Time-Sliced Perturbation Theory: A new approach to Large Scale Sctructure

Sergey Sibiryakov

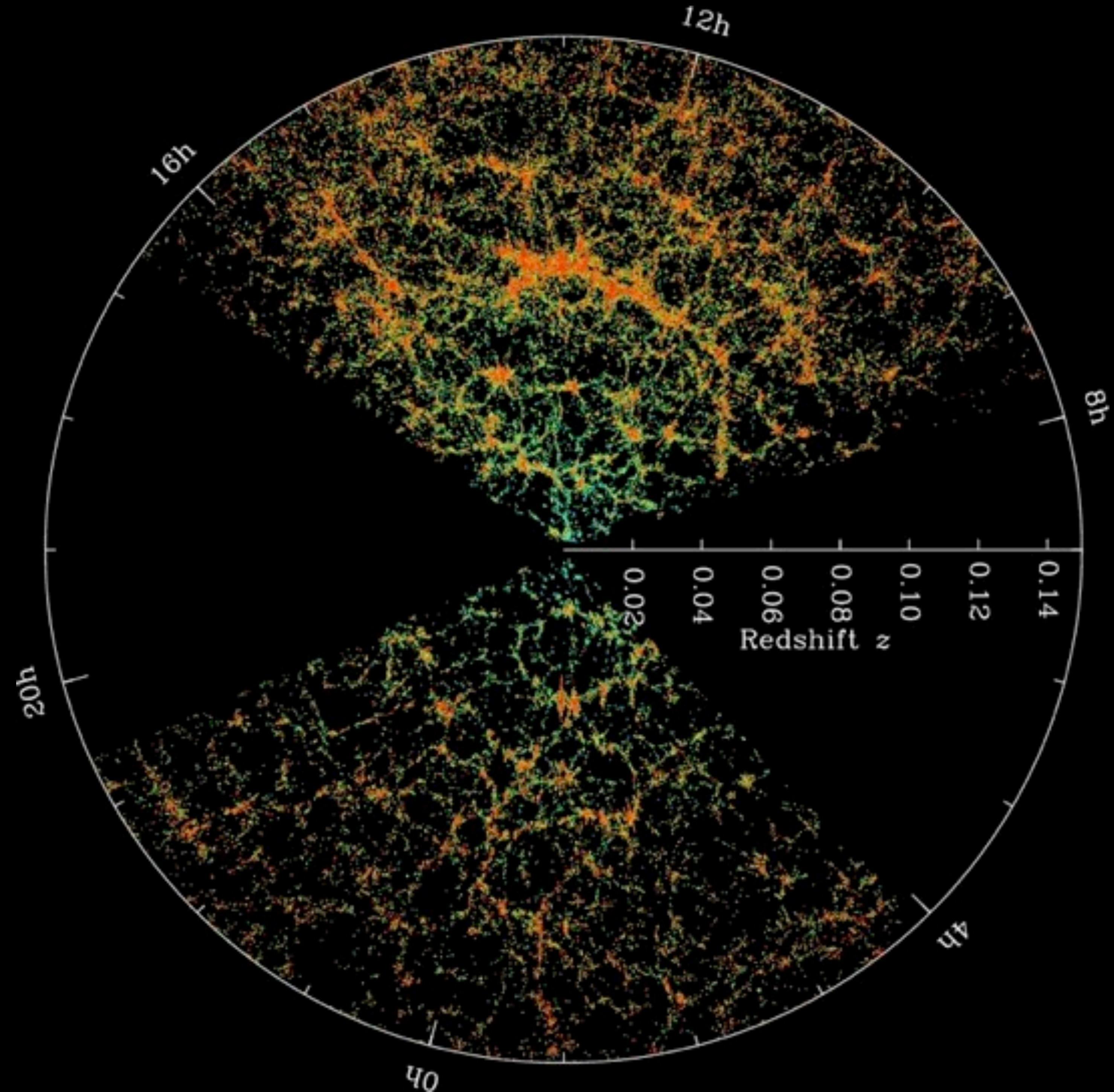
with D. Blas, M. Garny and M.M. Ivanov

arXiv: 1512.05807, 1603.xxxxx

+ work in progress

Rencontres de Moriond, La Thuile, 2016

The beautiful Universe of SDSS



Physics with LSS

- primordial non-gaussianity

  ➡ interactions in the inflationary sector

- baryon acoustic oscillations = standard ruler in the Universe

  ➡ dark energy equation of state

- evolution of perturbations

  ➡ neutrino mass

  properties of dark matter (e.g. fifth force, WDM)
  and dark energy (e.g. clustering)

Challenges of non-linear dynamics

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Newtonian approximation ($l \ll 10^4$ Mpc)

+ fluid description ($l \gg 10$ Mpc)

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla \left[(1 + \delta_\rho) \mathbf{u} \right] = 0$$

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \phi$$

$$\nabla^2 \phi = \frac{3}{2} \Omega_m(\tau) \mathcal{H}^2(\tau) \delta_\rho$$

Non-perturbative method: N-body simulations

- advantage: “exact”, goes beyond fluid description
- drawback: too costly -- cannot be used to test many theories beyond the Standard Cosmological Model

Recall that fluid description appears valid up to $k \sim 0.5 \cdot h \cdot \mathrm{Mpc}^{-1}$

➡ use perturbation theory to solve the Euler - Poisson system

Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

$$\dot{\theta}(k) + \left(\frac{3\Omega_m}{2f^2} - 1\right)\theta(k) - \frac{3\Omega_m}{2f^2}\delta_\rho(k) = \int d^3q \; \beta(q, k-q) \; \theta(q)\theta(k-q)$$

Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

$$\dot{\theta}(k) + \left(\frac{3\Omega_m}{2f^2} - 1\right)\theta(k) - \frac{3\Omega_m}{2f^2}\delta_\rho(k) = \int d^3q \; \beta(q, k-q) \; \theta(q)\theta(k-q)$$

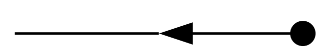
Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q \, \alpha(q, k-q) \, \theta(q)\delta_\rho(k-q)$$

$$\dot{\theta}(k) + \left(\frac{3\Omega_m}{2f^2} - 1\right)\theta(k) - \frac{3\Omega_m}{2f^2}\delta_\rho(k) = \int d^3q \, \beta(q, k-q) \, \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

