star \delta]$$

$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\beta[\theta \star \theta]$$

Matter Power Spectrum

$$P_{\mathrm{mm}}(k,t) = D^2(t)P_{\mathrm{lin}}(k) + D^4(t)\left[P_{22}(k) + 2P_{13}(k)\right]$$



Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \nabla \cdot \mathbf{v}$)

$$\delta' + \theta = -\alpha[\theta \star \delta]$$

$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\beta[\theta \star \theta]$$

Matter Power Spectrum

$$P_{\mathrm{mm}}(k,t) = D^2(t)P_{\mathrm{lin}}(k) + D^4(t)\left[P_{22}(k) + 2P_{13}(k)\right]$$

UV-behaviour of loop terms

$$P_{13} \overset{k\to 0}{\propto} k^2 P_{\mathrm{lin}}(k) \underbrace{\int_{\mathbf{q}} \frac{P_{\mathrm{lin}}(q)}{q^2}}_{\sigma_d^2} \qquad P_{22} \overset{k\to 0}{\propto} k^4 \int_{\mathbf{q}} \frac{P_{\mathrm{lin}}^2(q)}{q^4}$$

UV-divergent for $n > -1$ UV-divergent for $n > 1/2$

Problems of SPT

Large Densities

on small scales densities and velocities are large $\Rightarrow$ no clear expansion parameter and no control over the size of subleading corrections

Shell Crossing

shells cross and haloes virialize $\Rightarrow$ deviation from pressureless perfect fluid

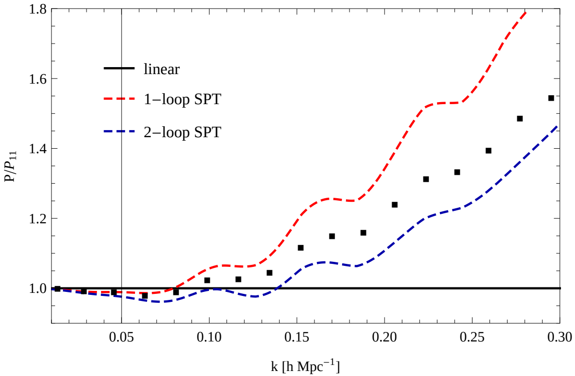
Divergencies

loops are sensitive to the UV and can diverge for particular sets of initial conditions

Performance

agreement with simulations is rather meagre and there are deviations even on very large scales

Performance of SPT for the Power Spectrum



Equations of Motion for the Long Modes

Coarse grained variables

$$f_\Lambda(\mathbf{x}) = \int \mathrm{d}^3x \; W_\Lambda(\mathbf{x}-\mathbf{x}')f(\mathbf{x}')$$

Integrating out the small scales

$$\delta'_\Lambda + \partial_j\left[(1+\delta_\Lambda)v_{\Lambda,j}\right] = 0$$

$$v'_{\Lambda,i} + \mathcal{H}v_{\Lambda,i} + \partial_i\phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j\tau_{\Lambda,ij}$$

Description of Short Scale Dynamics

$$(f_s g_s)_\Lambda = \langle f_s g_s\rangle_\Lambda + \frac{\partial\langle f_s g_s\rangle}{\partial\delta_\Lambda}\delta_\Lambda + f_s g_s - \langle f_s g_s\rangle_\Lambda + \ldots$$

Effective Stress Tensor - Parametrizing the Ignorance about small scales

$$\tau_{\Lambda,ij} = p\delta^{(K)}_{ij} + c_s^2\delta^{(K)}_{ij}\delta_\Lambda + c^2_{v,b}\delta^{(K)}_{ij}\partial_m v_{\Lambda,m} + c^2_{v,s}\left[\partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3}\delta^{(K)}_{ij}\partial_m v_{\Lambda,m}\right] + \Delta\tau_{ij}$$

$\Rightarrow$ all terms allowed by symmetries (second derivatives of the potential)

[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]

Equations of Motion for the Long Modes

Integrating out the small scales

$$\delta'_\Lambda + \partial_j \left[(1+\delta_\Lambda) v_{\Lambda,j}\right] = 0$$
$$v'_{\Lambda,i} + \mathcal{H} v_{\Lambda,i} + \partial_i \phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j \tau_{\Lambda,ij}$$

Description of Short Scale Dynamics

$$(f_s g_s)_\Lambda = \langle f_s g_s \rangle_\Lambda + \frac{\partial \langle f_s g_s \rangle}{\partial \delta_\Lambda}\delta_\Lambda + f_s g_s - \langle f_s g_s \rangle_\Lambda + \dots$$

Effective Stress Tensor - Parametrizing the Ignorance about small scales

$$\tau_{\Lambda,ij} = p\delta^{(K)}_{ij} + c_s^2 \delta^{(K)}_{ij}\delta_\Lambda + c_{v,b}^2 \delta^{(K)}_{ij}\partial_m v_{\Lambda,m} + c_{v,s}^2 \left[\partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3}\delta^{(K)}_{ij}\partial_m v_{\Lambda,m}\right] + \Delta\tau_{ij}$$

$\Rightarrow$ all terms allowed by symmetries (second derivatives of the potential)

[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]

Equations of Motion for the Long Modes

Integrating out the small scales

$$\delta'_\Lambda + \partial_j \left[(1+\delta_\Lambda) v_{\Lambda,j}\right] = 0$$
$$v'_{\Lambda,i} + \mathcal{H} v_{\Lambda,i} + \partial_i \phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j \tau_{\Lambda,ij}$$

