Caustics in Horndeski theory

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Scalar theories

- k-essence
- Galileons
- Horndeski theory
- Fab-four
- ...

Gravity is not included

Canonical kinetic term + quadratic mass:

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \right)$$

$$\rightarrow \eta^{\mu\nu}\nabla_\mu\nabla_\nu\phi + m^2\phi = 0$$

Non-linear kinetic term: K-essence

$$S_\phi = \int d^4x\sqrt{-g}K(X) \qquad X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

k-essence, k-inflation, DBI-inflation...

(for example this nonlinearity gives speed of scalar perturbations in inflation, different from speed of gravity)

More non-linear : Horndeski/Galileons

Nicolis et al'09
+ many others

shift-symmetric theories

k-essence

DPG-like term

$$\mathcal{L}_2 = K(X)$$

$$\mathcal{L}_3 = g_3(X)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\rho\sigma}\nabla_\mu\phi\nabla_\alpha\phi(\nabla_\nu\nabla^\beta\phi),$$

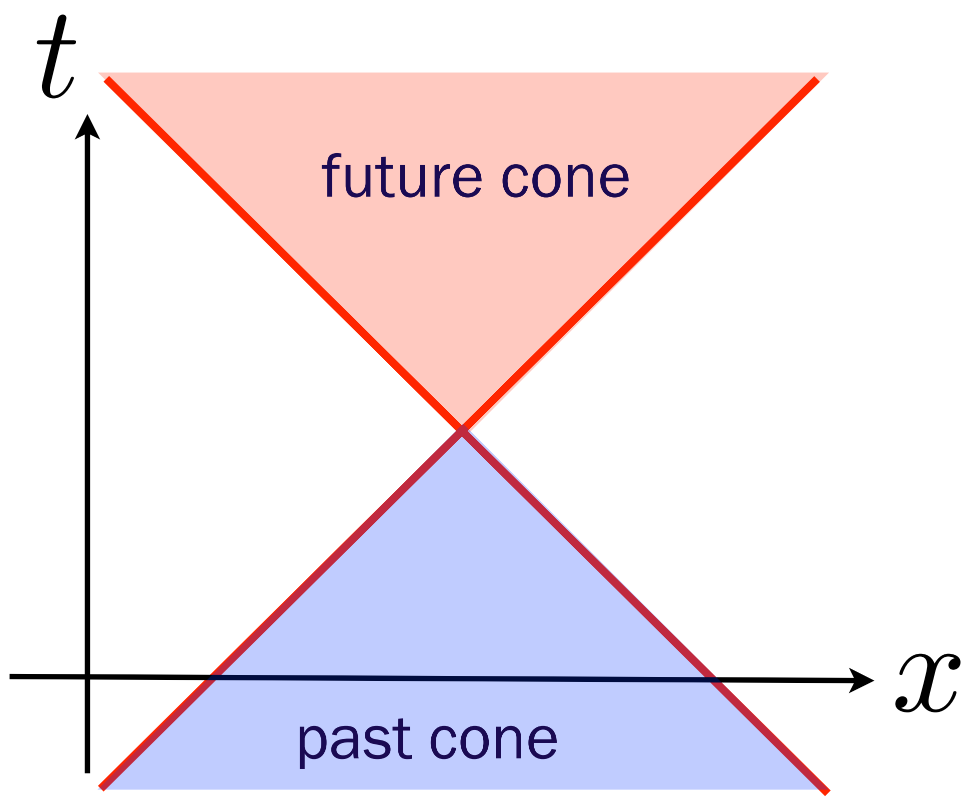
$$\mathcal{L}_4 = g_4(X)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\sigma}\nabla_\mu\phi\nabla_\alpha\phi(\nabla_\nu\nabla^\beta\phi)(\nabla_\rho\nabla^\gamma\phi),$$

$$\mathcal{L}_5 = g_5(X)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}\nabla_\mu\phi\nabla_\alpha\phi(\nabla_\nu\nabla^\beta\phi)(\nabla_\rho\nabla^\gamma\phi)(\nabla_\sigma\nabla^\delta\phi)$$