$\psi^{(1)} =$  ψ_0

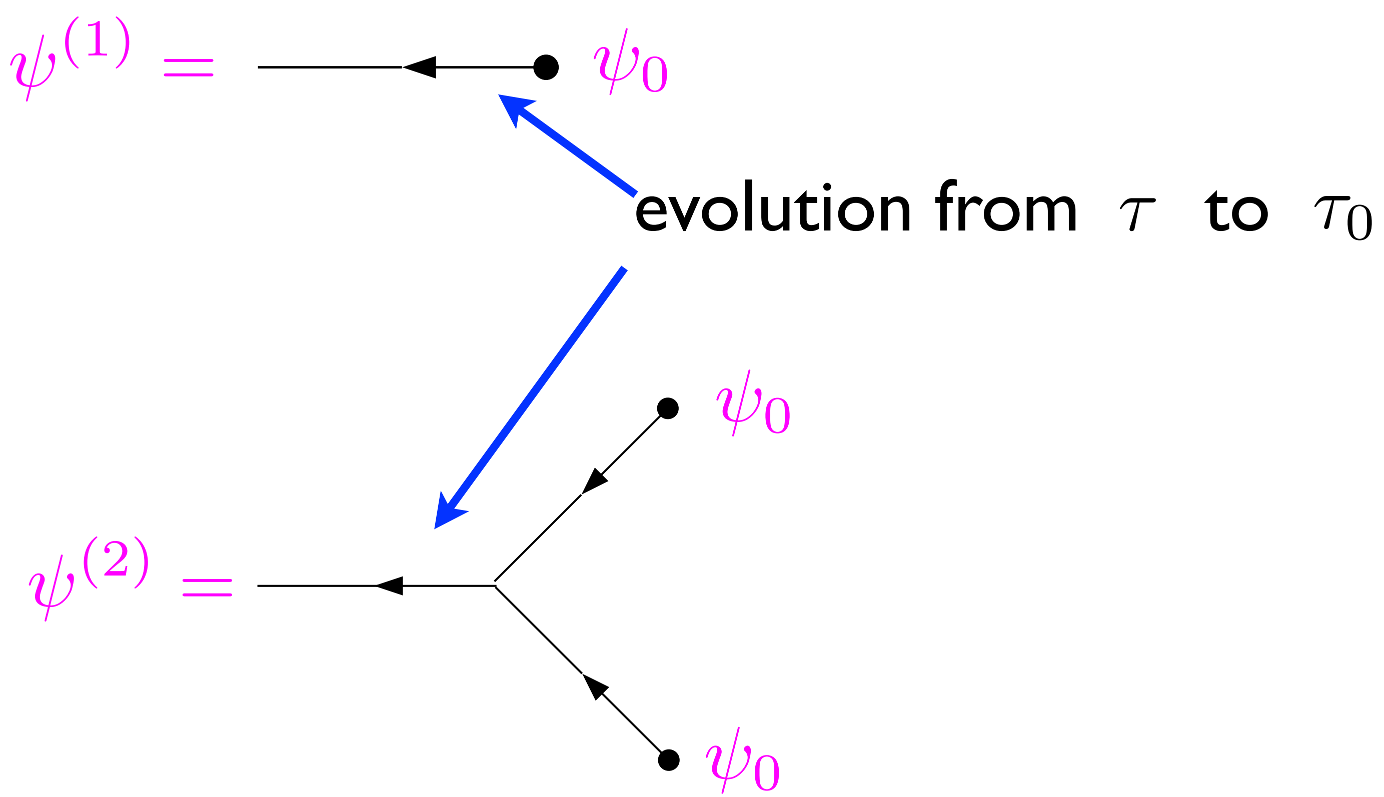
evolution from τ to τ_0

Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

$$\dot{\theta}(k) + \left(\frac{3\Omega_m}{2f^2} - 1\right)\theta(k) - \frac{3\Omega_m}{2f^2}\delta_\rho(k) = \int d^3q \; \beta(q, k-q) \; \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

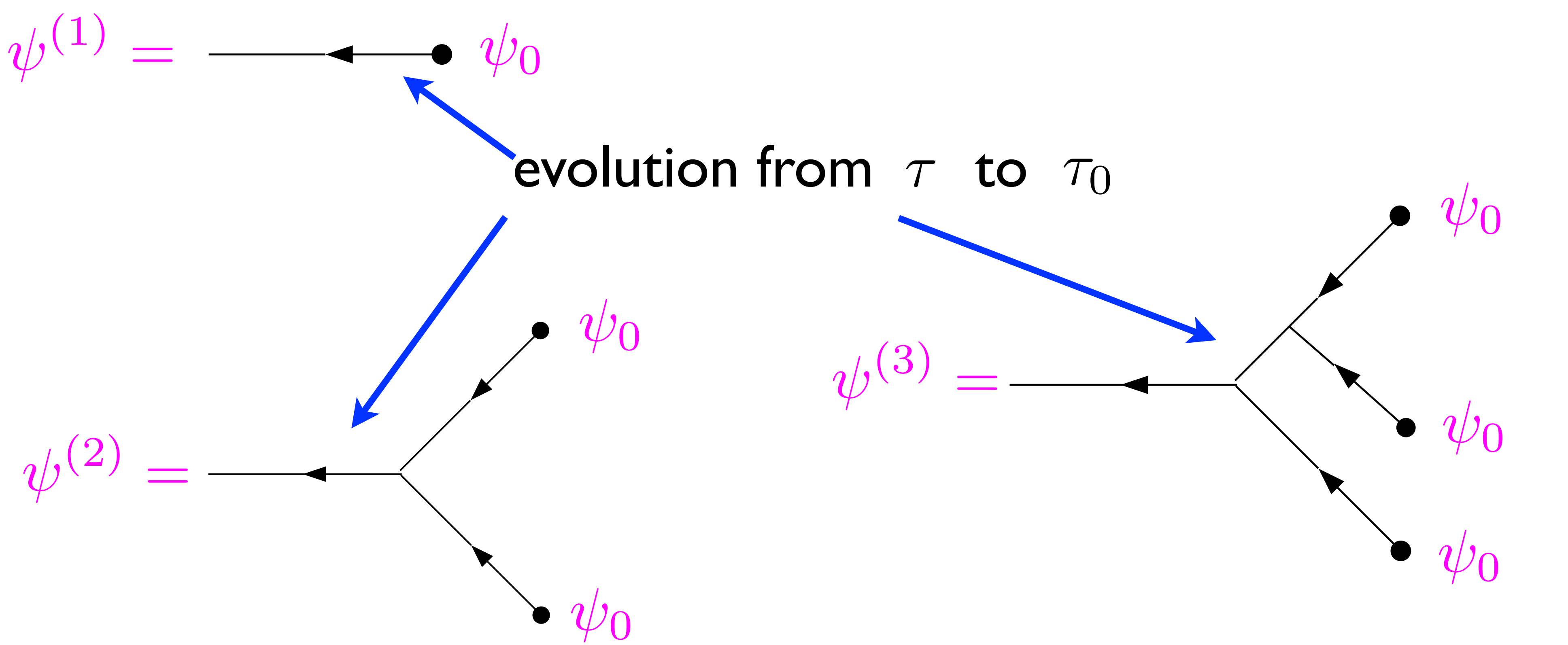


Standard perturbation theory (SPT)

$$\dot{\delta}_\rho(k) - \theta(k) = \int d^3q\, \alpha(q, k-q)\, \theta(q)\delta_\rho(k-q)$$

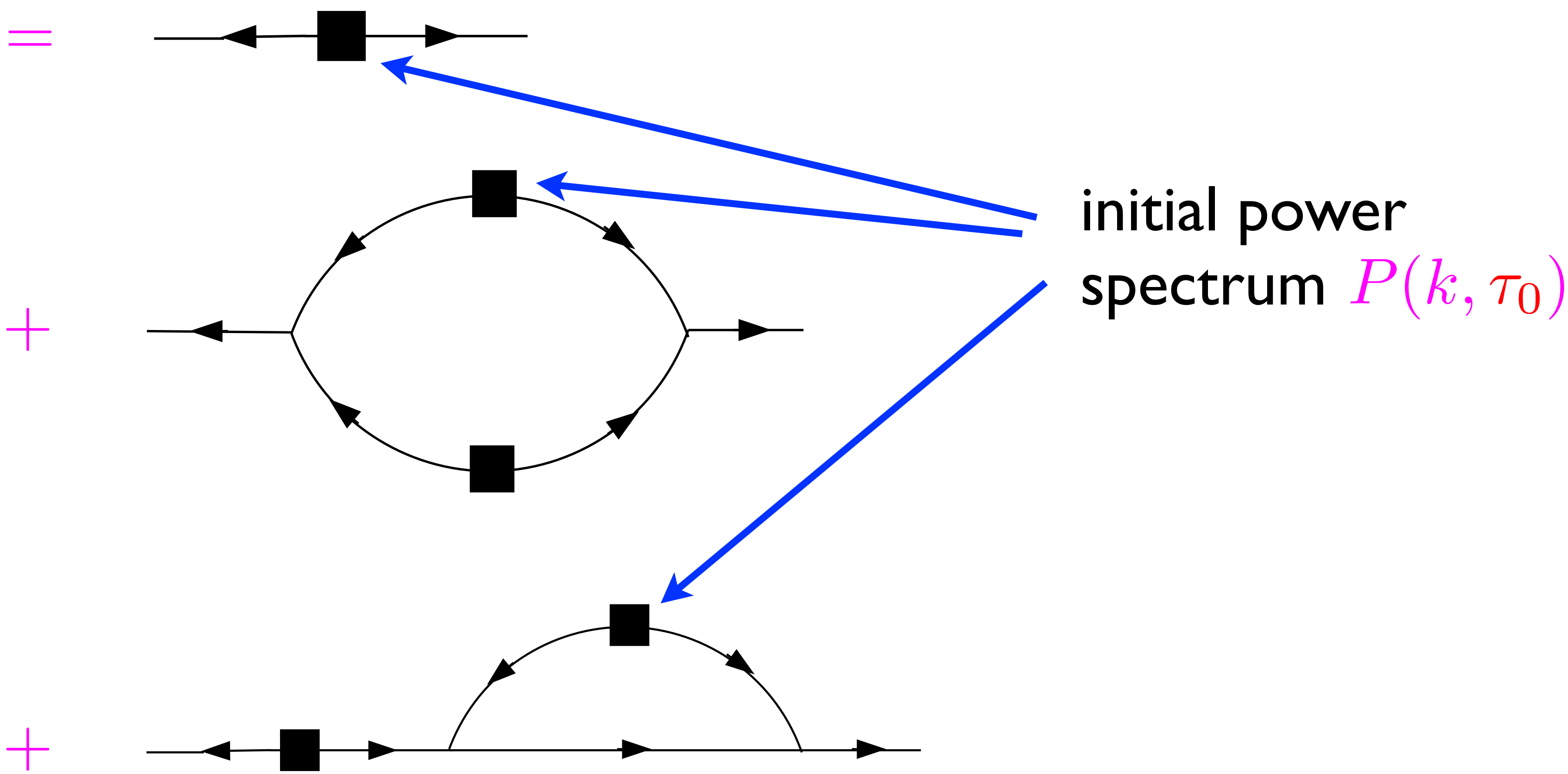
$$\dot{\theta}(k) + \left(\frac{3\Omega_m}{2f^2} - 1\right)\theta(k) - \frac{3\Omega_m}{2f^2}\delta_\rho(k) = \int d^3q\, \beta(q, k-q)\, \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$