Description of Short Scale Dynamics

$$(f_s g_s)_\Lambda = \langle f_s g_s \rangle_\Lambda + \frac{\partial \langle f_s g_s \rangle}{\partial \delta_\Lambda}\delta_\Lambda + f_s g_s - \langle f_s g_s \rangle_\Lambda + \ldots$$

Effective Stress Tensor - Parametrizing the Ignorance about small scales

$$\tau_{\Lambda,ij} = p\delta_{ij}^{(\mathrm{K})} + c_s^2 \delta_{ij}^{(\mathrm{K})}\delta_\Lambda + c_{v,b}^2 \delta_{ij}^{(\mathrm{K})}\partial_m v_{\Lambda,m} + c_{v,s}^2\left[\partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3}\delta_{ij}^{(\mathrm{K})}\partial_m v_{\Lambda,m}\right] + \Delta\tau_{ij}$$

⇒ all terms allowed by symmetries (second derivatives of the potential)

[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]

Equations of Motion for the Long Modes

Integrating out the small scales

$$\delta'_\Lambda + \partial_j \left[(1+\delta_\Lambda) v_{\Lambda,j}\right] = 0$$
$$v'_{\Lambda,i} + \mathcal{H} v_{\Lambda,i} + \partial_i \phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j \tau_{\Lambda,ij}$$

Description of Short Scale Dynamics

$$(f_s g_s)_\Lambda = \langle f_s g_s \rangle_\Lambda + \frac{\partial \langle f_s g_s \rangle}{\partial \delta_\Lambda}\delta_\Lambda + f_s g_s - \langle f_s g_s \rangle_\Lambda + \dots$$

Effective Stress Tensor - Parametrizing the Ignorance about small scales

$$\tau_{\Lambda,ij} = p\delta^{(K)}_{ij} + c_s^2 \delta^{(K)}_{ij}\delta_\Lambda + c_{v,b}^2 \delta^{(K)}_{ij}\partial_m v_{\Lambda,m} + c_{v,s}^2 \left[\partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3}\delta^{(K)}_{ij}\partial_m v_{\Lambda,m}\right] + \Delta\tau_{ij}$$

$\Rightarrow$ all terms allowed by symmetries (second derivatives of the potential)

[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]

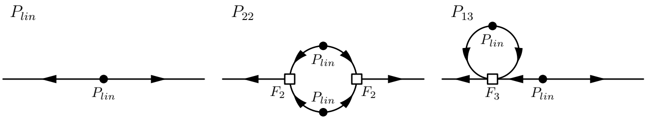
Effective Field Theory of LSS for Power Spectrum

Equations of Motion for Pressureless Perfect Fluid

$$\delta'_\Lambda + \theta_\Lambda = -\,\alpha[\theta_\Lambda \star \delta_\Lambda]$$
$$\theta'_\Lambda + \mathcal{H}\theta_\Lambda + \frac{3}{2}\mathcal{H}^2\delta_\Lambda = -\,\beta[\theta_\Lambda \star \theta_\Lambda]$$

Matter Power Spectrum

$$P_{\mathrm{mm}}(k,t) = P_{\mathrm{lin}}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda)$$



Effective Field Theory of LSS for Power Spectrum

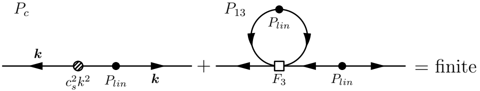
Equations of Motion including Effective Stress

$$\delta'_\Lambda + \theta_\Lambda = -\alpha[\theta_\Lambda \star \delta_\Lambda]$$

$$\theta'_\Lambda + \mathcal{H}\theta_\Lambda + \frac{3}{2}\mathcal{H}^2\delta_\Lambda = -\beta[\theta_\Lambda \star \theta_\Lambda] + \tilde{c}_s^2 k^2 \delta_\Lambda^{(1)}$$

Matter Power Spectrum

$$P_{\mathrm{mm}}(k,t) = P_{\mathrm{lin}}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)D^2(t)k^2P_{\mathrm{lin}}(k)$$



Sending cutoff to ∞ in ΛCDM

Calculation of the Power Spectrum

$$P(k) = P_{11}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)k^2P_{11}(k)$$

⇒ Result is Λ-independent

Running of the speed of sound

$$c_{s,\infty}^2 = c_s^2(\Lambda) - \frac{61}{210}\frac{1}{6\pi^2}\int_\Lambda^\infty \mathrm{d}q\ P(q)$$

$c_{s,\infty}^2$ contains errors of PT and microscopic stress

Scaling of Corrections for EdS power law

$$P_{l-\text{loop}} \propto P_{11}\Delta^l \propto P_{11}\left(\frac{k}{k_{\text{NL}}}\right)^{(n+3)l} \qquad P_{\text{stoch}} \propto \left(\frac{k}{k_{\text{NL}}}\right)^4$$

These scalings are important ingredients for forecasting – they form the theoretical error

Sending cutoff to ∞ in ΛCDM

Calculation of the Power Spectrum

$$P(k) = P_{11}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)k^2 P_{11}(k)$$

$\Rightarrow$ Result is Λ-independent

Running of the speed of sound

$$c_{s,\infty}^2 = c_s^2(\Lambda) - \frac{61}{210}\frac{1}{6\pi^2}\int_\Lambda^\infty \mathrm{d}q\ P(q)$$