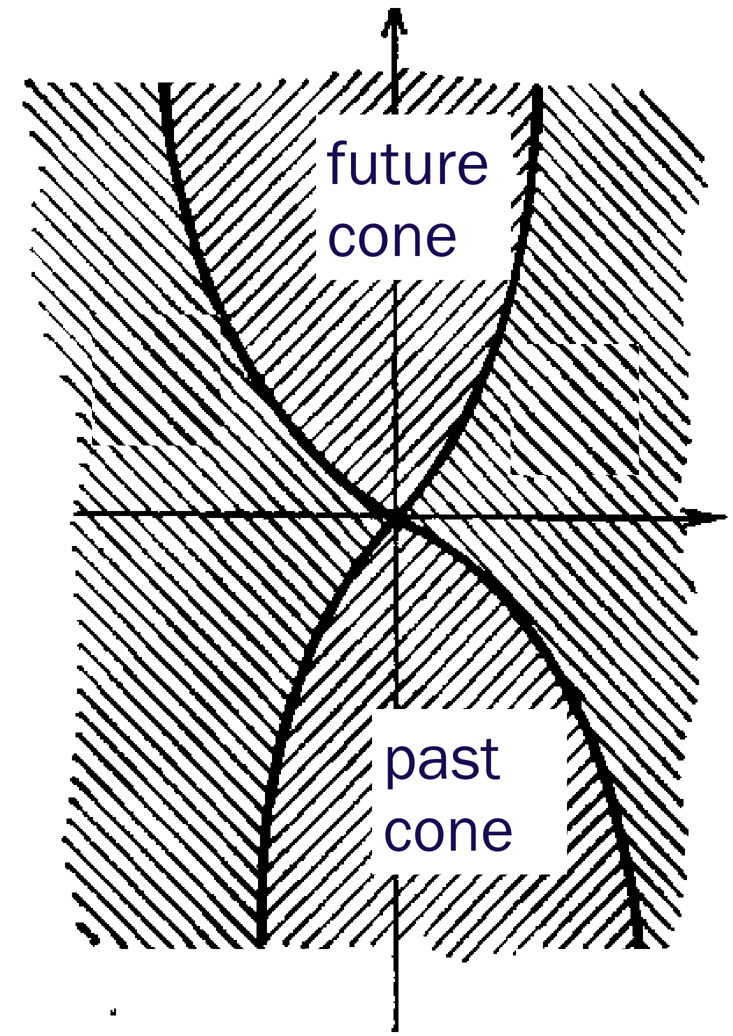
Linear vs nonlinear theory

causal structure

$$-\ddot{\phi} + \phi'' = -m^2\phi$$



k-essence, galileons, massive gravity...



- Perturbations propagate along characteristics
- Signals (wave fronts) propagate along characteristics
- Cones of influence are defined by characteristics

Non-linearity (mild)

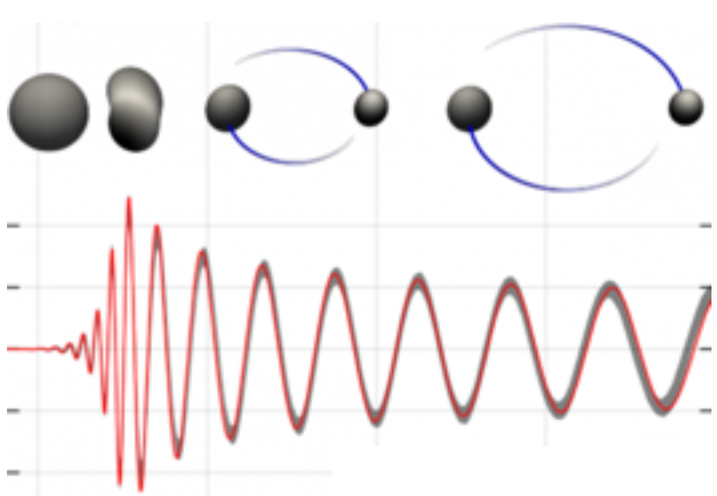
Non-linear potential term

Canonical kinetic term + arbitrary potential:

$$\rightarrow \qquad \nabla_\mu \nabla^\mu \phi + \frac{dV(\phi)}{d\phi} = 0$$

However, because the kinetic term is canonical, the characteristic structure is the same.

massive gravity:



$$\rightarrow$$

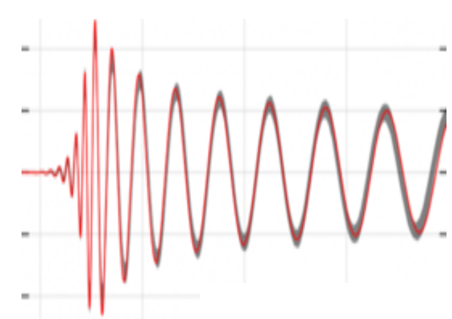


Fig taken from Ligo paper

Applications of modification of gravity

↙ ↘

Cosmology:

Homogeneous time-dependent background + small perturbations

Static (or quasistatic) configurations:

stars, black holes.

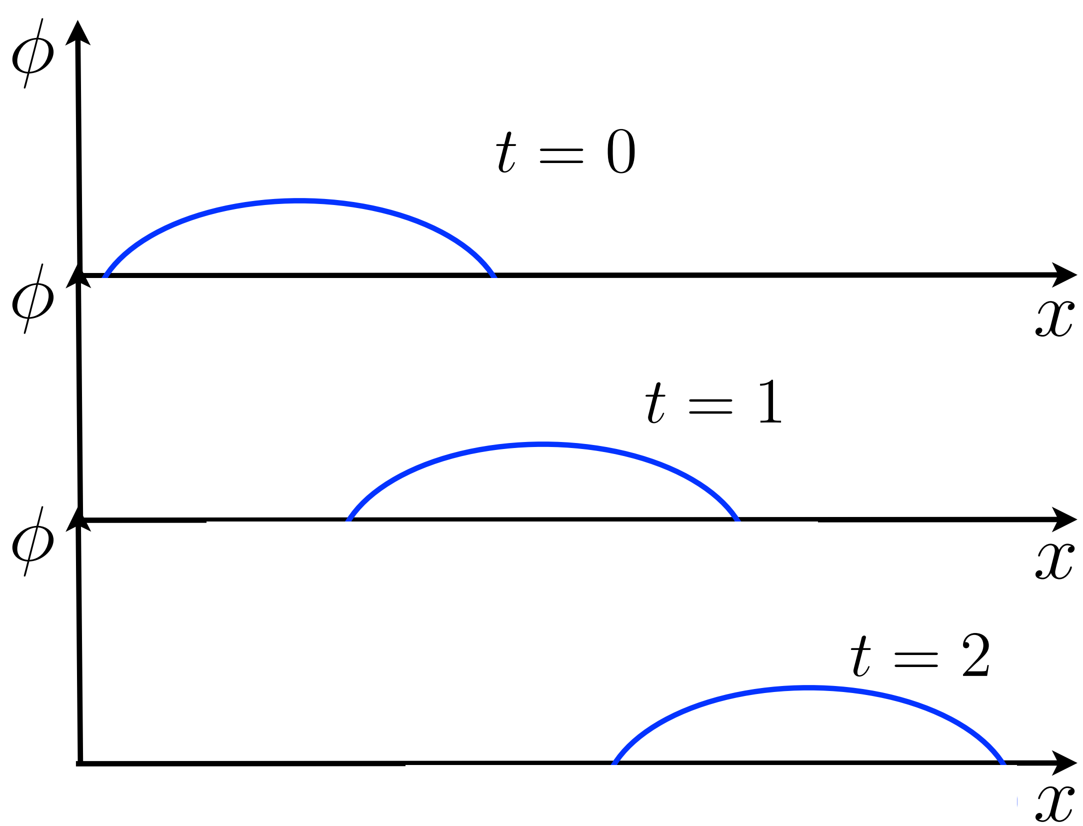
Let us try more dynamical situations.
For example, waves.