Average over the ensemble of initial conditions:

$$\langle \psi(k_1, \tau) \psi(k_2, \tau) \rangle = \langle \psi^{(1)} \psi^{(1)} \rangle + \langle \psi^{(2)} \psi^{(2)} \rangle + 2 \langle \psi^{(1)} \psi^{(3)} \rangle + \ldots =$$



Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

➡ EFT of LSS *Carrasco, Hertzberg, Senatore (2012)*

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

➡ EFT of LSS *Carrasco, Hertzberg, Senatore (2012)*

Complications:

- counterterms have non-local time-dependence

Abolhasani, Mirbabayi, Pajer (2015)

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

➡ EFT of LSS *Carrasco, Hertzberg, Senatore (2012)*

Complications:

- counterterms have non-local time-dependence

Abolhasani, Mirbabayi, Pajer (2015)

- treatment of stochastic terms is unclear

Problems of SPT

"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

➡ individual loop diagrams diverge at small momenta

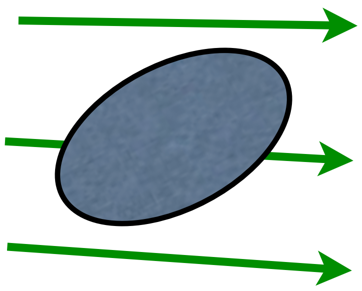
When summed, the divergences cancel in equal-time correlators

Problems of SPT

"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

→ individual loop diagrams diverge at small momenta

When summed, the divergences cancel in equal-time correlators



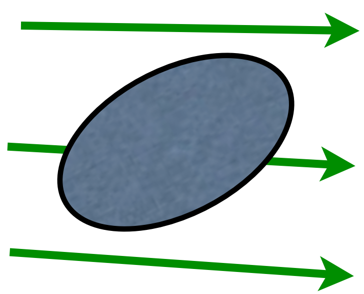
overdensity is moved by an almost homogeneous flow → accumulation of the effect with time

Problems of SPT

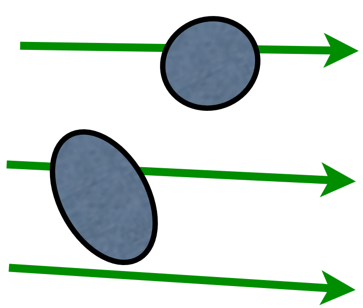
"Infrared" Kernels α , β in the e.o.m.'s behave as $1/q$

→ individual loop diagrams diverge at small momenta

When summed, the divergences cancel in equal-time correlators



overdensity is moved by an almost homogeneous flow → accumulation of the effect with time



two overdensities will move (almost) identically → cancellation in equal-time correlators

TSPT: time-sliced perturbation theory

Main ideas: Focus on equal-time correlators

Evolve the whole statistical ensemble in time

TSPT: time-sliced perturbation theory

Main ideas: Focus on equal-time correlators

Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Rightarrow \quad \psi(\tau;\psi_0)$$

TSPT: time-sliced perturbation theory

Main ideas: Focus on equal-time correlators

Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m$$

$$\psi(\tau;\psi_0)$$

SPT: $\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$

$$\Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$$

TSPT: time-sliced perturbation theory

Main ideas: Focus on equal-time correlators

Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Rightarrow \quad \psi(\tau;\psi_0)$$

SPT: $\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$ $\qquad \Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$

TSPT: $\int d\psi \, e^{-\Gamma[\psi;\tau]}\psi^2$ $\qquad \Gamma[\psi;\tau] = \sum_n \frac{\Gamma_n(\tau)}{n!}\,\psi^n$

Two integrals must coincide

→ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

→ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{} + A_{n+1}$$

contains only $\Gamma_{n'}$ with $n' < n$

Two integrals must coincide

➡ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

➡ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1} + A_{n+1}}$$

contains only $\Gamma_{n'}$ with $n' < n$

The same logic for fields in space with the substitution:
integral $\Longrightarrow$ path integral

Generating functional for cosmological correlators

$$Z[J, J_\delta; \tau] = \int [\mathcal{D}\theta] \exp\left\{ -\Gamma[\theta;\tau] + \int \theta J + \int \delta_\rho[\theta;\tau] J_\delta \right\}$$

$$\Gamma = \frac{1}{2}\int \frac{\theta^2}{\hat{P}(k)} + \sum_{n=3}^{\infty} \frac{1}{n!} \int \Gamma_n(\tau)\theta^n$$

$$\delta_\rho = \sum_{n=1}^{\infty} \frac{1}{n!} \int K_n(\tau)\theta^n$$

TSPT - 3d Euclidean QFT vocabulary:

- Γ --- 1PI effective action
- δ_ρ --- composite source
- τ --- external parameter

Advantages

- For gaussian initial conditions the time dependence factorize

$$\Gamma = \frac{1}{g^2(\tau)}\bar{\Gamma}$$

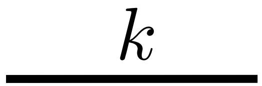
effective coupling constant

NB. For primordial NG

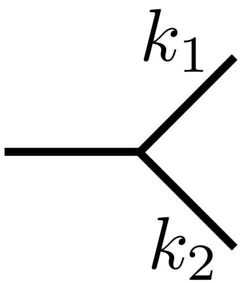
$$\Gamma = \frac{1}{g^2}\bar{\Gamma} + \frac{1}{g^3}\hat{\Gamma}$$

$\sim f_{NL} g_0$

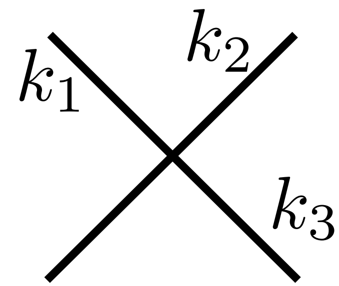
- Simplified diagrammatic technique



$$= g^2 \bar{P}(k)$$

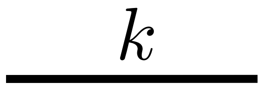


$$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$

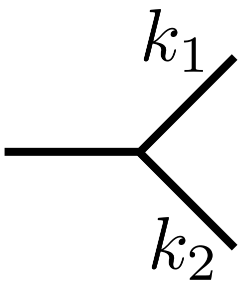


$$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$$

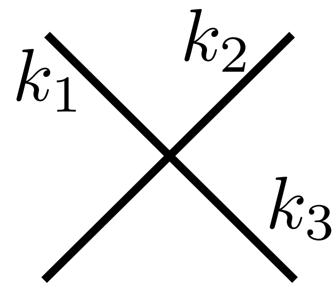
- Simplified diagrammatic technique



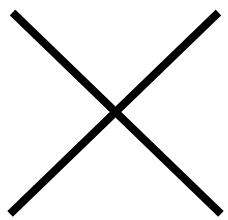
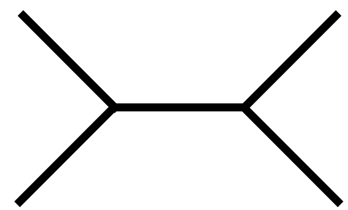
$= g^2 \bar{P}(k)$