$c_{s,\infty}^2$ contains errors of PT and microscopic stress

Scaling of Corrections for EdS power law

$$P_{l-\mathrm{loop}} \propto P_{11}\Delta^l \propto P_{11}\left(\frac{k}{k_{\mathrm{NL}}}\right)^{(n+3)l} \qquad P_{\mathrm{stoch}} \propto \left(\frac{k}{k_{\mathrm{NL}}}\right)^4$$

These scalings are important ingredients for forecasting – they form the theoretical error

Sending cutoff to ∞ in ΛCDM

Calculation of the Power Spectrum

$$P(k) = P_{11}(k) + P_{22}(k) + 2P_{13}(k) - 2c^2_{s,\infty} k^2 P_{11}(k)$$

⇒ Result is Λ-independent

Running of the speed of sound

$$c^2_{s,\infty} = c^2_s(\Lambda) - \frac{61}{210}\frac{1}{6\pi^2}\int_\Lambda^\infty \mathrm{d}q\ P(q)$$

$c^2_{s,\infty}$ contains errors of PT and microscopic stress

Scaling of Corrections for EdS power law

$$P_{l-\mathrm{loop}} \propto P_{11}\Delta^l \propto P_{11}\left(\frac{k}{k_{\mathrm{NL}}}\right)^{(n+3)l} \qquad P_{\mathrm{stoch}} \propto \left(\frac{k}{k_{\mathrm{NL}}}\right)^4$$

These scalings are important ingredients for forecasting – they form the theoretical error

Sending cutoff to ∞ in ΛCDM

Calculation of the Power Spectrum

$$P(k) = P_{11}(k) + P_{22}(k) + 2P_{13}(k) - 2c^2_{s,\infty} k^2 P_{11}(k)$$

$\Rightarrow$ Result is Λ-independent

Running of the speed of sound

$$c^2_{s,\infty} = c^2_s(\Lambda) - \frac{61}{210}\frac{1}{6\pi^2}\int_\Lambda^\infty \mathrm{d}q \; P(q)$$

$c^2_{s,\infty}$ contains errors of PT and microscopic stress

Scaling of Corrections for EdS power law

$$P_{l-\mathrm{loop}} \propto P_{11}\Delta^l \propto P_{11}\left(\frac{k}{k_{\mathrm{NL}}}\right)^{(n+3)l} \qquad P_{\mathrm{stoch}} \propto \left(\frac{k}{k_{\mathrm{NL}}}\right)^4$$

These scalings are important ingredients for forecasting – they form the theoretical error

Problems of SPT

Large Densities

on small scales densities and velocities are large ⇒ no clear expansion parameter and no control over the size of subleading corrections
smoothed fields are good expansion parameters ✔

Shell Crossing

shells cross and haloes virialize ⇒ deviation from pressureless perfect fluid effective stress tensor can capture shell crossing ✔

Divergencies

loops are sensitive to the UV and can diverge for particular sets of initial conditions effective corrections provide the required counterterms ✔

Performance

agreement with simulations is rather meagre and there are deviations even on very large scales

1 Introduction

2 Lagrangian Space

3 Eulerian Space

4 Summary

One-loop Lagrangian EFT

Displacement Field & Displacement Potentials

$$\boldsymbol{\Psi}_i = \mathbf{x}_i - \mathbf{q}_i \qquad \boldsymbol{\Psi} = \boldsymbol{\nabla}\phi + \boldsymbol{\nabla} \times \boldsymbol{\omega}$$

Displacement Potential with Counterterm

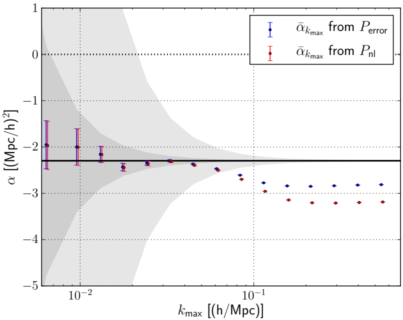
$$\phi_{1-\text{loop EFT}} = \left(1 + \alpha k^2\right)\phi^{(1)} + \phi^{(2)} + \phi^{(3)}$$

Measurement from Field Difference

$$\alpha_{\text{error}} = \frac{\phi_{\text{nl}} - \phi_{\text{3LPT}}}{k^2 \phi^{(1)}}$$

Measurement from Power Spectrum

$$\alpha_{\text{nl}} = \frac{1}{2}\frac{P_{\text{nl}} - P_{\text{3LPT}}}{k^2 P_{11}}$$



[Baldauf, Schaan, Zaldarriaga 2015]

One-loop Lagrangian EFT

Displacement Field & Displacement Potentials

$$\Psi_i = \mathbf{x}_i - \mathbf{q}_i \qquad \boldsymbol{\Psi} = \boldsymbol{\nabla}\phi + \boldsymbol{\nabla} \times \boldsymbol{\omega}$$

Displacement Potential with Counterterm

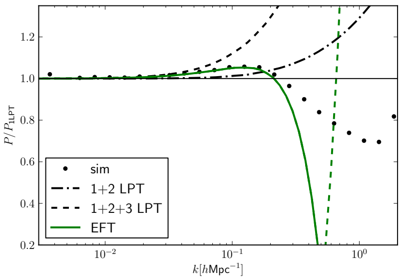
$$\phi_{1-\text{loop EFT}} = \left(1 + \alpha k^2\right)\phi^{(1)} + \phi^{(2)} + \phi^{(3)}$$

Measurement from Field Difference

$$\alpha_{\text{error}} = \frac{\phi_{\text{nl}} - \phi_{\text{3LPT}}}{k^2 \phi^{(1)}}$$

Measurement from Power Spectrum

$$\alpha_{\text{nl}} = \frac{1}{2}\frac{P_{\text{nl}} - P_{\text{3LPT}}}{k^2 P_{11}}$$



[Baldauf, Schaan, Zaldarriaga 2015]

Transfer Functions - How far can we possibly push in k?