Waves: canonical scalar

1+1 case:

$$-\ddot{\phi} + \phi'' = 0$$

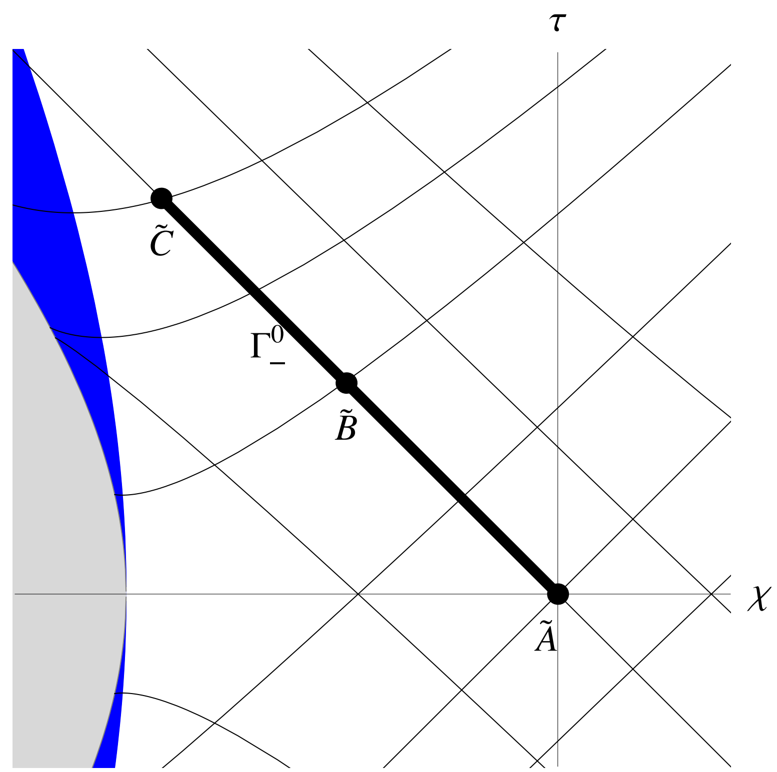
$$\phi = f_1(t-x) + f_2(t+x)$$

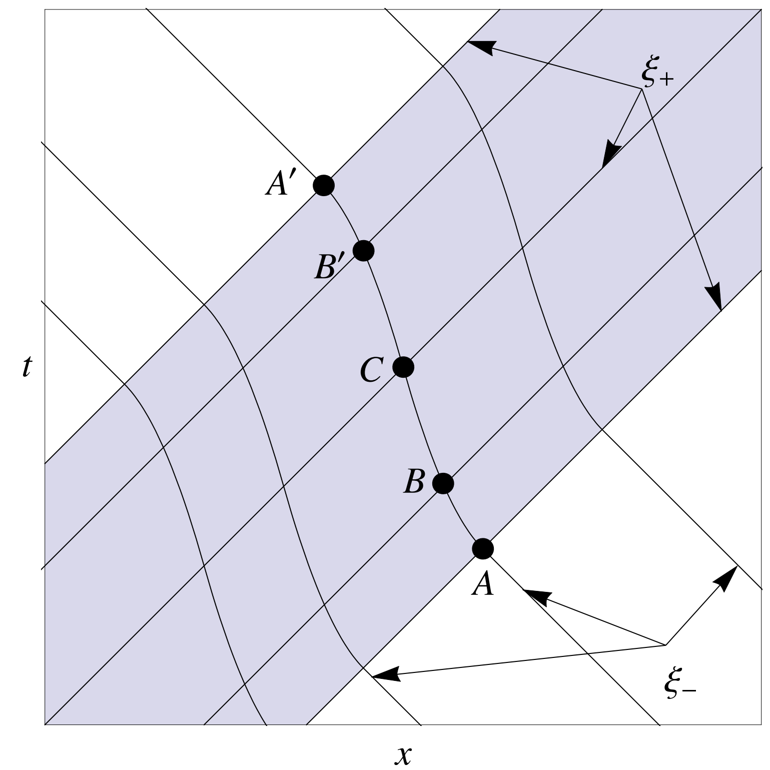


"Traveling" wave is also a solution for k-essence and galileons !

Are there other (more non-trivial) solutions ?