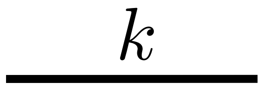
$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$



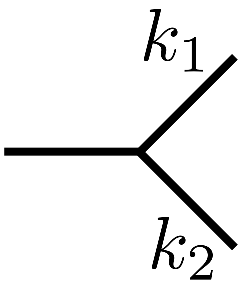
$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$

$\langle \theta\theta\theta\theta \rangle =$  + 

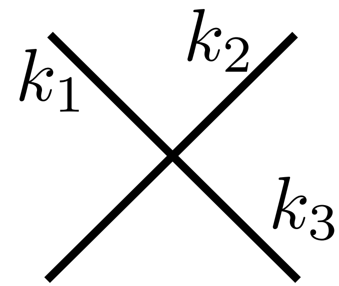
- Simplified diagrammatic technique



$= g^2 \bar{P}(k)$

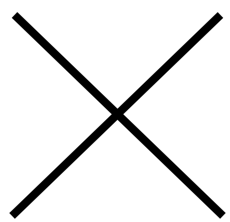


$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$

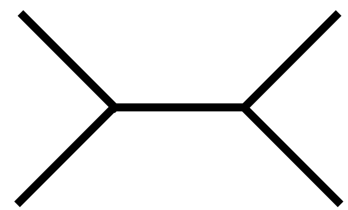


$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$

$\langle \theta\theta\theta\theta \rangle =$

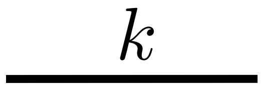


+

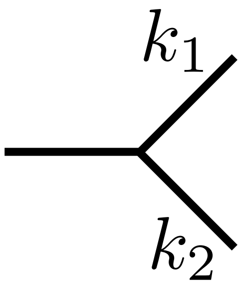


NB. Different from truncation of the Bogolyubov hierarchy

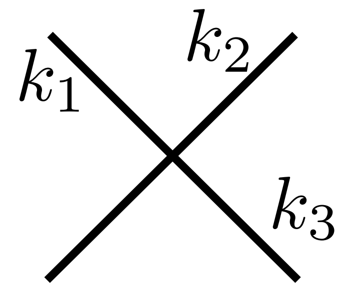
- Simplified diagrammatic technique



$= g^2 \bar{P}(k)$

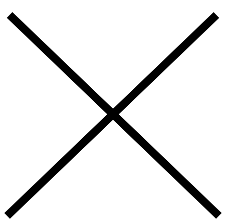


$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$

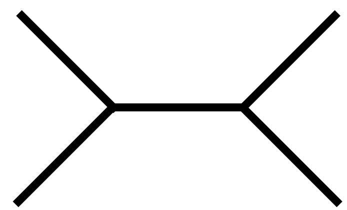


$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$

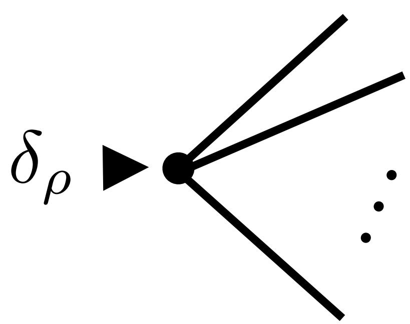
$\langle \theta\theta\theta\theta \rangle =$



+



NB. Different from truncation of the Bogolyubov hierarchy



$= K_n(k_1, k_2, \ldots, k_n)$

IR safety

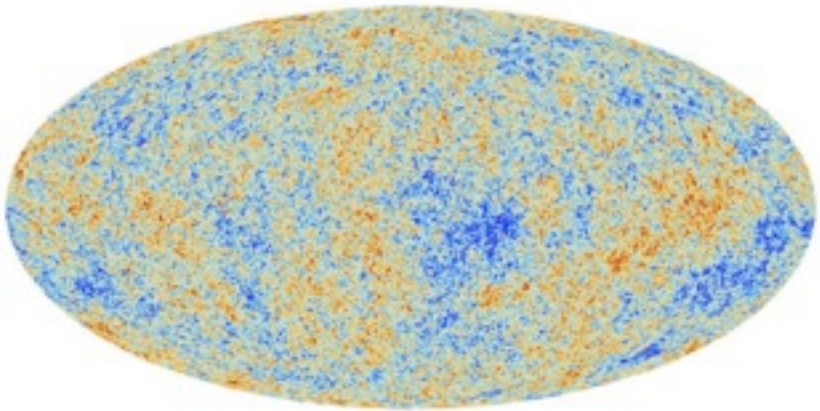
All Γ_n, K_n are finite for soft momenta

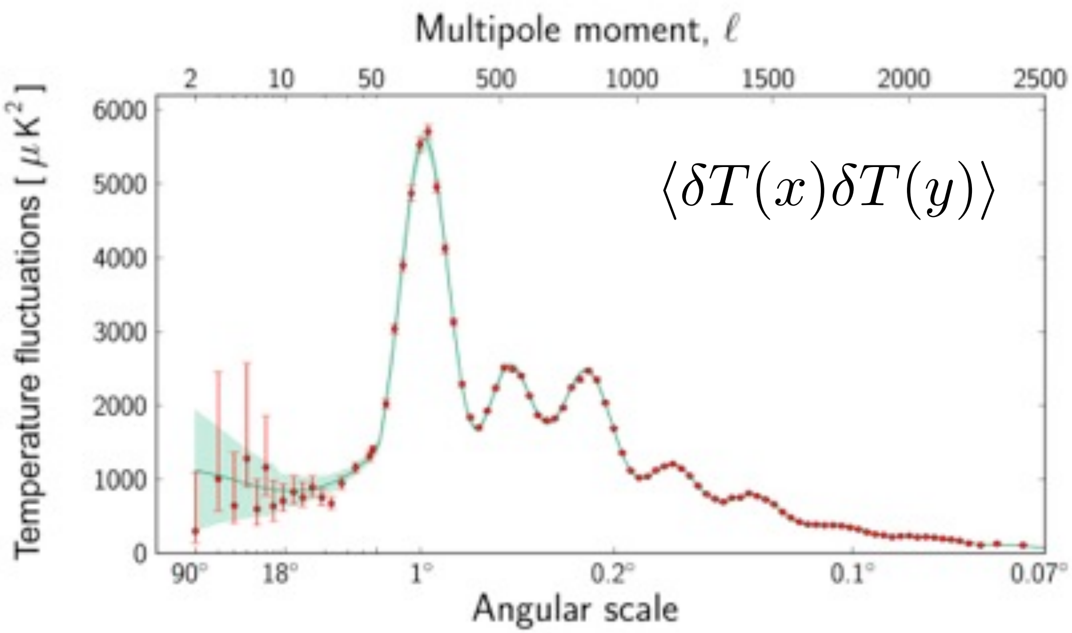
$$\lim_{\epsilon \to 0} \Gamma_n(k_1, \ldots, k_l, \epsilon q_1, \ldots, \epsilon q_{n-l}) < \infty$$

➡ no IR divergences in the individual loop diagrams

NB. Can be related to the equivalence principle / Galilean invariance of Γ through Ward identities