LPT provides a basis in field space

$$\phi_{n\text{tLPT}} = a_1^{\perp}\phi^{(1)\perp} + ... + a_n^{\perp}\phi^{(n)\perp} \qquad \phi_{\text{NL}} = \phi_{n\text{tLPT}} + \phi_{\text{error}}$$

minimizing $P_{\text{error}} = \left\langle |\phi_{\text{error}}|^2 \right\rangle = \left\langle (\phi_{\text{NL}} - \phi_{n\text{tLPT}})^2 \right\rangle$

$$a_1^{\perp} = 1 + \frac{P_{13}}{P_{11}} + \alpha k^2 + \dots \qquad a_1^{\perp} = \frac{\left\langle \phi^{(1)\perp} | \phi_{\text{NL}} \right\rangle}{\left\langle \phi^{(1)\perp} | \phi^{(1)\perp} \right\rangle}$$

[Baldauf, Schaan, Zaldarriaga 2015]

Transfer Functions - How far can we possibly push in k?

LPT provides a basis in field space

$$\phi_{n\mathrm{tLPT}} = a_1^{\perp}\phi^{(1)\perp} + ... + a_n^{\perp}\phi^{(n)\perp} \qquad \phi_{\mathrm{NL}} = \phi_{n\mathrm{tLPT}} + \phi_{\mathrm{error}}$$

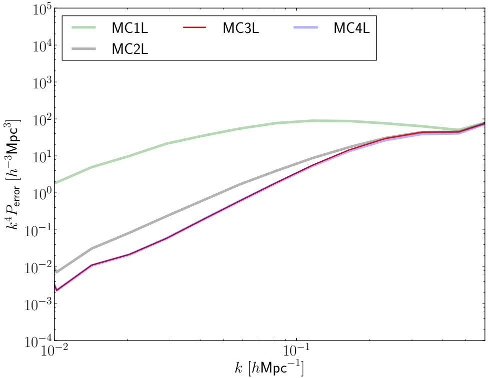
minimizing $P_{\mathrm{error}} = \left\langle |\phi_{\mathrm{error}}|^2 \right\rangle = \left\langle (\phi_{\mathrm{NL}} - \phi_{n\mathrm{tLPT}})^2 \right\rangle$

$$a_1^{\perp} = 1 + \frac{P_{13}}{P_{11}} + \alpha k^2 + \dots \qquad a_1^{\perp} = \frac{\left\langle \phi^{(1)\perp} | \phi_{\mathrm{NL}} \right\rangle}{\left\langle \phi^{(1)\perp} | \phi^{(1)\perp} \right\rangle}$$

[Baldauf, Schaan, Zaldarriaga 2015]

Stochastic Term

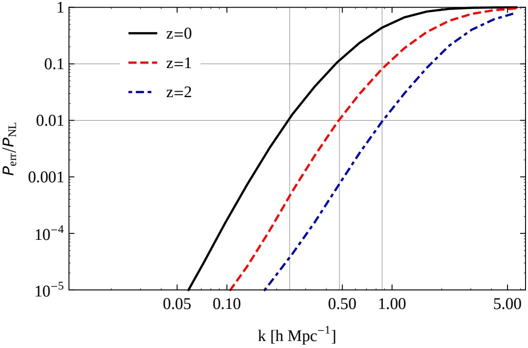
$$P_{\text{error}} = \left\langle |\phi_{\text{error}}|^2 \right\rangle = \left\langle (\phi_{\text{NL}} - \phi_{n\text{tLPT}})^2 \right\rangle \propto \left(\frac{k}{k_{\text{NL}}} \right)^4 \quad \text{with } k_{\text{NL}} \approx 0.32\, h\text{Mpc}^{-1}$$



[Baldauf, Schaan, Zaldarriaga 2015]

Stochastic Term in the Density Power Spectrum

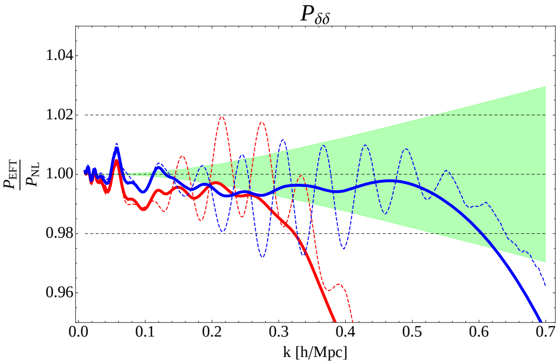
$$P_{\text{stoch}} = \langle \delta_{\text{NL}} - \delta_{\text{PT}} | \delta_{\text{NL}} - \delta_{\text{PT}} \rangle \quad \Rightarrow \quad P_{\text{NL}} = P_{\text{PT}} + P_{\text{stoch}}$$



$$k_{1\%} = 0.25 \; h\text{Mpc}^{-1} \text{ at } z = 0$$

[Baldauf, Schaan, Zaldarriaga 2015]

Promised percent accuracy to $k = 0.6\ h\mathrm{Mpc}^{-1}$ ...