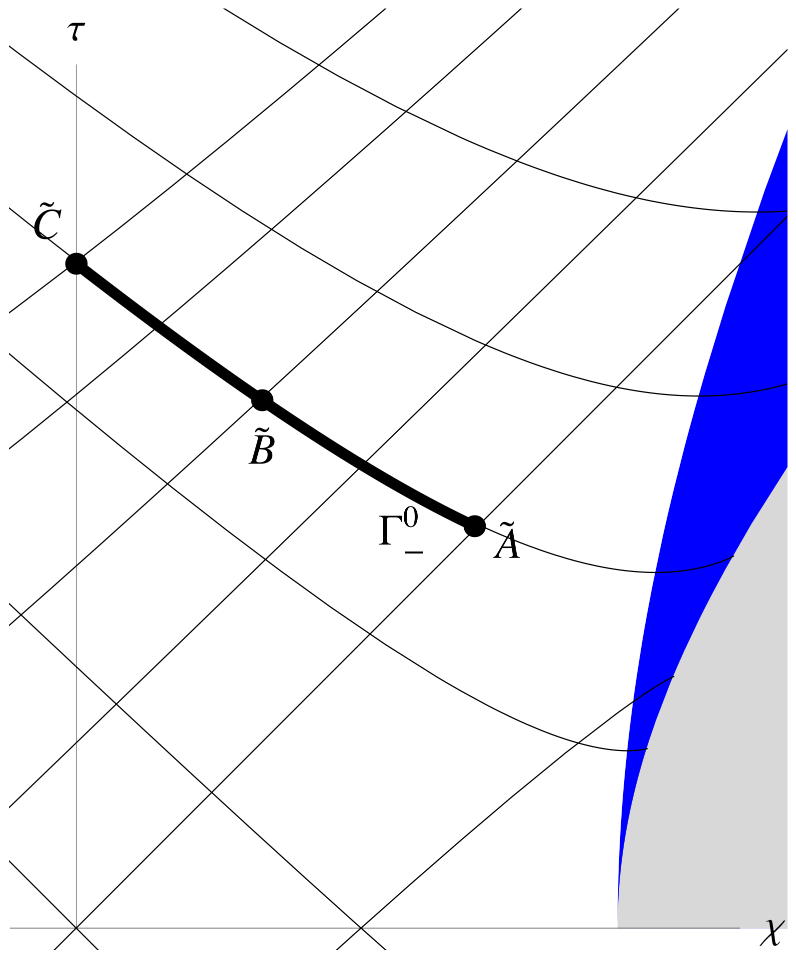
K-essence: traveling wave

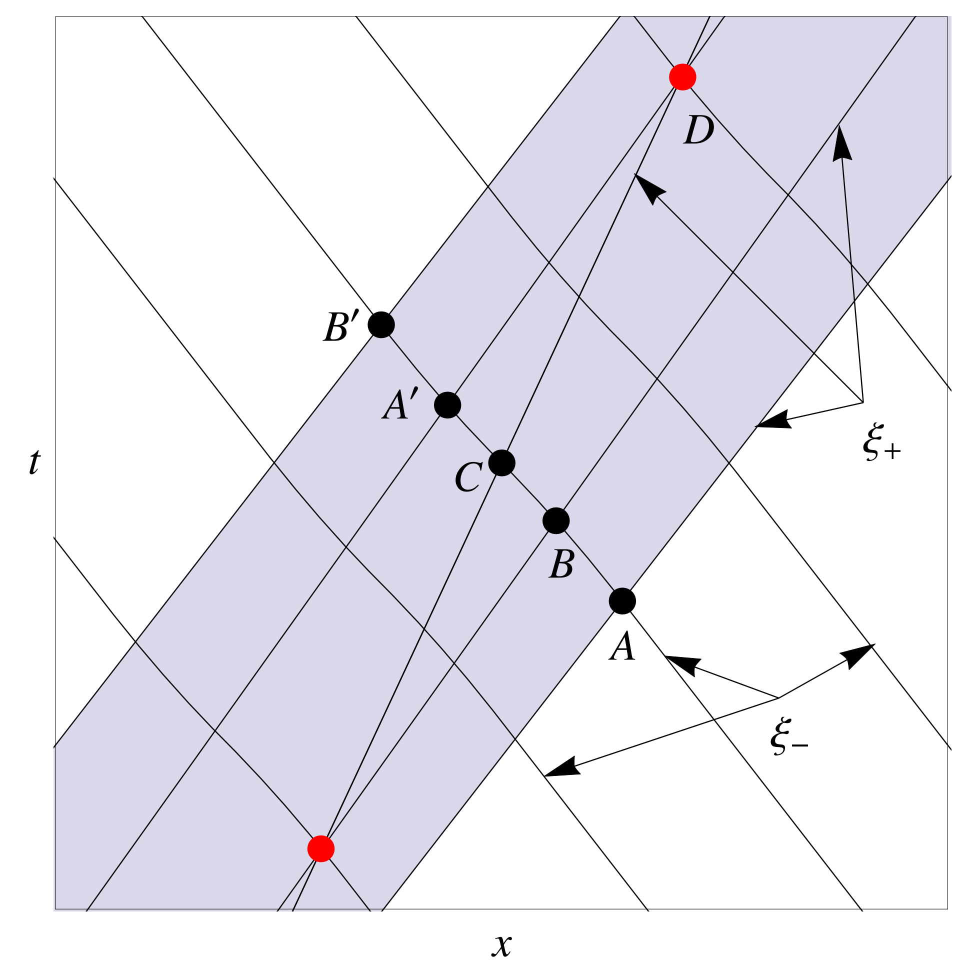




$$\tau = \dot{\phi}, \quad \chi = \phi' \qquad \rightarrow \quad \tau + \chi = 0 \rightarrow \quad \phi = \phi(t - x)$$

K-essence: generic wave





Caustics form

Waves in galileons

$$\mathcal{L}_2 = K(X)$$

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For k-essence wave solutions: $\ddot{\phi}\phi'' - (\dot{\phi}')^2 = 0.$

$\longrightarrow$ EOM for $\mathcal{L}_3$ vanishes

Waves in galileons

Any simple wave solution of
k-essence $\mathcal{L} = \mathcal{K}(X)$ is also a solution of a theory:

$$\mathcal{L} = \mathcal{K}(X) + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5.$$

Conclusions

- Travelling wave solutions in k-essence and in Horndeski theory exist, but they correspond to fine-tuned initial data.
- More general initial conditions lead to a propagating wave, which is not a traveling wave. Evolution of a generic simple wave leads to formation of caustics.
- Any wave solution for the k-essence is also a solution for the most general shift-symmetric galileon.
- More general solutions for galileons?
- Beyond Horndeski ?
- Physical consequences?