CMB Distortions due to Peculiar Motion and Intrinsic Anomalies

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March 2016, Moriond

[1] In collaboration with M.Quartin, O.Roldan and earlier work with R.Catena, M.Liguori, A.Renzi, L.Amendola, I.Masina, C.Quercellini.
arXiv:1603.02664, , arXiv:1510.08793,
JCAP 1509 (2015) 09, 050
JCAP 1506 (2015) 06, 047
JCAP 1501 (2015) 01, 008
JCAP 1403 (2014) 019
JCAP 1309 (2013) 036

JCAP 1202 (2012) 026; JCAP 1107 (2011) 027

CMB as a test of Global Isotropy

- Is the CMB statistically Isotropic?

- What is the impact of our peculiar velocity?

  $(\beta = \frac{v}{c} = 10^{-3})$

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- Can we disentangle them?

CMB spectrum

More precisely

- $T(\hat{n}) \to a_{\ell m}$

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Hypothesis of Gaussianity and Isotropy:

- $a_{\ell m}$ random numbers from a Gaussian of width C_ℓ^{th}.
- Physics fixes $C_\ell^{th} = \langle |a_{\ell m}|^2 \rangle$
- Uncorrelated: NO preferred direction

CMB: Peculiar Velocity and Anomalies

- Our velocity $\beta \equiv \frac{v}{c}$ breaks Isotropy introducing correlations in the CMB at *all* scales

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  $T'(\hat{n}) = T(\hat{n})\gamma(1 + \beta\cos\theta) \qquad (\cos(\theta) = \hat{n}\cdot\hat{\beta})$

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- Doppler:
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- Aberration:
  $T'(\hat{n}') = T(\hat{n})$
  $\theta - \theta' \approx \beta\sin\theta$

Peebles & Wilkinson '68, Challinor & van Leeuwen 2002, Burles & Rappaport 2006

In multipole space

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  $c^-_{\ell m} = -(\ell - 1 + 1)\sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$
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- Doppler (constant), aberration grows with ℓ!
- We can measure β (Kosowsky Kahniashvili, '2011, L. Amendola, Catena, Masina, A. N., Quartin'2011, Planck XXVII, 2013.)

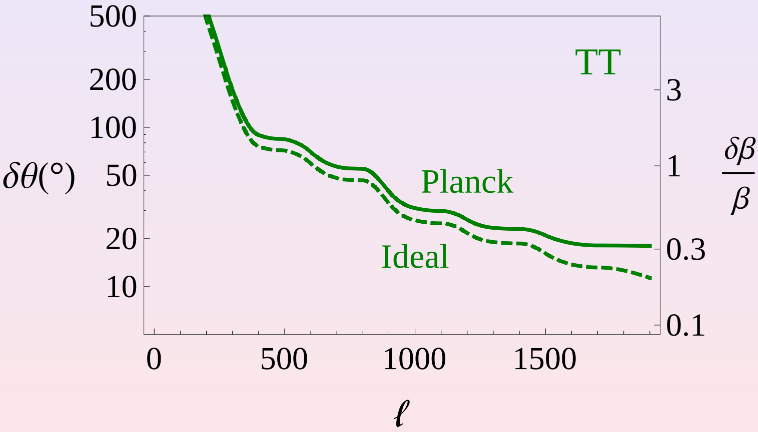
Expected sensitivity

CMB

CMB & Proper motion

Anomalies

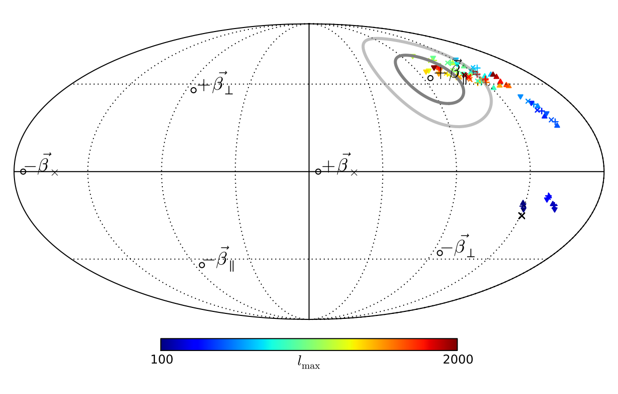
Frequency dependence



L.Amendola, R.Catena, I.Masina, A.N., M.Quartin, C.Quercellini 2011

Planck Measurement

$\beta = 384 km/s \pm 78 km/s$ (stat) $\pm 115 km/s$ (syst.)