[Senatore, Zaldarriaga 2014]

1 Introduction

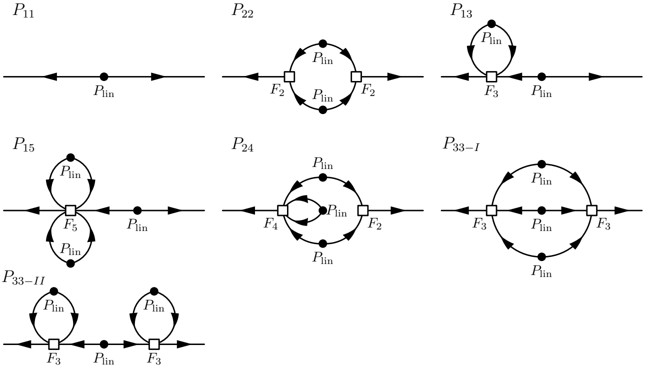
2 Lagrangian Space

3 Eulerian Space

4 Summary

Two-Loop Power Spectrum in SPT

$$P = P_{11} + \underbrace{P_{22} + 2P_{13}}_{1-loop} + \underbrace{2P_{15} + 2P_{24} + P_{33-I} + P_{33-II}}_{2-loop}$$



[Baldauf, Mercolli, Zaldarriaga 2015]

Principled approach to higher loops and *n*-point functions

Parametrize stress tensor to third order in the long modes

$$
\begin{aligned}
\tau_{ij}^{(1)} =& \epsilon_1^{(1)} \delta_{ij}^{(K)} \partial^2\phi + \epsilon_2^{(2)} \partial_i\partial_j\phi \\
\tau_{ij}^{(2)} =& \epsilon_1^{(2)} \delta_{ij}^{(K)} (\partial^2\phi)^2 + \epsilon_2^{(2)} \partial_i\partial_j\phi\, \partial^2\phi + \epsilon_3^{(2)} \partial_i\partial_l\phi\, \partial_l\partial_j\phi + \epsilon_4^{(2)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi \\
\tau_{ij}^{(3)} =& \epsilon_1^{(3)} \delta_{ij}^{(K)} (\partial^2\phi)^3 + \epsilon_2^{(3)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial^2\phi + \epsilon_3^{(3)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_m\partial_n\phi\, \partial_n\partial_l\phi \\
&+ \epsilon_4^{(3)} (\partial^2\phi)^2 \partial_i\partial_j\phi + \epsilon_5^{(3)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial_i\partial_j\phi \\
&+ \epsilon_6^{(3)} \partial_i\partial_l\phi\, \partial_l\partial_m\phi\, \partial_m\partial_j\phi + \epsilon_7^{(3)} \partial_i\partial_l\phi\, \partial_l\partial_j\phi\, \partial^2\phi
\end{aligned}
$$

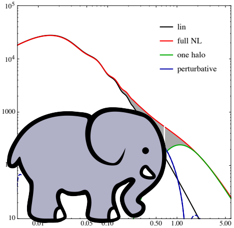
Principled approach to higher loops and *n*-point functions

Parametrize stress tensor to third order in the long modes

$$\tau_{ij}^{(1)} = \epsilon_1^{(1)} \delta_{ij}^{(K)} \partial^2\phi + \epsilon_2^{(2)} \partial_i\partial_j\phi$$

$$\tau_{ij}^{(2)} = \epsilon_1^{(2)} \delta_{ij}^{(K)} (\partial^2\phi)^2 + \epsilon_2^{(2)} \partial_i\partial_j\phi\, \partial^2\phi + \epsilon_3^{(2)} \partial_i\partial_l\phi\, \partial_l\partial_j\phi + \epsilon_4^{(2)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi$$

$$\begin{aligned}\tau_{ij}^{(3)} =& \epsilon_1^{(3)} \delta_{ij}^{(K)} (\partial^2\phi)^3 + \epsilon_2^{(3)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial^2\phi + \epsilon_3^{(3)} \delta_{ij}^{(K)} \partial_l\partial_m\phi\, \partial_m\partial_n\phi\, \partial_n\partial_l\phi \\ &+ \epsilon_4^{(3)} (\partial^2\phi)^2 \partial_i\partial_j\phi + \epsilon_5^{(3)} \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial_i\partial_j\phi \\ &+ \epsilon_6^{(3)} \partial_i\partial_l\phi\, \partial_l\partial_m\phi\, \partial_m\partial_j\phi + \epsilon_7^{(3)} \partial_i\partial_l\phi\, \partial_l\partial_j\phi\, \partial^2\phi\end{aligned}$$



With four parameters I can fit an elephant,
and with five I can make him wiggle his trunk.
John von Neumann

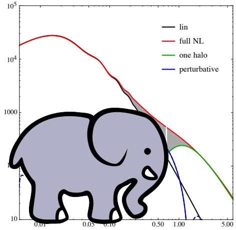
Principled approach to higher loops and *n*-point functions