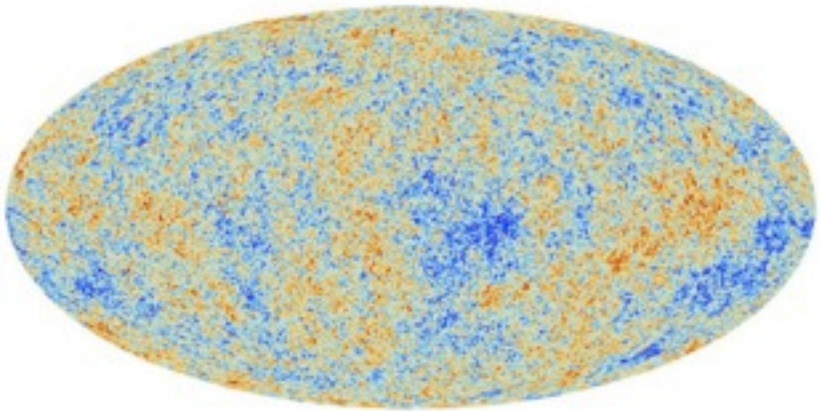
Baryon acoustic oscillations

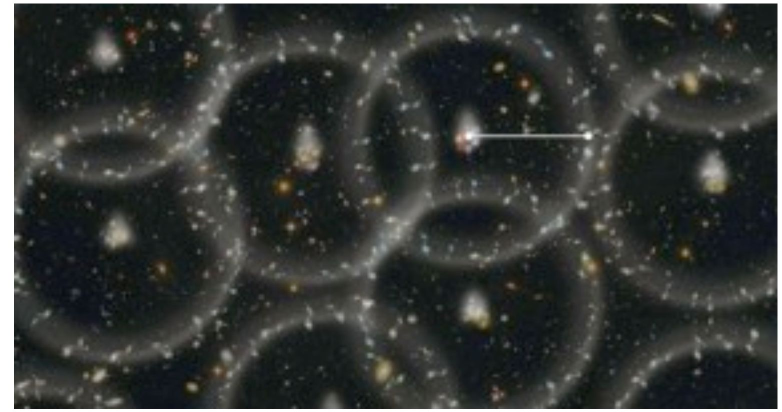


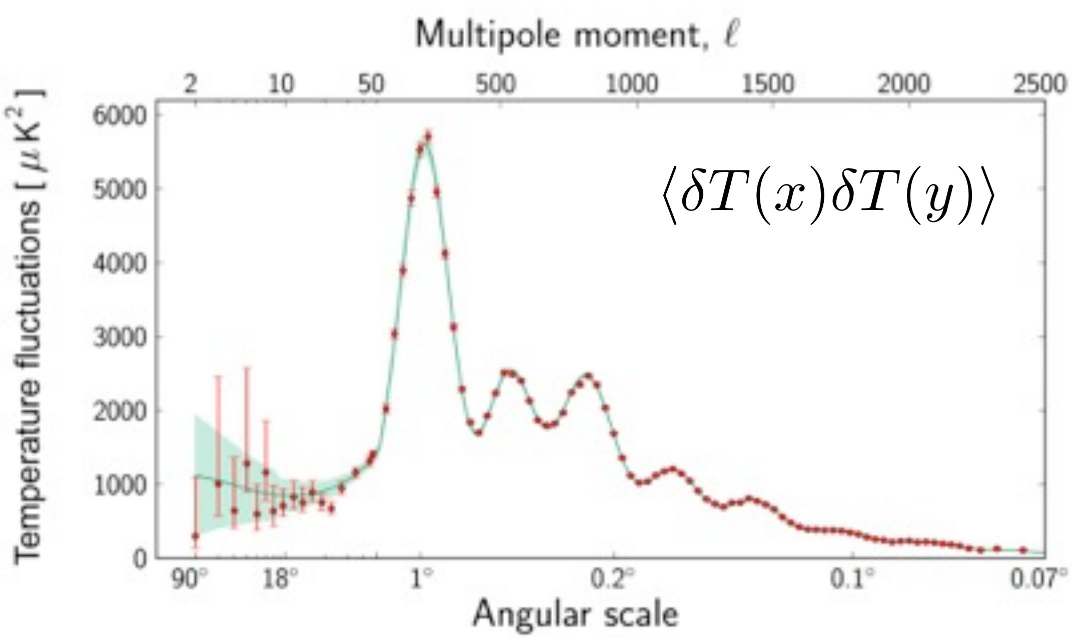


Credit: ESA and the Planck collaboration

Baryon acoustic oscillations

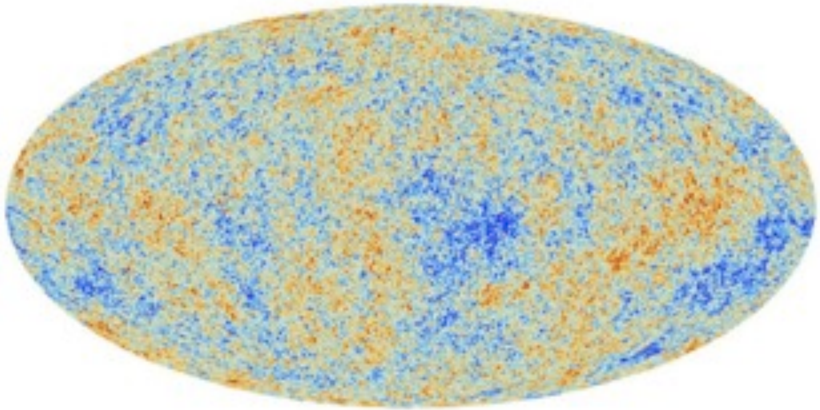


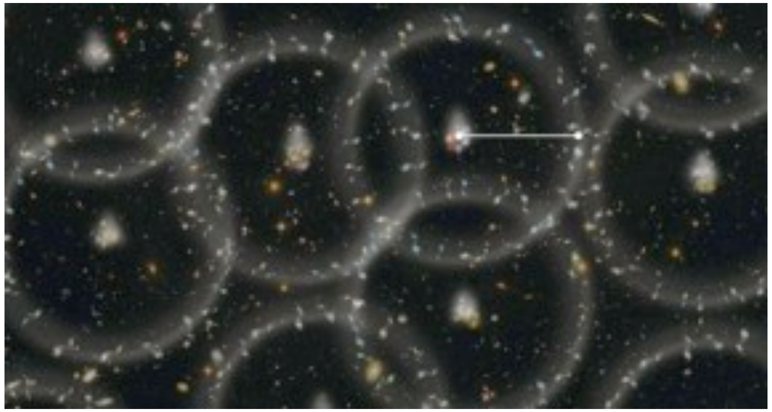