Planck Collaboration 2013, XXVII. Doppler boosting of the CMB: *Eppur si muove*

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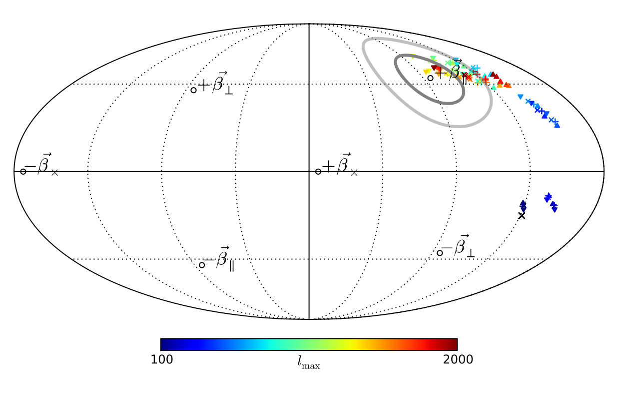

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Found both Aberration and Doppler

Is β degenerate with an Intrinsic Dipole?

- A dipolar large scale potential: $\Phi_L = \cos(\theta) f(r)$
- Produces[3] a CMB dipole Δ_1

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- It produces dipolar Lensing = Aberration
- Coefficient: generically non-degenerate with Aberration

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Testing Isotropy

- Given a map $T(\hat{n})$: mask half of the sky: $\tilde{T}(\hat{n}) = M(\hat{n})T(\hat{n})$
- We compute $\tilde{a}_{\ell m} \to \tilde{C}_\ell^M$

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Hemispherical asymmetry?

- In several papers: significant (about 3σ) hemispherical asymmetry at $\ell < \mathcal{O}(60)$

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- And also to the Planck data! (Up to which ℓ?)

  Planck Collaboration 2013, XIII. Isotropy and Statistics.

Planck asymmetry

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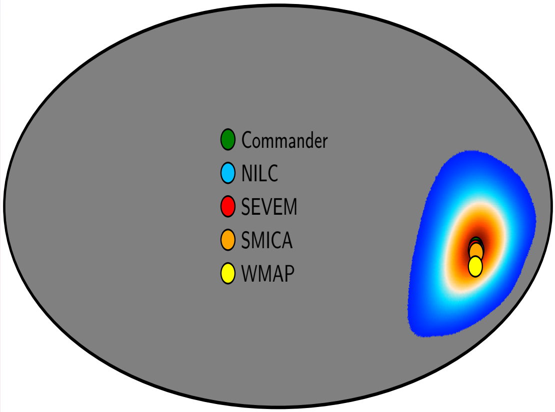
- Same as in WMAP



Hemispherical Asymmetry at high ℓ?

- A correct analysis has to include Doppler and Aberration (important at $\ell \gtrsim 1000$)

  A.N., M.Quartin & R.Catena, JCAP Apr. '13

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- We find between $2.5 - 3\sigma$ anomaly only at $\ell \lesssim 600$

  (A.N., M.Quartin & JCAP '14, Planck Collaboration 2013, XIII. Isotropy and Statistics)

Hemispherical Asymmetry due to Velocity

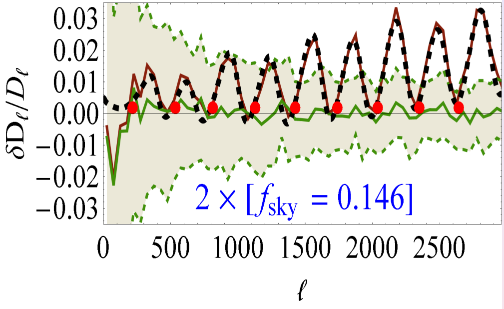

Figure: Discs along the Dipole direction

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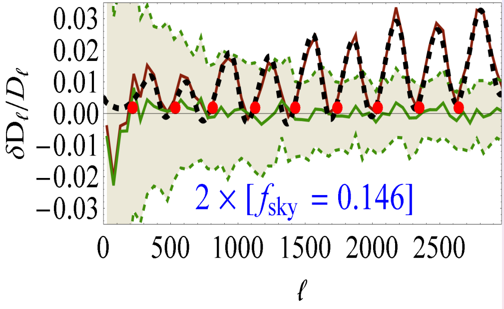

Figure: Discs along the Dipole direction

For a small disc:

$$\frac{\delta C_\ell}{C_\ell} \simeq 4\beta + 2\beta\ell C'_\ell$$

A.N., M.Quartin, R.Catena 2013

“Dipolar modulation"?