Parametrize stress tensor to third order in the long modes

$$\tau^{(1)}_{ij} = \epsilon^{(1)}_1 \delta^{(K)}_{ij} \partial^2\phi + \epsilon^{(2)}_2 \partial_i\partial_j\phi$$

$$\tau^{(2)}_{ij} = \epsilon^{(2)}_1 \delta^{(K)}_{ij} (\partial^2\phi)^2 + \epsilon^{(2)}_2 \partial_i\partial_j\phi\, \partial^2\phi + \epsilon^{(2)}_3 \partial_i\partial_l\phi\, \partial_l\partial_j\phi + \epsilon^{(2)}_4 \delta^{(K)}_{ij} \partial_l\partial_m\phi\, \partial_l\partial_m\phi$$

$$\begin{aligned}\tau^{(3)}_{ij} =& \epsilon^{(3)}_1 \delta^{(K)}_{ij} (\partial^2\phi)^3 + \epsilon^{(3)}_2 \delta^{(K)}_{ij} \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial^2\phi + \epsilon^{(3)}_3 \delta^{(K)}_{ij} \partial_l\partial_m\phi\, \partial_m\partial_n\phi\, \partial_n\partial_l\phi \\ &+ \epsilon^{(3)}_4 (\partial^2\phi)^2 \partial_i\partial_j\phi + \epsilon^{(3)}_5 \partial_l\partial_m\phi\, \partial_l\partial_m\phi\, \partial_i\partial_j\phi \\ &+ \epsilon^{(3)}_6 \partial_i\partial_l\phi\, \partial_l\partial_m\phi\, \partial_m\partial_j\phi + \epsilon^{(3)}_7 \partial_i\partial_l\phi\, \partial_l\partial_j\phi\, \partial^2\phi\end{aligned}$$

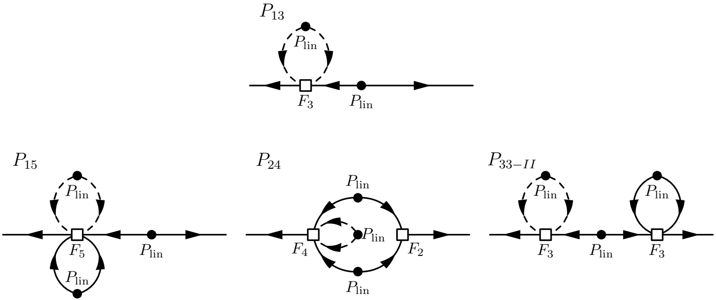


With four parameters I can fit an elephant,
and with five I can make him wiggle his trunk.
John von Neumann

Let's be less general and more pragmatic
→ focus on the UV-sensitivity of SPT

Two Loop Regularization

most UV-sensitive diagrams: single-hard diagrams containing an isolated power spectrum in the loop - all proportional to σ_d^2

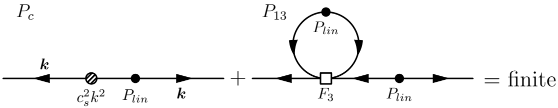


[Baldauf, Mercolli, Zaldarriaga 2015]

UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

$$P_{13}^{q_1\to\infty} = -k^2 P_{\rm lin} \frac{61}{630} \int_{\mathbf{q}} \frac{P_{\rm lin}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\rm lin}$$



Two Loop UV-Sensitivity

$$P_{15}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{24}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{33-II}^{q_1\to\infty} \propto \sigma_d^2$$

One Parameter Ansatz

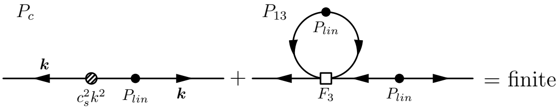
$$P_{\rm ctr} \propto c_s^2 \left[2P_{13}^{q_1\to\infty} + 2\bar{P}_{15}^{q_1\to\infty} + 2P_{24}^{q_1\to\infty} + P_{33-II}^{q_1\to\infty}\right]$$

[Baldauf, Mercolli, Zaldarriaga 2015]

UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

$$P_{13}^{q_1\to\infty} = -k^2 P_{\rm lin} \frac{61}{630} \int_{\mathbf{q}} \frac{P_{\rm lin}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\rm lin}$$



Two Loop UV-Sensitivity

$$P_{15}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{24}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{33-II}^{q_1\to\infty} \propto \sigma_d^2$$

One Parameter Ansatz

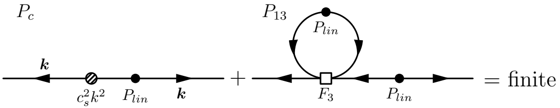
$$P_{\rm ctr} \propto c_s^2 \left[2P_{13}^{q_1\to\infty} + 2\bar{P}_{15}^{q_1\to\infty} + 2P_{24}^{q_1\to\infty} + P_{33-II}^{q_1\to\infty}\right]$$

[Baldauf, Mercolli, Zaldarriaga 2015]

UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

$$P_{13}^{q_1\to\infty} = -k^2 P_{\text{lin}} \frac{61}{630} \int_{\mathbf{q}} \frac{P_{\text{lin}}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\text{lin}}$$



Two Loop UV-Sensitivity

$$P_{15}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{24}^{q_1\to\infty} \propto \sigma_d^2 \quad P_{33-II}^{q_1\to\infty} \propto \sigma_d^2$$

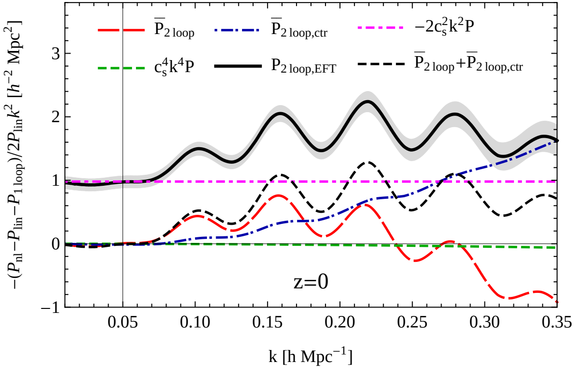
One Parameter Ansatz

$$P_{\text{ctr}} \propto c_s^2 \left[2P_{13}^{q_1\to\infty} + 2\bar{P}_{15}^{q_1\to\infty} + 2P_{24}^{q_1\to\infty} + P_{33-II}^{q_1\to\infty} \right]$$

[Baldauf, Mercolli, Zaldarriaga 2015]

Where can one measure c_s^2 in isolation?