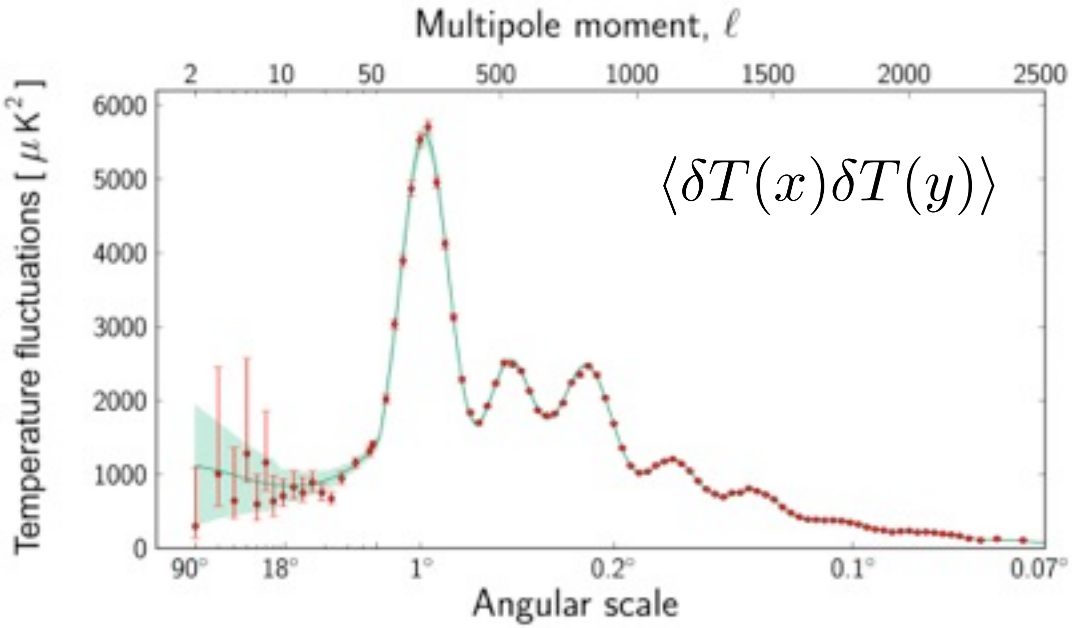


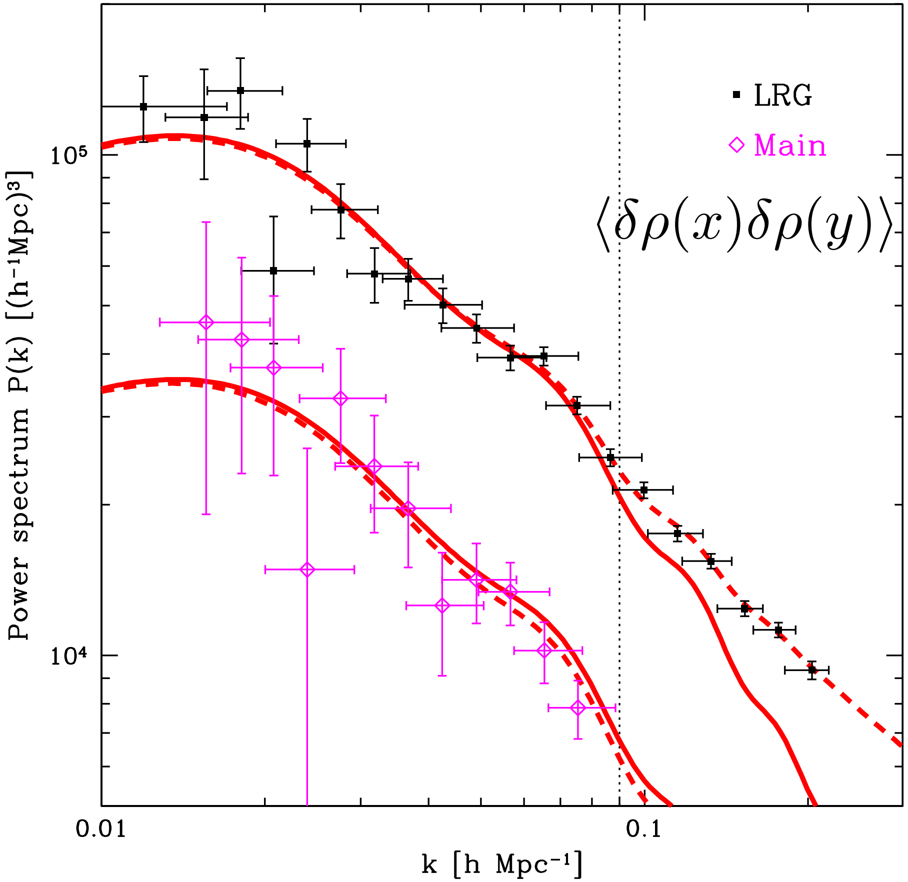
Credit: ESA and the Planck collaboration

Baryon acoustic oscillations





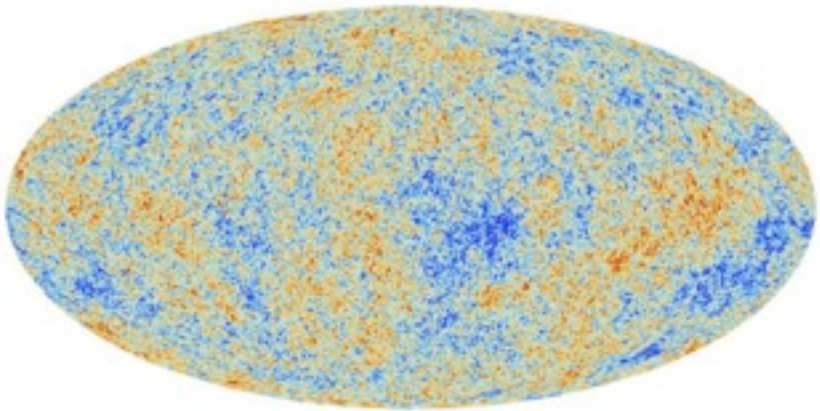


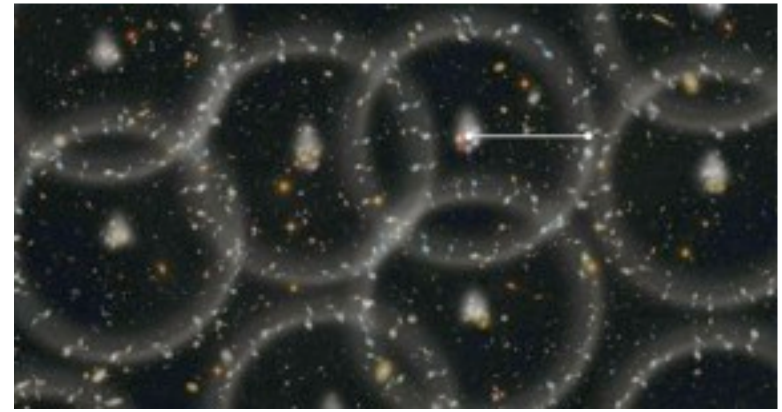


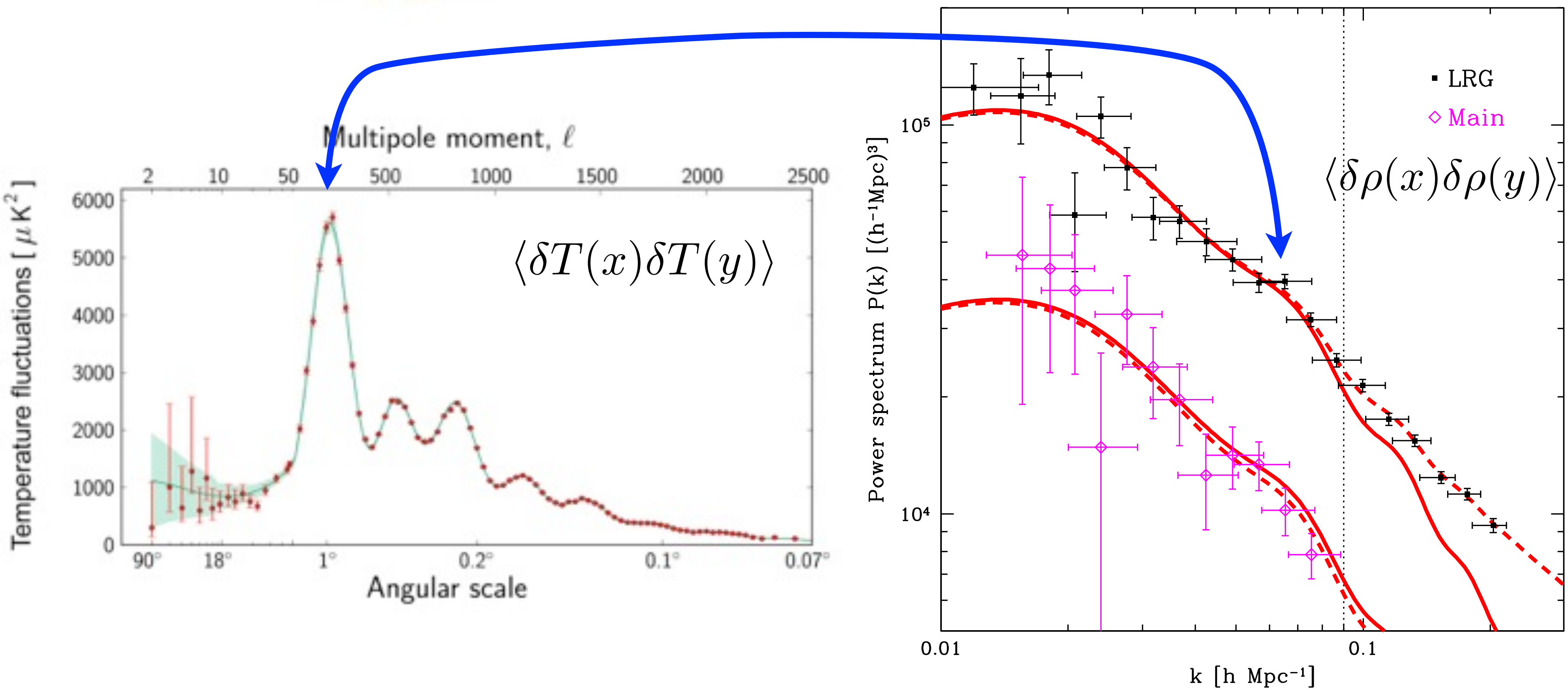
Credit: ESA and the Planck collaboration