- Several authors have studied the ansatz

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- 3-σ detection of A_{mod} along max. asymm. direction (For $\ell < 60$ or $\ell < 600$)

- $A_{\rm mod}$ 60 times bigger than β! (at $\ell < 60$)

Our Results on A

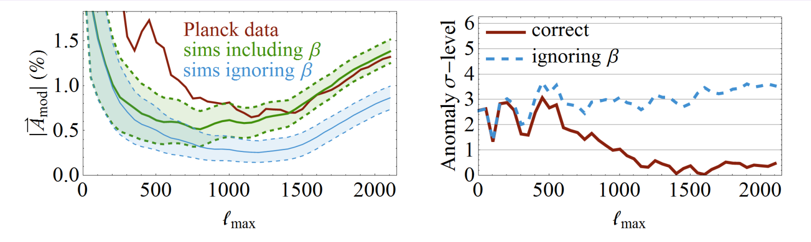

Figure: All simulations include Planck noise asymmetry.

A.N. & M.Quartin, 2014

Frequency dependence??

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- But, consider Intensity:

$$I(\nu) = \frac{2\nu^3}{e^{\frac{\nu}{T(\hat{\boldsymbol{n}})}} - 1} .$$

- Linearizing Intensity we get (WMAP, PLANCK...)

$$\Delta I(\nu, \hat{\boldsymbol{n}}) \approx \frac{2\nu^4 e^{\frac{\nu}{\nu_0}}}{T_0^2 \left(e^{\frac{\nu}{\nu_0}} - 1\right)^2} \Delta T(\hat{\boldsymbol{n}}) \equiv K \frac{\Delta T(\hat{\boldsymbol{n}})}{T_0} ,$$

Frequency dependence??

- At second order:

$$\frac{\Delta I}{K} = \frac{\Delta T(\hat{\boldsymbol{n}})}{T_0} + \left(\frac{\Delta T(\hat{\boldsymbol{n}})}{T_0}\right)^2 Q(\nu)\,,$$

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- Spurious y-distortion
- Degenerate with tSZ and primordial y-distortion
- Any T fluctuation produces this

Frequency dependence??

- Dominated by dipole Δ_1 [4]

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$$L(\nu, \hat{\boldsymbol{n}}) = \mu\Delta_1 + \frac{\delta T}{T_0} - \tilde{\beta}\mu \frac{\delta T}{T_0} + \tilde{\beta}\left(\frac{\delta T_{ab}}{T_0}\right) + \\ + \left[\left(\mu^2 - \frac{1}{3}\right)\Delta_1^2 + \frac{1}{3}\Delta_1^2 + 2\Delta_1\mu\frac{\delta T}{T_0}\right] Q(\nu) .$$

- Quadrupole (10^{-7})
- Monopole (10^{-7})
- Couplings (10^{-8})

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- Caveat : $\Delta_1 = \beta + \text{intrinsic dipole}$

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WMAP/Planck Quadrupole-Octupole alignments

Another anomaly:

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- $\hat{n}_2 \cdot \hat{n}_3 \approx 0.99$
- And also Dipole-Quadrupole-Octupole $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ aligned (*e.g.*Copi et al. '13)

Removing Doppler quadrupole

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- Using $Q_{\rm eff} \approx 1.7$ on SMICA 2013, (A.N. & M.Quartin, JCAP 2015)

  $\rightarrow 3.3\sigma$ for $\hat{n}_1 \cdot \hat{n}_2$

Planck Calibration?

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- WMAP calibrated using $\beta_{ORBITAL}$ ($\approx 10^{-4}$)
- Planck 2013 on β_{SUN} (using WMAP!)
- Planck 2015 calibrated on $\beta_{ORBITAL}$

Planck Calibration?

- Splitting $\beta_{TOT} = \beta_S + \beta_O$ (A.N. & M.Quartin '2015) :

$$\delta I_\nu = \frac{\delta T}{T_0} + \beta_S \cdot \hat{n} + \beta_O \cdot \hat{n} + \\ + Q(\nu)\left[(\beta_S \cdot \hat{n})^2 + (\beta_O \cdot \hat{n})^2 + 2(\beta_S \cdot \hat{n})(\beta_O \cdot \hat{n})\right]$$

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