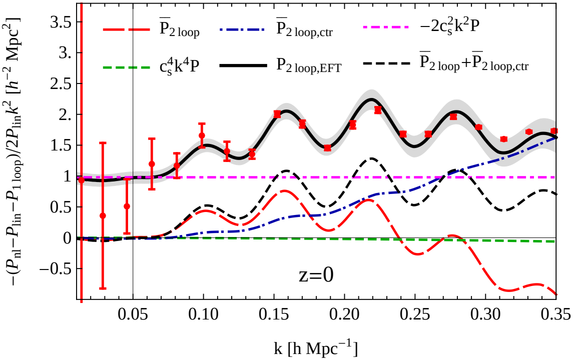
$$-\frac{P - P_{\text{lin}} - P_{\text{1loop}}}{2k^2 P_{\text{lin}}} = c_s^2 - \frac{\bar{P}_{\text{2loop}} + P_{\text{ctr,2loop}}}{2k^2 P_{\text{lin}}}$$



[Baldauf, Mercolli, Zaldarriaga 2015]

Where can one measure c_s^2 in isolation?

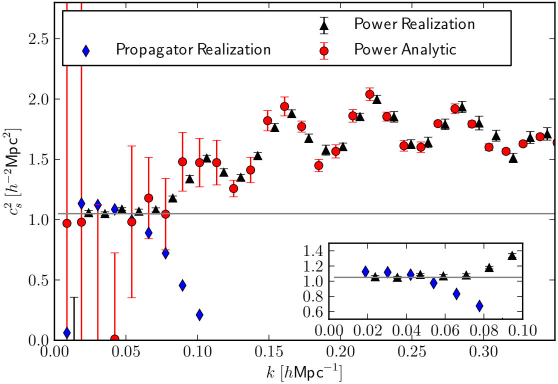
$$-\frac{P - P_{\text{lin}} - P_{\text{1loop}}}{2k^2 P_{\text{lin}}} = c_s^2 - \frac{\bar{P}_{\text{2loop}} + P_{\text{ctr,2loop}}}{2k^2 P_{\text{lin}}}$$



[Baldauf, Mercolli, Zaldarriaga 2015]

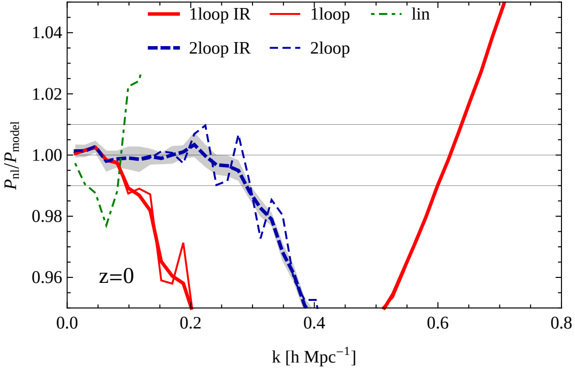
Propagator & Realization SPT Calculation

EFT can be evaluated for a given simulation initial condition & compared to the result ⇒ cancellation of cosmic variance & test at the field level



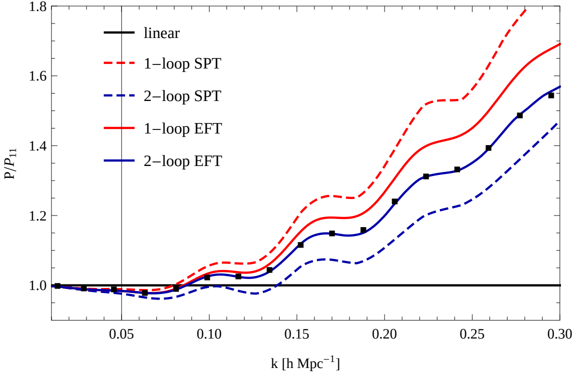
[Baldauf, Mercolli, Zaldarriaga 2015]

Performance for the Power Spectrum



[Baldauf, Mercolli, Zaldarriaga 2015], see also [Foreman, Perrier, Senatore 2015]

Performance for the Power Spectrum



[Baldauf, Mercolli, Zaldarriaga 2015], see also [Foreman, Perrier, Senatore 2015]

Problems of SPT

Large Densities

on small scales densities and velocities are large ⇒ no clear expansion parameter and no control over the size of subleading corrections
smoothed fields are good expansion parameters ✔

Shell Crossing

shells cross and haloes virialize ⇒ deviation from pressureless perfect fluid effective stress tensor can capture shell crossing ✔

Divergencies

loops are sensitive to the UV and can diverge for particular sets of initial conditions effective corrections provide the required counterterms ✔

Performance

agreement with simulations is rather meagre and there are deviations even on very large scales EFT parameters measured in the IR improve range in the UV ✔