From Tegmark et al. PRD 74, 123507

Baryon acoustic oscillations



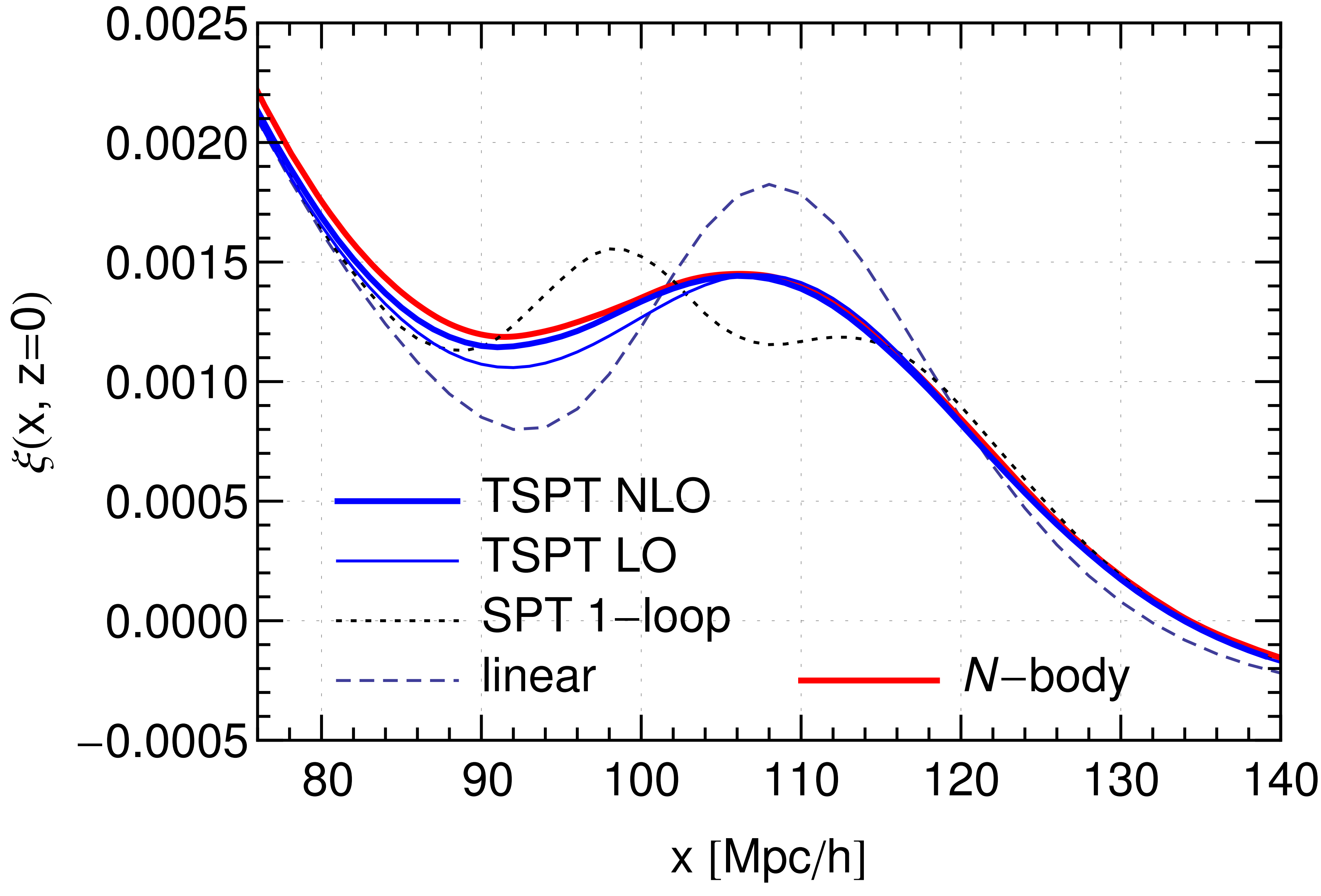




Credit: ESA and the Planck collaboration

From Tegmark et al. PRD 74, 123507

IR sensitivity of the BAO peak



IR resummation

In TSPT large IR contributions can be systematically resummed

Step 1: smooth + wiggly decomposition

$$P(k) = P_s(k) + P_w(k) \quad \Rightarrow \quad \Gamma(k) = \Gamma_s(k) + \Gamma_w(k)$$

IR resummation

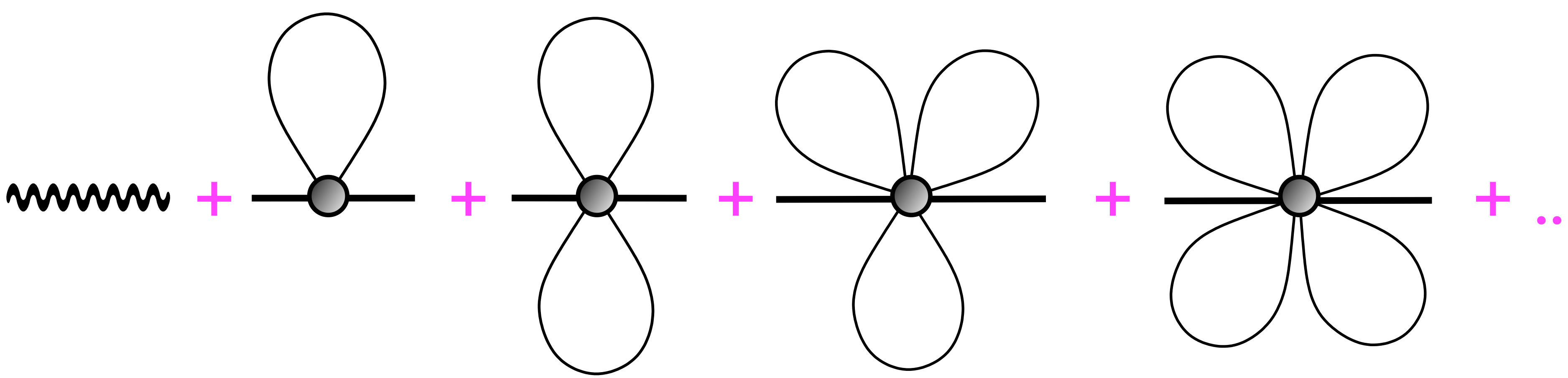
In TSPT large IR contributions can be systematically resummed

Step I: smooth + wiggly decomposition

$$P(k) = P_s(k) + P_w(k) \quad \Rightarrow \quad \Gamma(k) = \Gamma_s(k) + \Gamma_w(k)$$