Effective Field Theory of LSS for the Bispectrum

Equations of Motion including Effective Stress

$$\delta' + \theta = -\alpha[\theta \star \delta]$$

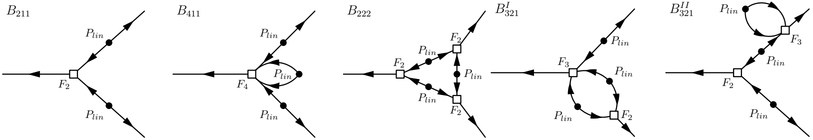
$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\beta[\theta \star \theta] + \tilde{c}_s^2 k^2 \delta^{(1)}$$

Matter Power Spectrum

$$P_{\rm mm}(k) = P_{\rm lin}(k) + P_{\rm 1loop}(k) - 2c_s^2 k^2 P_{\rm lin}(k)$$

Matter Bispectrum

$$B_{\rm mmm}(\mathbf{k}_j) = B_{\rm tree}(\mathbf{k}_j) + B_{\rm 1loop}(\mathbf{k}_j)$$



[Baldauf, Mercolli, Mirbabayi, Pajer 2014], , see also [Angulo, Foreman, Schmittfull, Senatore 2015]

Effective Field Theory of LSS for the Bispectrum

Equations of Motion including Effective Stress

$$\delta' + \theta = -\alpha[\theta \star \delta]$$

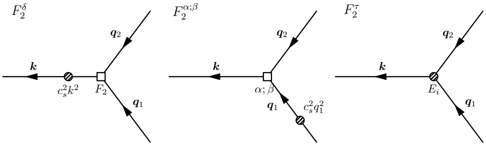
$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = -\beta[\theta \star \theta] + \tilde{c}_s^2 k^2 \delta^{(1)} + \tilde{c}_s^2 k^2 \delta^{(2)} + \tilde{e}_i E_i$$

Matter Power Spectrum

$$P_{\rm mm}(k) = P_{\rm lin}(k) + P_{\rm 1loop}(k) - 2c_s^2 k^2 P_{\rm lin}(k)$$

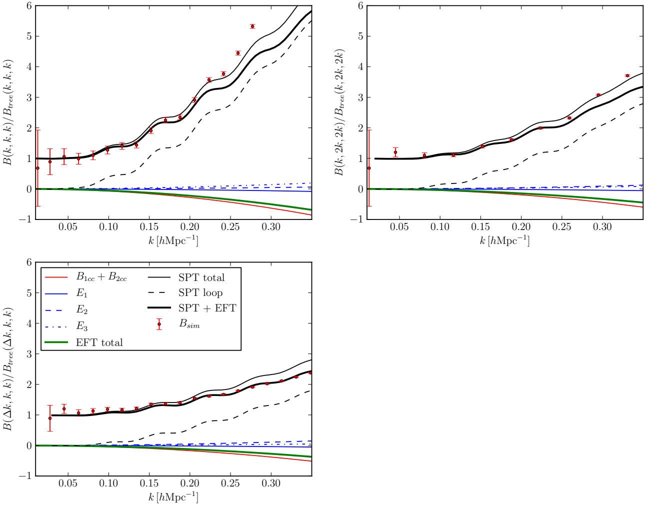
Matter Bispectrum

$$B_{\rm mmm}(\mathbf{k}_j) = B_{\rm tree}(\mathbf{k}_j) + B_{\rm 1loop}(\mathbf{k}_j) + B_{c_s}(\mathbf{k}_j) + B_{e_i}(\mathbf{k}_j)$$



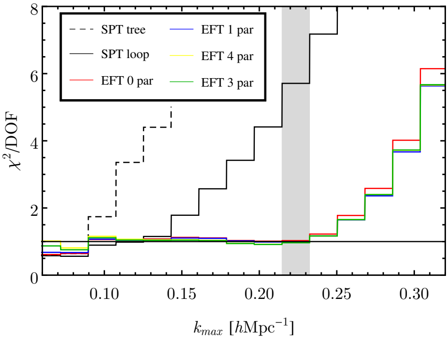
[Baldauf, Mercolli, Mirbabayi, Pajer 2014], , see also [Angulo, Foreman, Schmittfull, Senatore 2015]

Bispectrum Performance



[Baldauf, Mercolli, Mirbabayi, Pajer 2014]

Bispectrum Performance



[Baldauf, Mercolli, Mirbabayi, Pajer 2014]

1 Introduction

2 Lagrangian Space

3 Eulerian Space

4 Summary

Summary

Achievements

- detected non-zero counterterms for EFT at the field level in the low-k limit for densities and displacements
- detected the stochastic term $\Rightarrow$ leads to percent level corrections at $k = 0.3\ h\text{Mpc}^{-1}$
- One-loop Eulerian EFT $k_{1\%} = 0.1\ h\text{Mpc}^{-1}$, two-loop Eulerian EFT $k_{1\%} = 0.3\ h\text{Mpc}^{-1}$ at $z = 0$
- Improved description of the bispectrum using one-loop EFT

Outlook

- detailed measurements of the second order counterterms
- understanding of the origin of the "speed of sound" - microscopic or loop errors?

Don't get lost – Take Home Messages

1. solid constraints on the maximum range of validity of perturbative part of EFT approaches
2. solid detection of EFT corrections at the field level in the low-k
3. one-parameter two-loop EFT model for power spectrum
4. one-parameter one-loop EFT model for bispectrum
5. performance of EFT in constraining fundamental physics (Marko's talk)