Step II: identification of leading diagrams ⇒ daisies

$$P_w^{\mathrm{dressed}} =$$



Step III: add the smooth part

$$P(k) = P_s(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

BAO wavelength

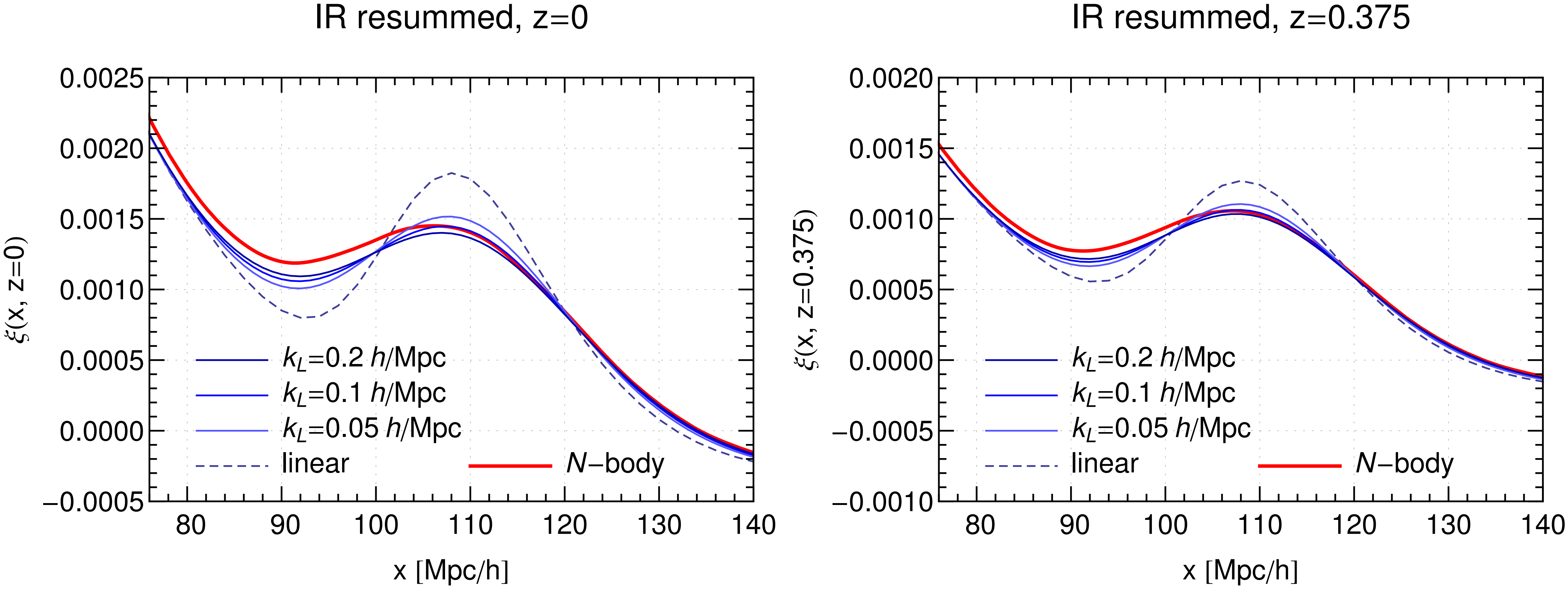
$$\Sigma_L^2 = \frac{4\pi}{3}\int_0^{k_L} dq\, P_s(q)\, \left(1 - j_0(qr_s) + 2j_2(qr_s)\right)$$

Baldauf, Mirbabayi, Simonovic, Zaldarriaga (2015)

Further developments:

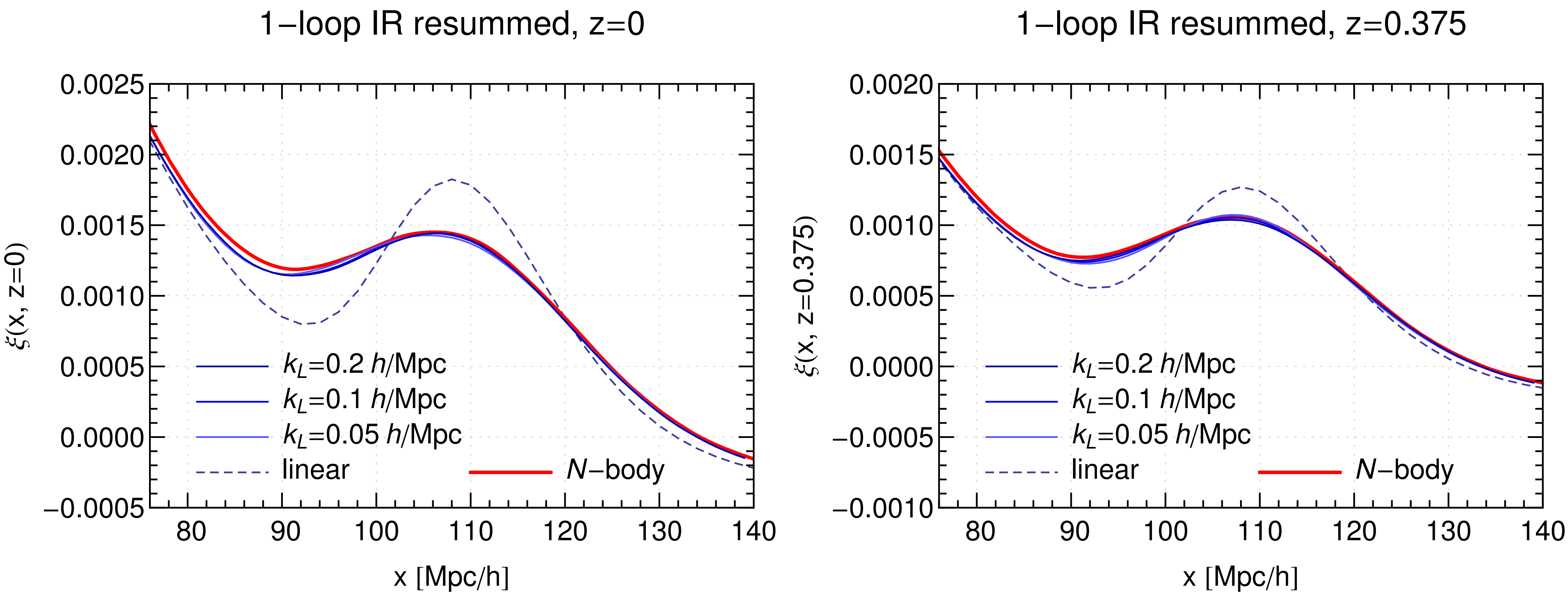
- take into account NLO IR corrections
- IR resummation of higher-point correlation functions

Comparison with N-body: Leading Order



k_L --- the IR separation scale

Comparison with N-body: NLO



k_L --- the IR separation scale

UV renormalization in TSPT

UV renormalization in TSPT

under development

UV renormalization in TSPT

under development

Introduce a hard cutoff:

$$\hat{P}(k) \mapsto \hat{P}^{\Lambda}(k) = \begin{cases} \hat{P}(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

UV renormalization in TSPT

under development

Introduce a hard cutoff:

$$\hat{P}(k) \mapsto \hat{P}^{\Lambda}(k) = \begin{cases} \hat{P}(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

Wilsonian RG:

$$\frac{d\Gamma_n^{\Lambda}}{d\Lambda} = \mathcal{F}_n[\hat{P}^{\Lambda}, \Gamma^{\Lambda}]$$

Boundary conditions = counterterms encapsulating the effects of short modes

Comparison with EFT of LSS

+ the cutoff dependence is treated systematically

+ counterterms are manifestly local in time

+ stochastic contributions are at the same footing as the dynamical ones

Comparison with EFT of LSS

+ the cutoff dependence is treated systematically

+ counterterms are manifestly local in time

+ stochastic contributions are at the same footing as the dynamical ones

- spatial locality is not manifest

→ exploit symmetries in momentum space
+ factorization ???

Summary and Outlook

- time-sliced perturbation theory (TSPT) is a promising approach to LSS in the mildly non-linear regime 20 Mpc < l < 100 Mpc
- free from IR divergences
- diagrammatic resummation of IR-enhanced contributions into the BAO feature
- UV renormalization à la Wilsonian RG

Summary and Outlook

- [x] time-sliced perturbation theory (TSPT) is a promising approach to LSS in the mildly non-linear regime
20 Mpc < l < 100 Mpc
- [x] free from IR divergences
- [x] diagrammatic resummation of IR-enhanced contributions into the BAO feature
- [x] UV renormalization à la Wilsonian RG
- [ ] classification of UV counterterms
- [ ] redshift space
- [ ] biases
- [ ] inclusion of baryons